

Corporate Leverage and Liquidity Dynamics^{*}

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Comments are welcome.

Abstract

We study leverage and liquidity dynamics when external equity is costly and shareholders cannot commit to future financing policies. Costly equity makes cash valuable, and servicing outstanding debt drains it. Shareholders therefore voluntarily repurchase term debt, even though they can issue equity and are free to borrow again. Retaining cash and retiring debt are alternative ways to meet future financing needs. When earnings are weak, however, shareholders strip the firm of its cash and default immediately. A payout covenant rules this out: the firm keeps operating and repurchases more debt. The covenant also overturns a standard result in models of corporate liquidity, that a dollar inside the firm is worth at least a dollar to shareholders.

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1 Introduction

“Optimal dynamic capital structure,” [DeMarzo \(2019\)](#) argues in his AFA presidential address, is “fundamentally, a commitment problem.” Shareholders choose leverage one decision at a time, each time in their own interest, and creditors price debt knowing this. What shareholders would like to promise about future borrowing, they cannot. Outstanding term debt creates a leverage ratchet ([Admati et al., 2018](#)), a form of debt overhang ([Myers, 1977](#)): when shareholders borrow more, existing creditors bear part of the cost; when they retire debt, existing creditors share the gain. In [DeMarzo and He \(2021\)](#), where debt transactions and equity financing are costless, shareholders never voluntarily repurchase debt, and the dilution creditors anticipate dissipates the tax benefits of debt.

Costly external equity changes this tradeoff. Asymmetric information ([Myers and Majluf, 1984](#)) and flotation costs make internal funds valuable.¹ Retaining cash helps meet future obligations without returning to the equity market—this motive is central to the liquidity-management literature.² Retiring term debt also reduces future financing needs: it uses cash today but lowers subsequent coupon and principal payments. This benefit can counteract the incentive to leave debt outstanding. [DeAngelo, Gonçalves, and Stulz \(2018\)](#) find that firms voluntarily deleverage using a combination of internally generated cash flows, corporate savings, and external equity.

To capture both voluntary deleveraging and debt dilution, we incorporate costly external equity into the setting of [DeMarzo and He \(2021\)](#), allowing the firm to retain cash, and show that shareholders can voluntarily repurchase debt even when equity financing remains available and they are free to borrow again. Debt dilution and costly external equity have largely been studied separately; their interaction jointly shapes cash retention, debt retirement, and payouts. Shareholders weigh the value of preserving liquidity against the benefit of reducing future debt service. In the quantitative analysis of our baseline model (Section 4), a firm repurchases debt when cash is sufficient and distributes cash above its payout target. When cash is exhausted and continua-

¹Evidence includes underwriting fees ([Altinkılıç and Hansen, 2000](#)), equity-offering announcement effects ([Asquith and Mullins, 1986](#)), and structural estimates of financing costs ([Hennessy and Whited, 2007](#)).

²[Froot, Scharfstein, and Stein \(1993\)](#) study costly external financing in a static setting. For discrete-time models with costly external equity, see, e.g., [Cooley and Quadrini \(2001\)](#), [Gomes \(2001\)](#), [Hennessy and Whited \(2005\)](#), [Hennessy and Whited \(2007\)](#), [Gamba and Triantis \(2008\)](#), [Riddick and Whited \(2009\)](#), and [Gryglewicz et al. \(2022\)](#). For the continuous-time corporate liquidity-management literature, see, e.g., [Bolton, Chen, and Wang \(2011\)](#), [Décamps et al. \(2011\)](#), [Hugonnier, Malamud, and Morellec \(2015\)](#), [Décamps et al. \(2016\)](#), [Della Seta, Morellec, and Zucchi \(2020\)](#), and [Abel and Panageas \(2023\)](#).

tion remains worthwhile, it instead issues costly equity. Repurchases and recapitalizations thus emerge as state-dependent responses to financing needs.

Our second set of results concerns payout covenants, which limit shareholders’ ability to extract assets at creditors’ expense (Jensen and Meckling, 1976; Smith and Warner, 1979; Kalay, 1982). In the baseline model of Sections 2–4, when earnings are low, the firm distributes all its cash and then immediately defaults. To prevent this opportunistic asset stripping, we introduce a payout covenant in Section 5. We show that restricting cash extraction naturally induces continued operation and debt repurchases. The covenant expands the repurchase region and increases repurchase intensity. Creditor protection thus operates partly through debt retirement, with cash holdings adjusting to the resulting financing needs.

The asset and debt structure of the baseline model in Section 2 follows the contingent-claims capital-structure literature (Leland, 1994; Goldstein, Ju, and Leland, 2001; DeMarzo and He, 2021). A long-lived asset generates uncertain earnings before interest and taxes (EBIT), which follow a geometric Brownian motion. Earnings shocks are therefore persistent. As in DeMarzo and He (2021), the firm has a single class of term debt that provides interest tax shields and requires coupon payments and gradual principal repayment. Newly issued debt has the same priority as outstanding debt, exposing existing creditors to future borrowing. Unlike in DeMarzo and He (2021), adjusting debt is costly in our model: as in Greenwood, Hanson, and Jin (2026), the firm incurs convex adjustment costs when issuing or repurchasing debt.³

For the equity part of our model, we build on the dynamic liquidity-management literature. Specifically, external equity issuance incurs fixed and proportional costs. These costs create a demand for internal funds, while an associated carrying cost implies that cash earns a return below the risk-free rate, limiting cash accumulation. Shareholders maximize equity value by choosing cash retention, payouts, debt issuance and repurchases, equity injections, and default, without commitment to future policies. Creditors anticipate these choices and price debt accordingly. We characterize a Markov perfect equilibrium with two scaled state variables, the cash-debt ratio

³Altinkılıç and Hansen (2000) report average underwriting fees of 1.09% of proceeds for straight bond offerings and 5.38% for seasoned equity offerings in their 1990–1997 sample. More recently, Bessembinder et al. (2022) report average gross underwriting spreads of approximately 0.65% for investment-grade bonds and 1.42% for high-yield bonds. These figures measure direct underwriting charges. Altinkılıç and Hansen also document rising marginal underwriting costs after controlling for issuer and offering characteristics, motivating convex debt issuance costs.

and the EBIT-debt ratio.

Section 3 characterizes the equilibrium. Debt prices determine what the firm receives when it borrows and what it pays when it repurchases debt, while shareholders' policies determine what creditors are paid. In DeMarzo and He (2021), equity value can be solved first and debt policy recovered afterwards. Convex debt adjustment costs break this separation. Equity value then satisfies a variational inequality, with regions for cash retention, payouts, lumpy equity issuance, and default, and the debt price solves a partial differential equation (PDE) whose form differs across these regions. The two are coupled through the debt policy and must be solved jointly. To the best of our knowledge, we are the first to analyze such a coupled PDE system in finance. Appendix B provides verification results and Appendix D describes the numerical method.

The payout covenant studied in Section 5 applies the commitment logic emphasized by DeMarzo (2019): restrictions on future policies can preserve funding benefits, but may sacrifice flexibility when costly to renegotiate. Our covenant prohibits payouts when the post-payout net-debt-to-EBIT ratio exceeds a threshold, while leaving debt issuance and repurchases available. The covenant also changes how shareholders value cash.

In the liquidity-management literature, the marginal value of cash is the central object: it summarizes how costly external financing shapes corporate financial decisions (Bolton, Chen, and Wang, 2024). A robust property across these models is that the marginal value of cash never falls below one, because a dollar inside the firm can always be paid out. Hugonnier, Malamud, and Morellec (2015) show that this result holds even when the standard barrier payout policy fails: wherever a dollar inside the firm would be worth less than one, shareholders pay it out. By contrast, a payout covenant in our model breaks this property. A dollar that cannot be distributed partly benefits creditors, so it can be worth less than a dollar to shareholders. Furthermore, the payout restriction can make the size of equity issuance smaller than in the baseline analysis in Sections 2–4.

Finally, Section 6 incorporates contemporaneous cash-flow shocks into the model with the payout covenant. In the preceding analysis, operating cash flow is locally deterministic. When earnings are sufficiently high relative to debt, the firm can meet current payments without a cash buffer. With contemporaneous shocks, cash can run out at any earnings level, and a firm without cash must issue equity or default. The firm therefore holds cash even when its earnings

are high, and relies on equity issuance over a wider range of earnings. The effects of the covenant are unchanged: it eliminates cash stripping followed by default, redirects cash toward debt repurchases, and can make the marginal value of cash fall below one.

Related literature. A closely related paper is [Lee and Ying \(2026\)](#), who also study leverage and liquidity without commitment and show that cash-rich firms repurchase debt. Their model uses consol debt, independent cash-flow increments with constant drift and volatility, zero interest on cash, and frictionless debt issuance and repurchases. They allow equity issuance, but their main analysis imposes a sufficiently high personal tax on dividends that shareholders optimally choose liquidation rather than an equity injection when cash is exhausted. Our model instead features amortizing term debt, persistent and stochastically growing EBIT as in [DeMarzo and He \(2021\)](#), interest-earning cash, fixed and proportional equity issuance costs, and convex debt adjustment costs.

These differences lead to three sets of new results. First, earnings are a state variable in our model but not in theirs, so whether the firm repurchases debt depends on its earnings as well as its cash. Repurchases arise at intermediate earnings; at low earnings, shareholders instead strip the firm of its cash and default. Second, shareholders issue costly equity in equilibrium to keep the firm operating, so debt repurchases and equity recapitalizations coexist. Third, we study a payout covenant, which is absent from their analysis. The covenant increases debt repurchases and can push the marginal value of cash below one.

With costless debt adjustment, [Lee and Ying \(2026\)](#) separate equity valuation from debt policy, as in [DeMarzo and He \(2021\)](#). Their perpetual debt remains fixed in the auxiliary no-trade valuation problem, leaving cash as its only evolving state variable and allowing an ordinary differential equation (ODE) characterization. In our model, amortizing debt adds the EBIT-debt ratio as a second state variable. We have to solve two coupled PDEs, one for equity and one for debt, in the cash-debt and EBIT-debt ratios.

Earlier models that integrate liquidity and debt policies include [Gryglewicz \(2011\)](#), [Bolton, Chen, and Wang \(2014\)](#), and [Della Seta, Morellec, and Zucchi \(2020\)](#). In these models, the amount of outstanding debt is either chosen once at the beginning or held fixed over time. The firm then manages its cash and payouts around this debt, with costly or no access to external

financing. More recently, [Hartman-Glaser, Mayer, and Milbradt \(2025\)](#) analyze a firm’s joint debt and liquidity policies under commitment and derive an optimal long-term contract with an intermediary. Unlike in these papers, shareholders in our model adjust debt continuously and cannot commit to future debt or cash policies.

Our paper builds on the contingent-claims capital-structure literature, see, e.g., [Fischer, Heinkel, and Zechner \(1989\)](#), [Leland \(1994, 1998\)](#), [Goldstein, Ju, and Leland \(2001\)](#), [Strebulaev \(2007\)](#), [DeMarzo and He \(2021\)](#), and [Benzoni et al. \(2022\)](#). Extending [DeMarzo and He \(2021\)](#) to allow for downward earnings jumps, [Malenko and Tsoy \(2026\)](#) show that debt repurchases arise through credibility: deviating from an announced debt policy worsens future financing terms. In our model (Section 4), shareholders voluntarily repurchase debt to reduce future debt service and the need for costly external equity.

Debt maturity shapes the commitment problem. [Abel \(2018\)](#) develops a dynamic tradeoff model with instantaneously maturing debt, which [Bolton et al. \(2026\)](#) also use to study leverage and credit risk with costly external equity and downward earnings jumps. In both models, outstanding debt is repaid before new debt is issued provided that the firm does not default, protecting creditors from subsequent dilution, and thus there is no debt ratcheting. By contrast, our term-debt holders remain exposed to future borrowing and payouts. We show how costly equity issuance can sustain voluntary debt repurchases despite the shareholders’ incentive to dilute the value of outstanding debt.

[Hu, Varas, and Ying \(2026\)](#) study debt maturity management with costless equity financing. Their central tradeoff is between short-term debt’s protection from dilution and long-term debt’s option to postpone default after large adverse shocks. Our focus is instead the joint management of liquidity and outstanding term debt when external equity is costly.

2 Baseline Model

In this section, we develop a model in which firms jointly choose cash holdings and term debt. We introduce costly external equity, as in [Bolton, Chen, and Wang \(2011\)](#), into the leverage-dynamics setting of [DeMarzo and He \(2021\)](#). Costly equity creates a precautionary demand for cash, while outstanding term debt exposes creditors to shareholders’ future financing and payout

decisions. Investors demand compensation for bearing systematic risk. We account for risk premia by specifying the firm's cash-flow processes directly under the risk-neutral measure $\mathbb{Q}$. This allows us to value claims by discounting their expected cash flows under $\mathbb{Q}$ at the constant risk-free rate $r > 0$. Unless otherwise stated, all expectations and dynamics are under $\mathbb{Q}$.

The firm has an asset in place that generates EBIT at rate Y_t , governed by a geometric Brownian motion (GBM):

$$dY_t = \mu Y_t dt + \sigma Y_t d\mathcal{B}_t, \quad Y_0 > 0, \quad (1)$$

where $\mu < r$ is the risk-adjusted drift, $\sigma > 0$ is the volatility, and $\{\mathcal{B}_t\}_{t \geq 0}$ is a standard Brownian motion under $\mathbb{Q}$. As in [Leland \(1998\)](#) and [DeMarzo and He \(2021\)](#), all outstanding debt has equal priority (*pari passu*) and amortizes at rate $\xi > 0$, implying an average maturity of $1/\xi$ absent default. For each unit of face value, the firm continuously pays coupons at a constant rate $c > 0$ until it defaults.

Debt issuance and repurchases. As in [DeMarzo and He \(2021\)](#), the firm dynamically adjusts its total outstanding debt balance by continuously issuing or repurchasing debt. Let G_t denote the debt issuance rate at t . When $G_t < 0$, the firm repurchases debt. The total face value of outstanding debt, F_t , evolves according to

$$dF_t = (G_t - \xi F_t) dt. \quad (2)$$

Thus, outstanding debt declines when issuance falls short of maturing principal, even if the firm makes no repurchases.

As in [Greenwood, Hanson, and Jin \(2026\)](#), we assume that it is costly to issue or repurchase debt, and this cost is smooth and convex in G , and homogeneous of degree one in G and F :

$$\Psi(G_t, F_t) = F_t \psi(g_t), \quad g_t = G_t/F_t, \quad (3)$$

where $\psi(\cdot)$ is the scaled cost function. The firm's debt is competitively priced. Let P_t denote the market price for a unit of the firm's outstanding debt at t . Net cash flows from debt issuance or repurchases over dt are $[G_t P_t - \Psi(G_t, F_t)] dt$.

Equity financing. The firm can also raise external funds through costly equity issuance. We assume that each equity issuance entails both a fixed cost Φ_t and a marginal cost γ . The fixed cost is assumed to be proportional to the firm's aggregate face value of outstanding debt, F_t , and is given by $\Phi_t = \phi F_t$, where $\phi > 0$ is a constant. Let H_t and X_t denote the firm's cumulative net proceeds from external equity issuance and cumulative issuance costs up to time t , respectively, with $H_0 = X_0 = 0$. Both processes are nondecreasing. Given the processes $\{H_t\}$ and $\{F_t\}$, cumulative equity issuance costs evolve according to

$$dX_t = \Phi_t \mathbf{1}_{\{dH_t > 0\}} + \gamma dH_t, \quad (4)$$

where $\mathbf{1}_A$ is an indicator function that equals one if event A occurs and zero otherwise. The fixed cost is paid once at each issuance. Net proceeds enter the firm's cash balance, while issuance costs are accounted for separately in shareholders' objective.

Cash accumulation and payouts. Let W_t denote the firm's cash inventory. Cash held inside the firm earns a net return $(r - \lambda)$, where $\lambda > 0$ is a per-unit carrying cost. The firm pays taxes at rate π on its interest income $(r - \lambda)W_t$ and operating earnings Y_t , while coupon payments cF_t are tax-deductible. Starting from $W_0 > 0$, cash evolves as

$$dW_t = \left[(1 - \pi)((r - \lambda)W_t + Y_t - cF_t) - \xi F_t + G_t P_t - \Psi(G_t, F_t) \right] dt + dH_t - dU_t, \quad (5)$$

where U_t denotes cumulative shareholder payouts through dividends or share repurchases. The three terms in brackets on the right-hand side of (5) are after-tax income, principal repayment, and net cash flow from debt issuance or repurchases, respectively. The increments dH_t and dU_t are nonnegative, and cash holdings must remain nonnegative before default.

Let τ_d denote the time at which shareholders terminate the firm and settle its debt. Equation (5) applies until that time. Any permitted payout immediately before liquidation is recorded in U ; remaining cash is allocated according to the priority rule specified below.

Equilibrium. The firm and its creditors interact continuously over time. Shareholders choose debt issuance, equity financing, payouts, and default, while creditors price debt based on their beliefs about these policies. Shareholders cannot commit to their future financial decisions.

Let $\Sigma = (\tau_d, \{G_t, H_t, U_t\}_{t \in (0, \tau_d)})$ denote the firm's strategy and $\tilde{\Sigma} = (\tilde{\tau}_d, \{\tilde{G}_t, \tilde{H}_t, \tilde{U}_t\}_{t \in (0, \tilde{\tau}_d)})$ creditors' belief about that strategy. Upon liquidation, operating assets have no recovery value. Remaining cash is used to repay debt up to its outstanding face value, with any surplus distributed to shareholders.

Given creditors' belief, the unit debt price is

$$P_t(\tilde{\Sigma}) = \mathbb{E}_t \left[\int_t^{\tilde{\tau}_d} e^{-(r+\xi)(s-t)} (c + \xi) ds + e^{-(r+\xi)(\tilde{\tau}_d-t)} \min \left\{ \frac{W_{\tilde{\tau}_d}}{F_{\tilde{\tau}_d}}, 1 \right\} \right], \quad (6)$$

where the default time and future cash and debt balances are evaluated under the conjectured strategy $\tilde{\Sigma}$. The price equals the present value of coupon and principal payments before default plus cash recovery at default. The additional discount rate ξ accounts for amortization of the debt claim.

For a strategy Σ and creditors' belief $\tilde{\Sigma}$, equity value is

$$V_t(\Sigma, \tilde{\Sigma}) = \mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} (dU_s - dH_s - dX_s) + e^{-r(\tau_d-t)} (W_{\tau_d} - F_{\tau_d})^+ \right], \quad (7)$$

where $(x)^+ \equiv \max\{x, 0\}$ denotes the positive part of x . Equity value includes shareholder payouts net of equity financing costs and any residual cash after debt is repaid at liquidation.

We use Markov strategies, which depend on the current state (W_t, Y_t, F_t) . Equity value and the unit debt price are therefore functions of that state:

$$\begin{aligned} V_t(\Sigma, \tilde{\Sigma}) &= V(W_t, Y_t, F_t; \Sigma, \tilde{\Sigma}), \\ P_t(\tilde{\Sigma}) &= P(W_t, Y_t, F_t; \tilde{\Sigma}). \end{aligned}$$

In Appendix A, we define the set of feasible Markov strategies, denoted by $\mathcal{S}$.

Definition 1. A Markov perfect equilibrium (MPE) is a Markov strategy Σ^* such that, for every state (W_t, Y_t, F_t) ,

$$V(W_t, Y_t, F_t; \Sigma^*, \Sigma^*) = \max_{\Sigma \in \mathcal{S}} V(W_t, Y_t, F_t; \Sigma, \Sigma^*). \quad (8)$$

The equilibrium debt price and equity value are $P(W_t, Y_t, F_t; \Sigma^*)$ and $V(W_t, Y_t, F_t; \Sigma^*, \Sigma^*)$, respectively.

In equilibrium, shareholders' optimal choices coincide with creditors' beliefs. We henceforth

suppress the strategy arguments and write equilibrium equity value and the unit debt price as $V(W, Y, F)$ and $P(W, Y, F)$, respectively.

3 Baseline Model Solution

We characterize the equilibrium with unrestricted shareholder payouts. The firm's cash holdings and earnings, relative to outstanding debt, determine whether shareholders retain cash while issuing or repurchasing debt, distribute funds, raise external equity, or default. These choices reflect the value of preserving liquidity against the costs of external financing and the obligations imposed by outstanding debt. Debt issuance and repurchases are chosen within the continuation region. Because both cash and earnings matter, the policy regions are described by boundaries in a two-dimensional state space. We explain the decisions in each region before bringing the valuation and boundary conditions together.

3.1 Cash Retention, Debt Issuance, and Repurchases: $dU_t = dH_t = 0$

Issuing and repurchasing debt while retaining cash. In this region, shareholders neither issue external equity nor make payouts, so $dH_t = dU_t = 0$. They retain funds inside the firm while choosing how much debt to issue or repurchase. Retaining cash helps meet future financing needs without paying equity issuance costs. Borrowing provides liquidity today but adds future debt service. By contrast, repurchasing debt reduces those obligations but uses cash that could be valuable later. The debt policy balances these uses of funds at the endogenous marginal value of cash.

Equity value, $V(W, Y, F)$, satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} rV = \max_G \big[& (1 - \pi)((r - \lambda)W + Y - cF) - \xi F + GP - \Psi(G, F) \big] V_W \\ & + (G - \xi F)V_F + \mu Y V_Y + \frac{1}{2} \sigma^2 Y^2 V_{YY}. \end{aligned} \quad (9)$$

Let G^* denote the optimal policy associated with (9). If $G^* > 0$, the firm issues debt and if $G^* < 0$, the firm repurchases debt. The first-order condition (FOC) for G^* is:

$$[P(W, Y, F) - \Psi_G(G^*, F)] V_W(W, Y, F) = -V_F(W, Y, F). \quad (10)$$

Two objects in (10) play a central role in what follows. The first is V_W , the *marginal equity value of cash*: the amount by which one more dollar of cash inside the firm raises equity value, holding earnings and debt fixed. In the baseline model, $V_W \geq 1$: a dollar can always be paid out, and retaining it can be worth more than one when it helps the firm meet future obligations and avoid costly external financing or premature default. For brevity, we refer to V_W as the marginal value of cash. The second is $-V_F$, the *marginal cost of debt to shareholders*: the reduction in equity value from an additional unit of debt face value, holding cash and earnings fixed. Its after-tax coupons and principal repayments consume cash, valued by shareholders at V_W at the time of payment. Higher debt also worsens the terms of future financing and brings default closer.

To interpret (10), suppose the firm issues one more unit of debt. It raises P in cash and pays the marginal adjustment cost $\Psi_G(G^*, F)$, and each dollar of the net proceeds is worth V_W to shareholders. This is the left side. The right side is the marginal cost of debt to shareholders, $-V_F$. The optimal debt policy equates the two sides.⁴ A repurchase is the mirror image: retiring one unit of debt saves shareholders $-V_F$ but costs them the cash paid, valued at V_W . The second-order condition for G^* is satisfied because $\Psi(G, F)$ is convex in G and $V_W > 0$.

When external equity is costless, the firm has no need to hold cash, and cash is no longer a state variable. Condition (10) then simplifies to $P(Y, F) = -V_F(Y, F) + \Psi_G(G^*, F)$, as in Greenwood, Hanson, and Jin (2026). If debt issuance and repurchases are also costless, we recover the condition in DeMarzo and He (2021): $P(Y, F) = -V_F(Y, F)$. In our model, costly external equity makes the marginal value of cash endogenous, so liquidity needs affect the incentive to issue or repurchase debt.

Joint determination of equity and debt values. Substituting (10) into (9) yields

$$rV = \left[(1 - \pi)((r - \lambda)W + Y - cF) - \xi F + \underbrace{G^* \Psi_G(G^*, F) - \Psi(G^*, F)}_{\text{effects of debt issuance/repurchases}} \right] V_W - \xi F V_F + \mu Y V_Y + \frac{1}{2} \sigma^2 Y^2 V_{YY}. \quad (11)$$

⁴The same marginal issue changes the unit debt price by $[P - \Psi_G(G^*, F)]P_W + P_F$: the proceeds raise cash, worth P_W per dollar to creditors, while the added face value dilutes them, $P_F < 0$. Whereas (10) is an optimality condition that holds with equality in equilibrium, the sign of this price impact is an equilibrium outcome. Without cash it is $P_F < 0$: issuance always dilutes existing creditors, as in DeMarzo and He (2021); with cash it can be positive, so that issuance benefits existing creditors. Absent adjustment costs ($\Psi \equiv 0$), its sign determines whether a smooth debt policy exists; see Appendix F.

With nonlinear adjustment costs, the underbraced term generally depends on the optimal debt policy. Debt prices determine the proceeds from issuance and the cost of repurchases, while the firm's financing and cash policies determine future default risk and hence debt prices. Equity valuation and debt pricing must therefore be determined jointly. This differs from [DeMarzo and He \(2021\)](#), where equity value can be characterized independently of debt issuance policy. Appendix C provides the formal interpretation of the debt adjustment term.

In this region, the price of debt per unit of face value, $P(W, Y, F)$, satisfies the following partial differential equation (PDE):

$$\begin{aligned} rP = & c + \xi(1 - P) + (G^* - \xi F)P_F + \mu Y P_Y + \frac{1}{2}\sigma^2 Y^2 P_{YY} \\ & + [(1 - \pi)((r - \lambda)W + Y - cF) - \xi F + G^* P - \Psi(G^*, F)] P_W. \end{aligned} \quad (12)$$

The term $c + \xi(1 - P)$ captures coupon payments and principal repayment at par, net of the value of the retired claim. The remaining terms reflect changes in debt value as face value, earnings, and cash evolve. The second line captures how liquidity affects debt prices; it disappears when costless equity financing eliminates cash holdings.

Scaling the problem. Using the model's homogeneity, we reduce the three-state-variable problem to two dimensions:

$$V(W, Y, F) = v(w, y)F, \quad P(W, Y, F) = p(w, y), \quad (13)$$

where $w = W/F$ is the cash-debt ratio and $y = Y/F$ is the EBIT-debt ratio.

In the continuation region, the cash ratio evolves according to

$$dw_t = \mu_w(w_t, y_t; g_t) dt, \quad (14)$$

where

$$\mu_w(w, y; g) = (1 - \pi)[(r - \lambda)w + y - c] - \xi + gp(w, y) - \psi(g) - (g - \xi)w. \quad (15)$$

The last term, $-(g - \xi)w$, captures the effect of net debt growth on the scaled cash balance w . The EBIT-debt ratio evolves as

$$dy_t = (\mu + \xi - g_t)y_t dt + \sigma y_t d\mathcal{B}_t. \quad (16)$$

Thus, the growth of earnings relative to debt depends on both earnings growth μ and net debt issuance ($g_t - \xi$).

Let g^* denote the optimal scaled debt issuance rate, with negative values indicating repurchases. Using (13) and (3), we obtain $-V_F = wv_w + yv_y - v$, $V_W = v_w$, $\Psi_G(G^*, F) = \psi'(g^*)$, and $P(W, Y, F) = p(w, y)$. The FOC (10) therefore becomes

$$p(w, y) = \frac{wv_w(w, y) + yv_y(w, y) - v(w, y)}{v_w(w, y)} + \psi'(g^*). \quad (17)$$

Using (13) to simplify the HJB equation (9) and evaluating at the optimal debt issuance rate g^* , we obtain the following PDE for the scaled equity value $v(w, y)$:

$$\mathcal{L}_{g^*} v(w, y) = 0, \quad (18)$$

where, for a given policy g ,

$$\mathcal{L}_g v = \mu_w(w, y; g)v_w + (\mu + \xi - g)yv_y + \frac{1}{2}\sigma^2 y^2 v_{yy} - (r + \xi - g)v. \quad (19)$$

The corresponding debt-pricing equation (12) reduces to

$$\mathcal{L}_{g^*} p(w, y) - g^* p(w, y) + c + \xi = 0. \quad (20)$$

Together with (17), these equations determine equity value, debt prices, and debt policy in this region. We use scaled values and policies in the remaining regions.

3.2 Equity Issuance and Recapitalization: $dH_t > 0$

When the value of replenishing internal funds justifies its cost, shareholders raise external equity. The fixed issuance cost makes it optimal to recapitalize in lumps: the firm raises enough to restore a cash buffer rather than financing each shortfall separately. The issuance decision therefore has two parts—when to raise equity and how much cash to hold afterward. Both depend on the value of continued operation and the firm's future financing needs.

When the firm raises net external equity of $dH_t = m_t F_t$ at time t , where $m_t > 0$ is the scaled net issuance amount, it incurs the fixed and proportional costs specified in (4): $dX_t = (\phi + \gamma m_t) F_t$.

Let $\mathcal{M}v(w, y)$ denote the scaled equity value conditional on an equity issuance:

$$\mathcal{M}v(w, y) = \sup_{m>0} \{v(w + m, y) - \phi - (1 + \gamma)m\}. \quad (21)$$

The first term on the right-hand side of (21) is the post-issuance scaled equity value. The remaining terms are the scaled net proceeds m and the total scaled issuance cost $\phi + \gamma m$.

Because the firm can issue external equity at any time, $v(w, y) \geq \mathcal{M}v(w, y)$. To economize on the fixed cost, it issues equity in lumps. When equity issuance is optimal,

$$v(w, y) = \mathcal{M}v(w, y), \quad (22)$$

and a differentiable interior optimal issuance amount m^* satisfies

$$v_w(w + m^*, y) = 1 + \gamma. \quad (23)$$

The post-issuance cash target is $w + m^*$, with earnings and outstanding debt unchanged at the instant of issuance. At this target, the marginal value of the last dollar raised equals the marginal shareholder outlay, $1 + \gamma$. The fixed cost affects the decision to issue and the amount raised, although it does not enter this marginal condition directly.

In the equity issuance region, creditors anticipate the injection, so the debt price immediately before issuance equals its value immediately afterward:

$$p(w, y) = p(w + m^*, y). \quad (24)$$

3.3 Payouts with Continuation: $dU_t > 0$

When continued operation remains valuable, shareholders distribute excess cash while preserving the firm as a going concern. A payout is optimal when the value of retaining an additional dollar no longer exceeds the dollar shareholders can receive immediately. Let $\bar{w}(y)$ denote the endogenous cash target at payout. At a given earnings ratio y , cash above this target is distributed, bringing the cash-debt ratio back to $\bar{w}(y)$. The target balances the value of liquidity for future financing needs against the cost of holding cash. It varies with earnings rather than representing a fixed cash ratio for all firms.

Equity value before the payout equals the distribution plus the value of the firm afterward.

For $w \geq \bar{w}(y)$,

$$v(w, y) = v(\bar{w}(y), y) + w - \bar{w}(y). \quad (25)$$

Taking the partial derivative with respect to w gives

$$v_w(w, y) = 1, \quad w \geq \bar{w}(y). \quad (26)$$

The marginal value of cash therefore equals one in this region, while equity value exceeds the cash balance, $v(w, y) > w$, because the firm retains continuation value. Section 3.4 discusses the other type of payout: distributing all cash and defaulting, for which $v(w, y) = w$.

Debt prices already reflect the payout that shareholders will make, so $p(w, y) = p(\bar{w}(y), y)$ in the payout region. Consequently,

$$p_w(w, y) = 0, \quad w \geq \bar{w}(y). \quad (27)$$

3.4 Cash Stripping and Default

Shareholders default when continued operation, including the option to raise equity, is no longer worthwhile. With unrestricted payouts, they never leave cash in the firm at default: any cash that would otherwise pass to creditors can instead be distributed to shareholders immediately before liquidation. Thus, if shareholders choose to terminate the firm at a state with positive cash, they first distribute all remaining cash and then default at zero cash. This cash-stripping result follows from their control over payouts and the absence of restrictions on pre-default distributions.

The distinction between the state before the terminal payout and the state at default matters for valuation. Shareholders receive the available cash, while creditors recover nothing because operating assets have no liquidation value. In the region where immediate cash stripping and default are optimal, the values before the terminal payout are

$$v(w, y) = w, \quad p(w, y) = 0. \quad (28)$$

Note that in this region, $v_w = 1$ also holds. However, this is the mathematically trivial case as the solution degenerates. The non-degenerate case where $v_w = 1$ is the region where the firm

makes an equity payout and then continues. Section 5 introduces a payout covenant that limits this cash extraction and preserves creditors' claim on cash that cannot be distributed before liquidation.

At $w = 0$, shareholders choose among feasible continuation, equity issuance, and default. Continuation without new equity requires a debt policy consistent with the nonnegative-cash constraint. If continuation is infeasible and the value of an equity injection does not cover its cost, shareholders default, giving $v(0, y) = p(0, y) = 0$. Appendix D.1 provides details. Because EBIT follows geometric Brownian motion, $Y = 0$ is an absorbing state. With no future operating income, shareholders distribute all remaining cash and default. Equity value before the payout therefore equals the cash balance, $v(w, 0) = w$, and debt has no recovery value, $p(w, 0) = 0$.

3.5 Equilibrium Characterization

The four policy regions—cash retention (Section 3.1), payouts with continuation (Section 3.3), equity issuance (Section 3.2), and cash stripping followed by default (Section 3.4)—are summarized by the following variational inequality:⁵

$$\max \left\{ \underbrace{\max_g \mathcal{L}_g v}_{\text{issue/repurchase debt}}, \underbrace{\mathcal{M}v - v}_{\text{issue equity}}, \underbrace{1 - v_w}_{\text{pay out and continue}}, \underbrace{w - v}_{\text{pay out and default}} \right\} = 0, \quad \text{for } w > 0 \text{ and } y > 0. \quad (29)$$

Appendix D provides the associated boundary conditions for (29). Each term in (29) is the gain from one of shareholders' four options: hold cash while adjusting debt, issue equity, pay out a dollar and continue, or pay out all cash and default. At each state, a zero term identifies an optimal action, while a negative term indicates that the corresponding action is suboptimal. In the cash-retention region, shareholders keep cash inside the firm and choose debt issuance or repurchases according to (17). In the equity-issuance region, they replenish liquidity, with $\mathcal{M}v = v$. In the payout-and-continuation region, they distribute excess cash while preserving

⁵Fixed equity issuance costs make issuance lumpy: the firm jumps to a new cash level, so the issuance term $\mathcal{M}v - v$ compares equity value at two different states. Mathematically, equity issuance in this case is an impulse control. An inequality with such a nonlocal term caused by an impulse control is a quasi-variational inequality; see Bensoussan and Lions (1987) and Øksendal and Sulem (2007) for textbook treatments. If equity issuance carried only a proportional cost, equity issuance would be a singular control rather than an impulse control, the nonlocal term would disappear, and equity value would satisfy a variational inequality. For brevity, we refer to both cases (with and without fixed-equity-issuance-induced non-local terms) as variational inequality.

the firm’s continuation value: $v_w = 1$ and $v > w$. Finally, in the payout-and-default region, they distribute all remaining cash and terminate the firm: $v = w$ and creditors recover nothing. Note that there are two payout regions where $v_w = 1$. However, economic predictions in these two regions are fundamentally different. For this reason, we separate these two payout cases in the variational inequality (29) with the understanding that the $1 - v_w(w, y)$ term represents the “payout with continuation” region.

The debt price $p(w, y)$ satisfies the nonlinear PDE (20) in the cash-retention region, (27) for payouts followed by continuation, and (24) in the equity issuance region. In the cash-stripping and default region, $p = 0$. In the limit as $y \rightarrow \infty$, we derive closed-form solutions for $\lim_{y \rightarrow \infty} p(w, y)$ and $\lim_{y \rightarrow \infty} v(w, y)$. See Appendix C.2 for details.

We characterize equilibrium through a coupled system comprising variational inequality (29) for equity value and PDE (C.3) for debt prices. Unlike in DeMarzo and He (2021) and Lee and Ying (2026), where equity value can be characterized independently of debt issuance policy, debt adjustment costs in our model prevent this separation and introduce highly nonlinear terms through the equilibrium debt policy. Debt prices affect equity value through debt issuance and repurchases, both of which are costly. In turn, shareholders’ policies determine the various regions in which debt is competitively priced. Economically, fixed equity issuance costs make equity issuance lumpy. Technically, this generates a nonlocal term in the variational inequality, which we must take into account when developing a numerical method to jointly solve the coupled PDE system for equilibrium equity and debt values in Appendix D.

4 Quantitative Analysis of Baseline Model

Using the equilibrium characterization in Section 3, we examine how costly external equity shapes the joint cash and debt dynamics. A central result is voluntary debt repurchase: when earnings are intermediate relative to debt and cash is sufficient, the firm uses cash to repurchase debt even though it can still borrow.

Parameter choices. We set the risk-free rate to $r = 0.05$, the risk-neutral EBIT drift to $\mu = 0.02$, EBIT volatility to $\sigma = 0.35$, and the corporate tax rate to $\pi = 0.20$. These choices provide an illustrative benchmark within the contingent-claims capital-structure framework of

Table 1: **Summary of Parameter Values**

Parameter	Symbol	Value
Risk-free interest rate	r	0.05
EBIT drift	μ	0.02
EBIT volatility	σ	0.35
Corporate tax rate	π	0.20
Debt coupon rate	c	0.06
Debt amortization rate	ξ	0.15
Debt adjustment cost	θ	5
Cash-carrying cost	λ	0.02
Fixed equity issuance cost	ϕ	0.03
Proportional equity issuance cost	γ	0.1

Leland (1994), Goldstein, Ju, and Leland (2001), and DeMarzo and He (2021). Rates are annualized where applicable. For our quantitative analysis, we use a quadratic debt adjustment cost specification, $\psi(g) = \theta g^2/2$, which is symmetric around zero issuance.

For the term debt, we set its annual coupon rate to $c = 0.06$, its financing cost parameter to $\theta = 5$, and its amortization rate to $\xi = 0.15$. The implied average maturity is $1/\xi \approx 6.7$ years absent default, so outstanding term debt creates persistent payment obligations.

For external equity, the fixed cost per issuance is 3% of outstanding debt face value, $\phi = 0.03$, and the proportional cost is $\gamma = 0.10$. These costs capture direct underwriting fees and other financing frictions as in Hennessy and Whited (2007), Eckbo, Masulis, and Norli (2007), and Bolton, Chen, and Wang (2011). The fixed equity cost makes recapitalizations lumpy. Costly external equity gives the firm an incentive to retain cash, while a carrying cost limits cash accumulation. We set the annual cash-carrying cost to $\lambda = 0.02$ in line with the literature, see, e.g., Hennessy and Whited (2007). Table 1 summarizes the parameter values for an illustration of our baseline model solution.

Policy regions. Figure 1 demonstrates the solution regions in the two-dimensional space defined by the cash-debt ratio $w = W/F$ (on the horizontal axis) and the EBIT-debt ratio $y = Y/F$ (on the vertical axis). The six regions are: (i) debt issuance (blue) and (ii) debt repurchases (pink), in which the firm retains cash and neither pays out nor issues equity, $dU_t = dH_t = 0$; (iii) payout with continuation (yellow), $dU_t > 0$; (iv) payout of all remaining cash followed by default

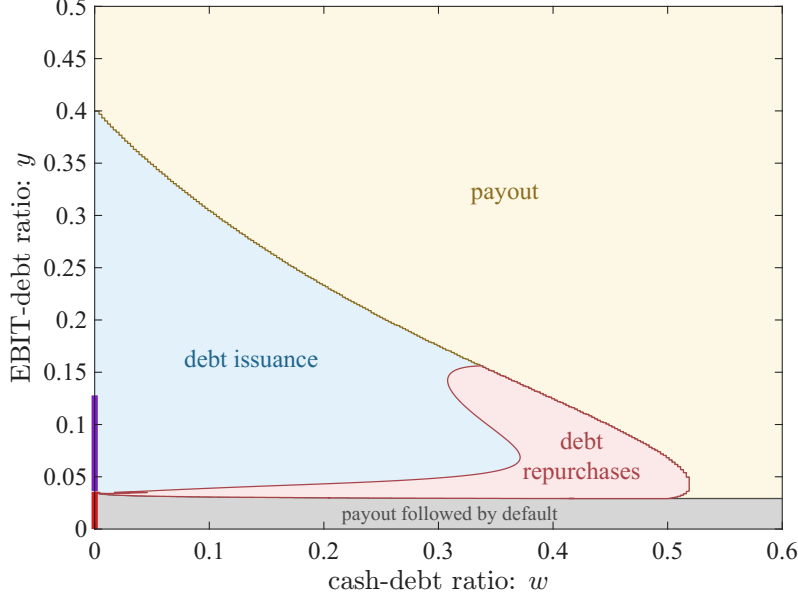


Figure 1: **Equilibrium solution regions.** This figure illustrates the six-region solution for the baseline model in the two-dimensional space defined by the cash-debt ratio $w = W/F$ and the EBIT-debt ratio $y = Y/F$. Parameter values are reported in Table 1.

(gray); and, on the zero-cash axis ($w = 0$), (v) equity issuance (purple segment), $dH_t > 0$, and (vi) default (red segment).

Next, we discuss our model's economic implications along the dimension of the EBIT-debt ratio y . The main focus is the intermediate- y region: debt service, $(c+\xi)F$, is substantial relative to earnings. In this case, the level of cash holdings determines whether shareholders issue debt, repurchase debt, or pay out funds.

High y : payouts and cash retention. When earnings are sufficiently high relative to debt, current operating cash flow covers debt service and the firm optimally holds no cash. In our numerical example, for $y \geq 0.40$, $\bar{w}(y) = 0$: shareholders distribute all available cash, leaving $w = 0$, and the marginal value of cash equals one. This is because high profitability allows the firm to service its debt without relying on internal funds. In Section 6, we show this result no longer holds when profitability is subject to an additional contemporaneous shock and explain the mechanism.

For approximately $0.16 < y < 0.40$, shareholders instead retain a cash buffer. Below the endogenous payout boundary $\bar{w}(y)$, the firm issues debt to capture tax benefits and at the same

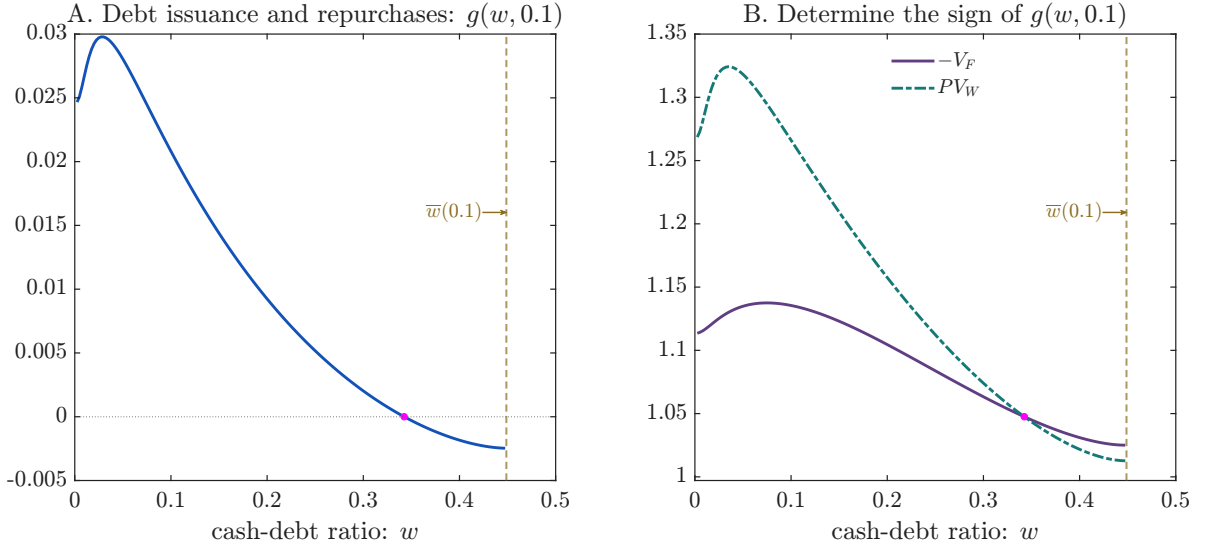


Figure 2: **Debt issuance and repurchases when the EBIT-debt ratio y is intermediate.** In both panels, the EBIT-debt ratio is fixed at $y = 0.10$ and the vertical dashed lines mark the payout boundary: $\bar{w}(0.10) = 0.45$. Panel A plots $g^*(w, 0.10)$: positive values indicate debt issuance and negative values indicate debt repurchases. Panel B plots $-V_F$ (solid purple) and PV_W (dash-dotted teal). By the first-order condition (10) for debt issuance and repurchases, the two curves cross at $w = 0.34$ where $g^* = 0$ and the firm repurchases debt if and only if $-V_F > PV_W$.

time retains all internally generated profits. For w exceeding $\bar{w}(y)$, excess cash $w - \bar{w}(y)$ is paid out. The boundary slopes downward over this range: stronger earnings relative to debt reduce the cash buffer shareholders choose to hold.

Intermediate y : voluntary debt repurchases. At intermediate EBIT-debt ratios, debt service absorbs a substantial share of earnings, making the allocation of cash particularly important. When cash is limited, the firm borrows to capture tax benefits and continues its operations. With ample cash, shareholders instead repurchase debt to reduce the firm's future liabilities. Panel A of Figure 2 illustrates this choice at $y = 0.10$. The firm issues debt if $w < 0.343$ and repurchases debt if $0.343 < w < 0.449$. At the endogenous payout boundary, $\bar{w}(0.10) = 0.449$, shareholders begin to distribute excess cash $w - \bar{w}(0.10)$.

Panel B explains the switch by comparing PV_W , the cash proceeds from issuing one unit of debt valued by shareholders, with $-V_F$, the marginal cost of debt to shareholders. Under our quadratic specification, the FOC (10) becomes $\theta g^* V_W = PV_W + V_F$. Since $V_W > 0$, the firm

issues debt when $PV_W > -V_F$ and repurchases debt when $PV_W < -V_F$. At low cash holdings, the value of liquidity favors issuance. The PV_W and $-V_F$ curves cross at $w = 0.34$, where $g^* = 0$. For cash-debt ratios above $w = 0.34$ but below the payout boundary $\bar{w}(y)$ where $y = 0.1$, shareholders repurchase debt. In [DeMarzo and He \(2021\)](#) where external equity is costless, cash is not a state variable and the firm never repurchases its debt (mathematically, $PV_W < -V_F$ never holds as $V_W = 1$ and $P = -V_F$).

Retaining an additional dollar increases the liquidity buffer; using it to repurchase debt instead reduces outstanding obligations. Both choices help meet future financing needs, but through different channels. Their relative value depends on earnings and cash holdings, as well as the equilibrium price of debt.

This mechanism counteracts the leverage ratchet. Shareholders bear the cost of retiring debt, while some of the benefit accrues to remaining creditors. Costly external equity nevertheless gives shareholders a reason to reduce debt obligations when their cash buffer is ample. They therefore choose active repurchases, $g^* < 0$, even though they cannot commit to refrain from borrowing again. These repurchases are distinct from passive debt reduction through amortization: face value can decline whenever $g^* < \xi$, including when new issuance remains positive.

Equity recapitalization is the optimal response when cash is exhausted and keeping the firm as a going concern is optimal for shareholders. On the purple segment of the zero-cash axis in [Figure 1](#), shareholders inject liquidity and restore a cash buffer. Fixed equity issuance costs make these liquidity injections lumpy.

Low y : cash extraction and default. When y is sufficiently low, retaining cash and continuing the firm is no longer optimal for shareholders. In the gray region of [Figure 1](#), they distribute all remaining cash and then default, so $v(w, y) = w$ and $p(w, y) = 0$. At zero cash, liquidation is immediate: continued operation, including the option to raise equity, offers shareholders no greater value. The option to extract cash therefore limits their willingness to preserve the firm for creditors.

In summary, costly external equity makes shareholders willing to repurchase debt when the firm has an ample cash buffer and continuing its operation is worthwhile. This is because under these circumstances reducing future debt service can be more valuable than increasing

corporate savings by one more unit. This is why voluntary debt repurchases can be optimal absent shareholders' *ex ante* commitment to future financing policies. When earnings become too low, however, shareholders instead extract cash and default.

Section 5 examines how restricting this cash extraction changes shareholders' incentives. By limiting payouts when net debt is high relative to earnings, the covenant can redirect internal funds toward debt retirement.

5 Covenants

Dividend restrictions can mitigate conflicts between shareholders and creditors (Jensen and Meckling, 1976) by limiting shareholders' ability to extract assets after debt has been issued (Smith and Warner, 1979). In our baseline analysis in Sections 2–4, shareholders can distribute the firm's remaining cash to themselves before default, thus leaving none of it for creditors. We introduce a covenant that rules out this opportunistic behavior and show that this payout restriction significantly mitigates shareholders' commitment problem. Below, we first introduce this covenant, analyze the model solution, and then compare its economic implications with those predicted by our baseline model in Section 4.

5.1 Payout Restriction and Equilibrium

The covenant permits a payout to shareholders only if the firm's net debt, $F - W$, does not exceed η times its contemporaneous EBIT, Y , where $\eta > 0$ is a contractual threshold. In scaled variables, the condition $(F - W)/Y \leq \eta$ reads

$$w + \eta y \geq 1. \tag{30}$$

A payout is permitted only if the firm still satisfies (30) immediately after the payout. The line $w + \eta y = 1$ therefore splits the state space in two: on one side payouts are prohibited; on the other they are permitted, subject to the post-payout test (30). Covenants of this form, written on debt relative to EBITDA, are widely used in practice (Lian and Ma, 2021).⁶

⁶Bräuning, Ivashina, and Ozdagli (2025) study incurrence covenants, which restrict the borrower's actions once a threshold is crossed but leave control with shareholders. Our payout covenant shares this feature: failure to satisfy the payout test prevents payouts but does not itself trigger default or transfer control to creditors.

We define the Markov perfect equilibrium with the covenant by restricting the admissible strategies in Definition 1 to payout policies that satisfy (30). Let $\hat{v}$, $\hat{p}$, and $\hat{g}$ denote the scaled equity value, the unit debt price, and the debt policy in this equilibrium. We characterize $\hat{v}$ by taking the two sides of the covenant line in turn.

Payouts prohibited ($w + \eta y < 1$). Shareholders have three options. They can (i) hold cash and continue, issuing or repurchasing debt as (w, y) dictates; (ii) issue equity; or (iii) default. These three options are respectively captured by the three non-positive terms in the following variational inequality for scaled equity value $\hat{v}$:

$$\max \left\{ \underbrace{\max_g \mathcal{L}_g \hat{v}}_{\text{issue/repurchase debt}}, \underbrace{\mathcal{M}\hat{v} - \hat{v}}_{\text{issue equity}}, \underbrace{-\hat{v}}_{\text{default}} \right\} = 0. \quad (31)$$

The term associated with the optimal strategy is zero.

Compared with the variational inequality (29) for the baseline model without covenants, there are two key changes. First, the payout term $1 - \hat{v}_w$ is gone. In the baseline, shareholders can always pay a dollar to themselves, so a dollar inside the firm is worth at least one to them, as in the liquidity-management literature (see Bolton, Chen, and Wang, 2024); this is what the term $1 - \hat{v}_w$ in (29) encodes. Here the covenant blocks that payment, and part of a retained dollar goes to creditors, now or later. As a result, $\hat{v}_w$ can fall below one. Second, the default payoff is zero rather than w . Because $w + \eta y < 1$ implies $w < 1$, cash falls short of the face value of debt. Creditors are senior, and the covenant prevents shareholders from paying the cash out before defaulting, so creditors receive all of it. The last term in (31) is therefore $-\hat{v}$ rather than $w - \hat{v}$.

Payouts permitted ($w + \eta y \geq 1$). Shareholders may now pay out cash, as long as the firm satisfies (30) after the payout. The variational inequality becomes

$$\max \left\{ \underbrace{\max_g \mathcal{L}_g \hat{v}}_{\text{issue/repurchase debt}}, \underbrace{\mathcal{M}\hat{v} - \hat{v}}_{\text{issue equity}}, \underbrace{1 - \hat{v}_w}_{\text{pay out and continue}} \right\} = 0. \quad (32)$$

Violations of maintenance covenants instead shift control to creditors. Following such violations, net debt issuance declines sharply and persistently (Roberts and Sufi, 2009), and shareholder payouts and leverage fall (Nini, Smith, and Sufi, 2012). Bolton, Chen, and Wang (2014) find that a payout covenant increases cash holdings and optimal initial leverage in their quantitative analysis. Hartman-Glaser, Mayer, and Milbradt (2025) derive covenants that transfer control to creditors in distress as part of an optimal contract between the firm and an intermediary.

The first two terms are the same as in (31). There are two differences. First, the payout term $1 - \hat{v}_w$ returns, so $\hat{v}_w \geq 1$ as in Section 3 for the baseline analysis. Second, there is no default term. Default would give shareholders $(w - 1)^+$, and they can always do at least as well by continuing. Intuitively speaking, the option value of keeping the firm alive is strictly positive.⁷

Two more ingredients complete the characterization. The first is the debt policy: the optimal issuance or repurchase rate $\hat{g}$ satisfies the first-order condition (17), with $\hat{v}$ and $\hat{p}$ in place of v and p . The second is debt pricing: as in Section 3, debt is competitively priced given shareholders' policies. Appendix D.2 gives the boundary conditions for (31) and (32) and the equation for $\hat{p}$. As in the baseline, we solve the coupled system for $\hat{v}$ and $\hat{p}$ numerically.

By limiting cash extraction, the covenant has a direct effect and an indirect effect. The direct effect is on default. Shareholders can no longer pay out the firm's cash before defaulting, so default leaves them with nothing. They therefore continue operating in states where they would strip cash and default in the baseline. The indirect effect works through the marginal value of cash. A dollar that cannot be paid out can be worth less than one to shareholders, which makes repurchasing debt with cash cheaper and replenishing cash through costly equity issuance less attractive. The firm thus repurchases more debt and raises less equity. We next quantify these effects by comparing the covenant equilibrium with the baseline.

5.2 Effects of the Payout Covenant

We hold the baseline parameters of Table 1 fixed and set $\eta = 3.5$, the median debt-to-earnings covenant threshold reported by Lian and Ma (2021). A payout must therefore leave $w + 3.5y \geq 1$. At $y = 0.1$, for example, it must leave $w \geq 0.65$: cash of at least 65% of the face value of debt.

Figure 3 shows the equilibrium regions under the covenant in the (w, y) space of Figure 1. There are five: (i) debt issuance (blue) and (ii) debt repurchases (pink), where the firm holds cash and neither pays out nor issues equity, $dU_t = dH_t = 0$; (iii) payout with continuation (yellow), $dU_t > 0$; and, on the zero-cash axis, (iv) equity issuance (purple segment), $dH_t > 0$, and (v) default (red segment). The sixth region of the baseline, payout followed by default, is gone. The dashed line is the covenant line. We now compare the two equilibria (with and without

⁷If $w \geq 1$, they can pay out $w - 1$, which satisfies (30), and keep the firm going with $w = 1$; by limited liability, $\hat{v}(w, y) \geq \hat{v}(1, y) + w - 1 \geq w - 1$. If $w < 1$, default yields zero because creditors are senior, and continuing is worth at least zero. The default term $(w - 1)^+ - \hat{v}$ is thus non-positive and can be omitted.

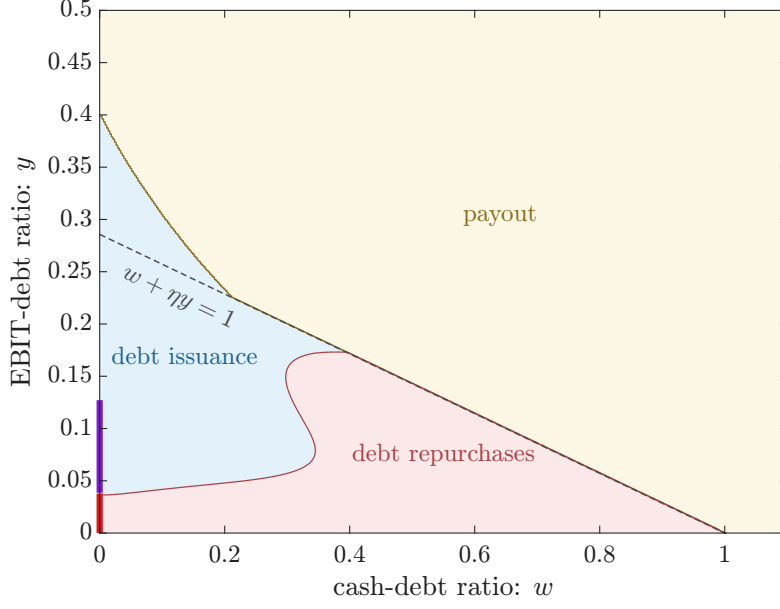


Figure 3: **Equilibrium solution regions under the payout covenant.** This figure illustrates the five-region solution under the payout covenant in the (w, y) space. The dashed line marks the covenant boundary, $w + \eta y = 1$ where $\eta = 3.5$. Parameter values are reported in Table 1.

covenants) along four dimensions: cash extraction, debt repurchases, the marginal value of cash, and equity issuance.

Cash extraction followed by default disappears. Panel A of Figure 4 shows that absent the covenant the firm distributes its cash and defaults (gray) when its EBIT-debt ratio is low, e.g., $y \leq 0.03$. Panel B demonstrates the effect of introducing the covenant: in these low- y states the firm continues operating and repurchases debt. Even though nothing in the covenant requires continuation, shareholders choose to do so because default leaves them with nothing (Section 5.1), so the cash they cannot extract is now used to keep the firm alive and to retire debt.

The covenant enlarges the debt repurchase region and increases the repurchase size. Panels A and B also show that the covenant expands the set of states in which shareholders repurchase debt. Some of these states are the ones just discussed. Absent covenants the firm pays out and defaults there. In the others it pays out and continues. Take $y = 0.10$. The payout boundary moves from $w = 0.45$ to $w = 0.65$. With the covenant the firm repurchases debt for

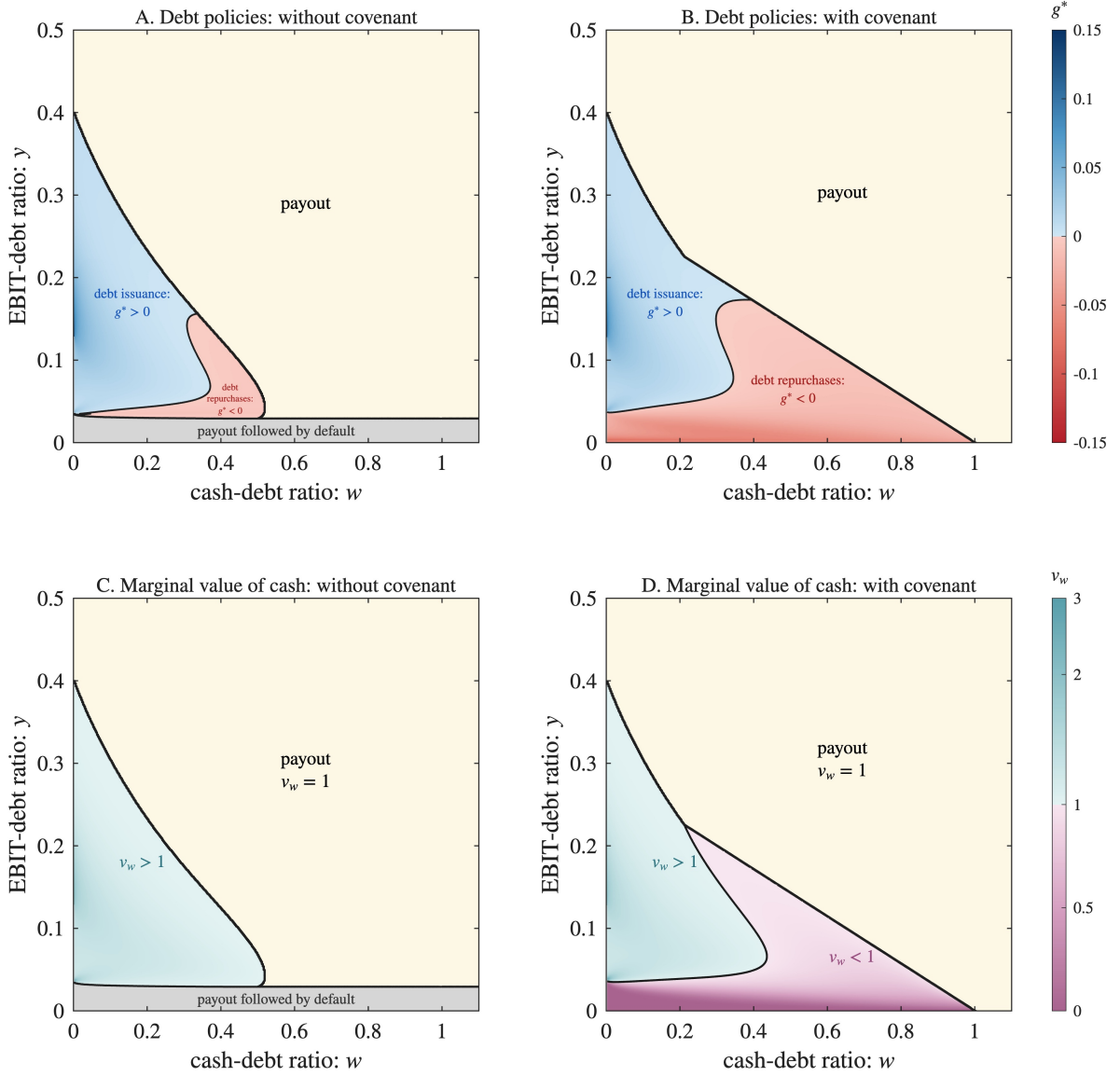


Figure 4: **Effects of payout covenants on debt issuance and repurchases, $g^*(w, y)$, and the marginal equity value of cash, $v_w(w, y)$.** Restricting payouts expands the debt repurchase region and increases repurchase sizes, especially at low EBIT-debt ratios, and also creates a region where the marginal (equity) value of cash v_w falls below one.

$0.45 < w < 0.65$. Absent covenants it pays out its excess cash instead. The darker shading at low y in Panel B shows that repurchases are also larger. The mechanism is the one identified in Section 5.1. Cash that cannot be paid out is cheap to shareholders. Spending it on debt lowers future debt service and net debt and brings the firm back toward the states where payouts are permitted.

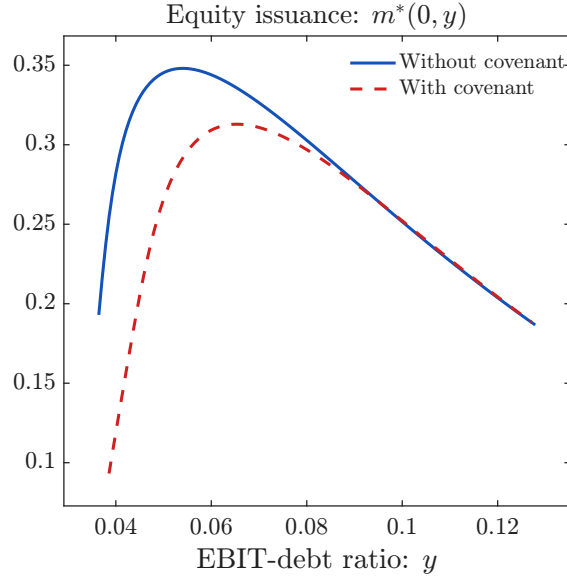


Figure 5: **Equity issuance with and without payout covenants.** The curves plot the cash-debt ratio w immediately after the firm issues equity as its cash reaches zero: $m^*(0, y)$. The blue solid and red dashed lines denote the solution without and with covenants, respectively.

The covenant can reduce the marginal value of cash v_w below one. Panels C and D compare the marginal value of cash. With no covenant (Panel C), $v_w \geq 1$ everywhere, with $v_w = 1$ in the payout region. With the covenant (Panel D), v_w falls below one in a broad region on the prohibited side of the covenant line, and it approaches zero at very low y , where almost every retained dollar ends up with creditors. Shareholders in this region would rather pay out a dollar than keep it; they keep it only because the covenant dictates so. The region where $v_w < 1$ nearly coincides with the repurchase region in Panel B, which links the restriction on payouts to the incentive to repurchase debt.

The covenant reduces the size of equity issuance as v_w can be less than one. Figure 5 plots the cash-debt ratio immediately after an equity issuance at $w = 0$, $m^*(0, y)$. With the covenant, the firm raises less equity at low EBIT-debt ratios: at $y = 0.05$, for example, the post-issuance cash-debt ratio is about 0.29 instead of 0.35. This is consistent with Panels C and D in Figure 4. A dollar raised at a cost of more than one for shareholders is worth less than one to shareholders in states where the firm repurchases debt. Therefore, the firm issues less equity as part of the benefits accrue to creditors. For y above about 0.09, where the payout restriction

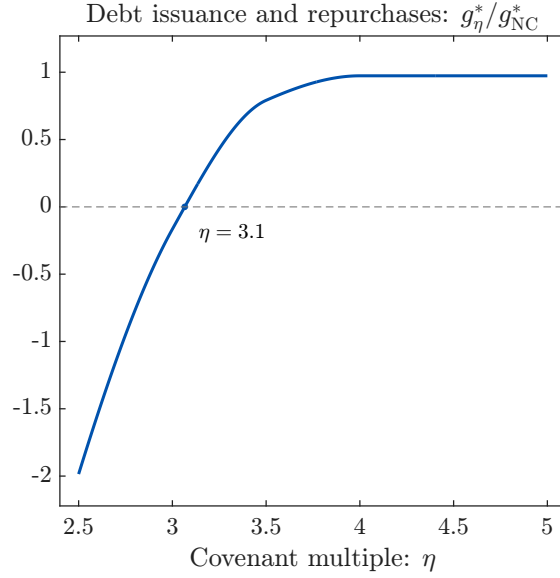


Figure 6: **Covenant tightness and debt issuance/repurchases.** This figure shows that the firm repurchases debt: $g_\eta^*(w, y) < 0$ if the covenant is tight ($\eta < 3.1$), and issues debt, $g_\eta^*(w, y) > 0$ otherwise ($\eta > 3.1$). Under no covenant, the firm issues debt: $g_{NC}^*(w, y) = 0.058\%$ where $(w, y) = (0.28, 0.15)$. All parameter values (other than η) are reported in Table 1.

is unlikely to bind, the firm issues similar amounts of equity with and without the covenant. This is because debt repurchases in the foreseeable future are unlikely.

5.3 Covenant Tightness

The preceding comparison shows that a payout covenant can redirect cash toward debt retirement. Next, we examine how this response varies with covenant tightness. A lower η tightens the payout condition $w + \eta y \geq 1$, requiring more cash or less outstanding debt before payouts are permitted.

Figure 6 plots g_η^*/g_{NC}^* at $(w, y) = (0.28, 0.15)$: the firm's debt issuance rate under the covenant, g_η^* , relative to its rate without the covenant, $g_{NC}^* > 0$. For $\eta \geq 4$, the ratio is flat and just below one: the covenant is loose, and the firm borrows almost as much as without it. For $3.1 < \eta < 4$, the firm still issues debt, but its issuance rate falls quickly as the covenant tightens, reaching zero at $\eta = 3.1$. For $\eta < 3.1$, the firm switches to repurchasing debt, and further tightening increases the repurchase rate. A tighter restriction on payouts thus gives shareholders a stronger incentive to reduce future debt service and restore their ability to pay out.

6 Contemporaneous Cash-Flow Shocks

In this section, we extend the baseline model of Section 2 to include contemporaneous cash-flow risk alongside persistent earnings risk.⁸ In our preceding analyses, after-tax operating cash flow over $(t, t + dt)$ is $(1 - \pi)Y_t dt$. With contemporaneous cash-flow shocks, after-tax operating cash flow becomes

$$Y_t[(1 - \pi)dt + \varepsilon d\mathcal{Z}_t], \quad (33)$$

where Y_t is given in (1), $\mathcal{Z}$ is a new standard Brownian motion and $\varepsilon > 0$ measures (after-tax) cash-flow volatility. Let ρ denote the correlation coefficient between $\mathcal{Z}$ and the Brownian motion $\mathcal{B}$ driving the Y_t process. In this extended model, $Y_t dt$ should be interpreted as expected EBIT. Contemporaneous cash-flow shocks capture unexpected cash receipts or payments, such as an emergency repair payment, that change the firm's cash holdings.

Once we incorporate contemporaneous cash-flow shocks, unlike in our preceding analyses in Sections 2–3, the firm's cash accumulation becomes instantaneously random:

$$dW_t = [(1 - \pi)((r - \lambda)W_t + Y_t - cF_t) - \xi F_t + G_t P_t - \Psi(G_t, F_t)]dt + \varepsilon Y_t d\mathcal{Z}_t + dH_t - dU_t. \quad (34)$$

In the continuation region where the firm chooses to neither pay dividends nor raise external equity, its equity value satisfies

$$\begin{aligned} rV = \max_G & [(1 - \pi)((r - \lambda)W + Y - cF) - \xi F + GP - \Psi(G, F)]V_W \\ & + (G - \xi F)V_F + \mu Y V_Y + \frac{1}{2}\sigma^2 Y^2 V_{YY} + \frac{1}{2}\varepsilon^2 Y^2 V_{WW} + \rho\varepsilon\sigma Y^2 V_{WY}. \end{aligned} \quad (35)$$

Compared with (9), two terms are new. The V_{WW} term captures contemporaneous cash-flow risk, while the V_{WY} term captures its correlation with EBIT risk. The debt-pricing equation also features the corresponding terms involving P_{WW} and P_{WY} . Since the diffusion coefficients do not depend on debt issuance, the first-order condition for $g(w, y)$ is the same as (10). In this section, we focus on the case with payout covenants. To keep notation light, we drop the hats used for the covenant model in Section 5 and write V , P , v , and p for values under the covenant. Appendix E provides the no-covenant analysis for the model with contemporaneous cash-flow shocks.

⁸Décamps et al. (2016) show that permanent and transitory cash-flow shocks have distinct implications for financing and liquidity management. Related analyses include Bolton et al. (2026) and Chen et al. (2026).

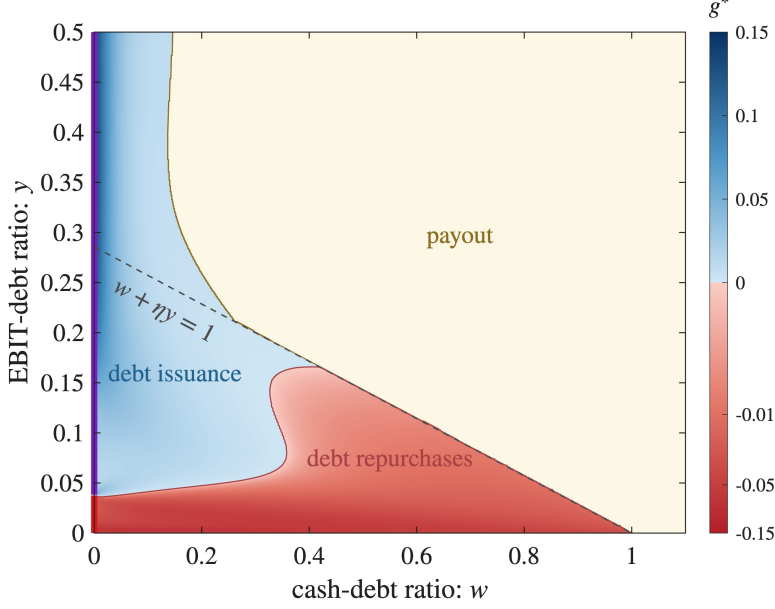


Figure 7: **Equilibrium regions with cash-flow shocks and a payout covenant.** The figure plots policy regions for $\varepsilon = 0.20$, $\rho = 0$, and $\eta = 3.5$. Blue-color intensity indicates the debt issuance ($g^* > 0$) rate and red-color intensity indicates the debt repurchase ($g^* < 0$) rate. The dashed line marks the covenant line: $w + \eta y = 1$. Other parameters are given in Table 1.

With contemporaneous cash-flow shocks ($\varepsilon > 0$), a firm with no cash cannot rely on its earnings, however high. The diffusion term in (34) can produce an operating loss over the next instant, and debt issuance, which is smooth and of order dt , cannot cover it. The firm must therefore issue equity or default.⁹ Because equity issuance is costly, shareholders hold cash to avoid it, unless Y is so low that default is optimal. Without such shocks, cash accumulation is locally deterministic. This is why, when earnings are high relative to debt, operating cash flows cover current debt payments and the firm needs no cash buffer (with or without covenants).

Figure 7 plots the five-region solution in the (w, y) space.¹⁰ Introducing contemporaneous cash-flow shocks eliminates the high- y region (on the vertical axis as $w = 0$) in the $\varepsilon = 0$ setting where the firm pays out and holds no cash. That is, even at high EBIT-debt ratios, the firm voluntarily holds cash to guard against contemporaneous cash-flow shocks. This is because the firm has to issue external equity when it runs out of cash no matter how high its earnings are,

⁹Equity value at zero cash is $v(0, y) = \max\{\mathcal{M}v(0, y), 0\}$. The first term is the value of issuing equity and continuing and the second is shareholders' payoff upon default.

¹⁰Using the homogeneity property, Appendix E provides valuation equations for scaled equity and debt values together with scaled cash dynamics.

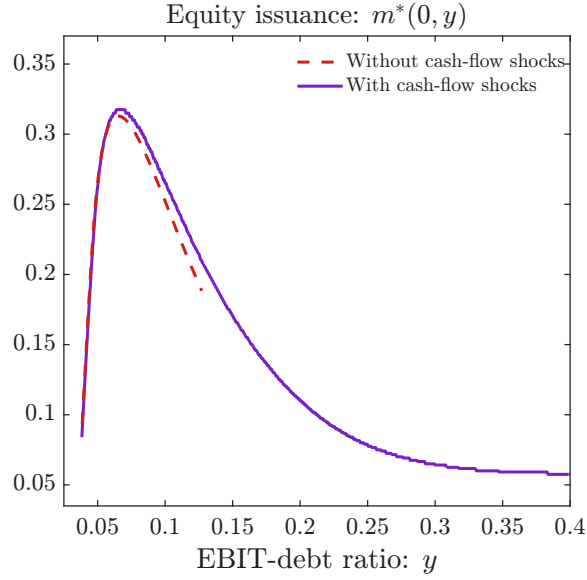


Figure 8: **Equity issuance under the payout covenant: the effect of cash-flow shocks.** The curves plot $m^*(0, y)$ where equity issuance is optimal at zero cash. Both models impose $\eta = 3.5$. The red dashed curve has $\varepsilon = 0$; the purple solid curve has $\varepsilon = 0.20$ and $\rho = 0$. The no-shock curve ends where equity issuance ceases to be optimal. Other parameters are given in Table 1.

if it wants to survive. In addition, the volatility of the scaled cash balance, εy , increases with y , so the firm wants to hold more (scaled) cash. This explains why the firm may hold more cash when its y is higher (see the top left region in Figure 7). Debt repurchases occur for intermediate EBIT-debt ratios when cash holdings are ample, and the firm makes an equity payout when its cash holdings are larger still. As in Section 5.2, the covenant can push the marginal value of cash below one. Near zero cash, $v_w > 1$ because cash helps the firm avoid costly equity issuance. At higher cash holdings where payouts are prohibited, v_w can fall below one, as a dollar that cannot be distributed partly benefits creditors (figure not reported).

Figure 8 compares equity issuance at zero cash with and without contemporaneous cash-flow shocks, holding the covenant fixed. The main difference is the range of earnings over which the firm issues equity. With this contemporaneous shock, a firm that runs out of cash must issue equity to continue, however high its earnings. Without this shock, a firm with $y \geq 0.128$ continues at zero cash without an equity injection. Where both firms issue, the amounts are similar at low y and larger with shocks at intermediate y : the firm wants a larger cash buffer

against contemporaneous cash-flow shocks.

7 Conclusion

Capital structure is a commitment problem, and costly external equity changes how it plays out. When raising equity is costly, cash inside the firm is valuable, and outstanding debt drains it. Shareholders then have a reason to retire debt that is absent when equity is costless: they voluntarily repurchase term debt, even though they can issue equity and are free to borrow again. Retaining cash and retiring debt are two ways to meet future financing needs, and the firm uses both. However, when earnings are weak enough, shareholders choose to strip the firm of its cash and then default immediately.

A payout covenant rules out this opportunistic behavior. The firm then keeps operating in states where it would otherwise be stripped of cash, and it uses the retained cash to repurchase more debt. The covenant also changes what cash is worth. In models of corporate liquidity, a dollar inside the firm is worth at least a dollar to shareholders, because it can always be paid out. Under the covenant, cash can be worth less, since part of a dollar that cannot be distributed benefits creditors.

Our analysis takes the covenant as given for a firm with debt already in place. A natural next step is to ask which covenant the firm would choose when it first issues debt. Our results suggest the answer is not obvious. A tighter covenant protects creditors and leads to more debt repurchases, but it also lowers what cash is worth to shareholders and weakens their incentive to inject equity. How these forces balance at issuance, we leave for future research.

References

- Abel, Andrew B, 2018, Optimal debt and profitability in the trade-off theory, *Journal of Finance* 73, 95–143.
- , and Stavros Panageas, 2023, Precautionary saving in a financially constrained firm, *Review of Financial Studies* 36, 2878–2921.
- Admati, Anat R, Peter M DeMarzo, Martin F Hellwig, and Paul Pfleiderer, 2018, The leverage ratchet effect, *Journal of Finance* 73, 145–198.
- Altinkılıç, Oya, and Robert S Hansen, 2000, Are there economies of scale in underwriting fees? Evidence of rising external financing costs, *The Review of Financial Studies* 13, 191–218.
- Asquith, P., and D. W. Mullins, 1986, Equity issues and offering dilution, *Journal of Financial Economics* 15, 61–89.
- Bensoussan, Alain, and Jacques-Louis Lions, 1987, *Impulse Control and Quasi-Variational Inequalities*. Modern Applied Mathematics Series (John Wiley & Sons Canada, Limited).
- Benzoni, Luca, Lorenzo Garlappi, Robert S Goldstein, and Chao Ying, 2022, Debt dynamics with fixed issuance costs, *Journal of Financial Economics* 146, 385–402.
- Bessembinder, Hendrik, Stacey Jacobsen, William Maxwell, and Kumar Venkataraman, 2022, Overallocation and secondary market outcomes in corporate bond offerings, *Journal of Financial Economics* 146, 444–474.
- Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of Tobin’s q , corporate investment, financing, and risk management, *Journal of Finance* 66, 1545–1578.
- , 2014, Debt, taxes, and liquidity, Discussion paper National Bureau of Economic Research.
- , 2024, The marginal value of cash: corporate savings, investment, and financing, *Annual Review of Financial Economics* 16, 295–324.

- Bolton, Patrick, Zhaoli Jiang, Neng Wang, and Jinqiang Yang, 2026, Leverage and credit risk dynamics under costly equity issuance, Working paper.
- Bolton, Patrick, Neng Wang, and Jinqiang Yang, 2019, Investment under uncertainty with financial constraints, *Journal of Economic Theory* 184, 104912.
- Bräuning, Falk, Victoria Ivashina, and Ali Ozdagli, 2025, High-yield debt covenants and their real effects, *Review of Financial Studies*, forthcoming.
- Chen, Nan, Xavier Giroud, Ling Qin, and Neng Wang, 2026, What drives corporate savings, Discussion paper National Bureau of Economic Research.
- Cooley, T. F., and V. Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.
- Dai, Min, Yanwei Jia, and Hanqing Jin, 2026, Dynamic mean-variance portfolio selection with transaction costs, *Mathematics of Operations Research*, forthcoming.
- Dai, Min, and Yifei Zhong, 2010, Penalty methods for continuous-time portfolio selection with proportional transaction costs, *The Journal of Computational Finance* 13, 1–31.
- Davis, M. H. A., 1993, *Markov Models and Optimization* (Chapman & Hall: London).
- DeAngelo, H., A. S. Gonçalves, and R. M. Stulz, 2018, Corporate deleveraging and financial flexibility, *The Review of Financial Studies* 31, 3122–3174.
- Décamps, Jean-Paul, Sebastian Gryglewicz, Erwan Morellec, and Stéphane Villeneuve, 2016, Corporate policies with permanent and transitory shocks, *Review of Financial Studies* 30, 162–210.
- Décamps, Jean-Paul, Thomas Mariotti, Jean-Charles Rochet, and Stéphane Villeneuve, 2011, Free cash flow, issuance costs, and stock prices, *Journal of Finance* 66, 1501–1544.
- Della Seta, Marco, Erwan Morellec, and Francesca Zucchi, 2020, Short-term debt and incentives for risk-taking, *Journal of Financial Economics* 137, 179–203.

- DeMarzo, Peter M, 2019, Presidential address: Collateral and commitment, *Journal of Finance* 74, 1587–1619.
- , and Zhiguo He, 2021, Leverage dynamics without commitment, *Journal of Finance* 76, 1195–1250.
- Eckbo, B. E., R. W. Masulis, and O. Norli, 2007, Security offerings, in B. E. Eckbo, ed.: *Handbook of Corporate Finance: Empirical Corporate Finance*, vol. 1 . pp. 233–373 (Elsevier/North-Holland).
- Fischer, Edwin O, Robert Heinkel, and Josef Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44, 19–40.
- Froot, Kenneth A, David S Scharfstein, and Jeremy C Stein, 1993, Risk management: Coordinating corporate investment and financing policies, *the Journal of Finance* 48, 1629–1658.
- Gamba, Andrea, and Alexander Triantis, 2008, The value of financial flexibility, *Journal of Finance* 63, 2263–2296.
- Goldstein, Robert, Nengjiu Ju, and Hayne Leland, 2001, An EBIT-based model of dynamic capital structure, *The Journal of Business* 74, 483–512.
- Gomes, J. F., 2001, Financing investment, *American Economic Review* 91, 1263–1285.
- Greenwood, Robin, Samuel G Hanson, and Lawrence J Jin, 2026, Reflexivity in credit markets, Working paper, forthcoming in the *Journal of Finance*.
- Gryglewicz, Sebastian, 2011, A theory of corporate financial decisions with liquidity and solvency concerns, *Journal of Financial Economics* 99, 365–384.
- , Lorian Mancini, Erwan Morellec, Enrique Schroth, and Philip Valta, 2022, Understanding cash flow risk, *Review of Financial Studies* 35, 3922–3972.
- Hartman-Glaser, Barney, Simon Mayer, and Konstantin Milbradt, 2025, A theory of cash flow-based financing with distress resolution, *Review of Economic Studies* 92, 3995–4025.

- Hennessy, Christopher A, and Toni M Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.
- , 2007, How costly is external financing? Evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.
- Hu, Yunzhi, Felipe Varas, and Chao Ying, 2026, Debt maturity management, *Review of Financial Studies*.
- Hugonnier, Julien, Semyon Malamud, and Erwan Morellec, 2015, Capital supply uncertainty, cash holdings, and investment, *Review of Financial Studies* 28, 391–445.
- Jensen, M. C., and W. H. Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of Financial Economics* 3, 305–360.
- Kalay, Avner, 1982, Stockholder-bondholder conflict and dividend constraints, *Journal of Financial Economics* 10, 211–233.
- Lee, Jangwoo, and Chao Ying, 2026, Leverage dynamics and liquidity management without commitment, *Available at SSRN 6193739*.
- Leland, Hayne E, 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.
- , 1998, Agency costs, risk management, and capital structure, *Journal of Finance* 53, 1213–1243.
- Lian, C., and Y. Ma, 2021, Anatomy of corporate borrowing constraints, *The Quarterly Journal of Economics* 136, 229–291.
- Malenko, Andrey, and Anton Tsoy, 2026, Optimal time-consistent debt policies, *Review of Financial Studies*.
- Myers, S. C., 1977, Determinants of corporate borrowing, *Journal of Financial Economics* 5, 147–175.

- Myers, Stewart C, and Nicolas S Majluf, 1984, Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* 13, 187–221.
- Nini, Greg, David C. Smith, and Amir Sufi, 2012, Creditor control rights, corporate governance, and firm value, *Review of Financial Studies* 25, 1713–1761.
- Øksendal, Bernt, and Agnès Sulem, 2007, *Applied Stochastic Control of Jump Diffusions* (Springer: Berlin) 2nd edn.
- Riddick, Leigh A, and Toni M Whited, 2009, The corporate propensity to save, *Journal of Finance* 64, 1729–1766.
- Roberts, Michael R., and Amir Sufi, 2009, Control rights and capital structure: An empirical investigation, *Journal of Finance* 64, 1657–1695.
- Smith, Clifford W, and Jerold B Warner, 1979, On financial contracting: An analysis of bond covenants, *Journal of Financial Economics* 7, 117–161.
- Strebulaev, Ilya A, 2007, Do tests of capital structure theory mean what they say?, *Journal of Finance* 62, 1747–1787.

Appendices

A Markov Strategies

This appendix defines the feasible Markov strategies used in Definition 1.

Definition 2. Markov strategies. A strategy $\Sigma = (\tau_d, \{G_t, H_t, U_t\}_{t \in (0, \tau_d)})$ consists of the default time, debt issuance policy, equity issuance policy, and payout policy. It is a feasible Markov strategy if the following conditions hold:

- (i) It is adapted to the filtration $\{\mathcal{F}_t\}_{t \geq 0}$ generated by $\{\mathcal{B}_t\}_{t \geq 0}$.
- (ii) The debt process in (2) and cash process in (5) have unique strong solutions.
- (iii) Expected discounted payouts and financing costs are finite:

$$\mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} dU_s \right] < \infty, \quad \mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} (dH_s + dX_s) \right] < \infty.$$

- (iv) There are sets $\mathcal{D}$, $\mathcal{H}$, and $\mathcal{U}$ in $\mathbb{Z} = \{(W, Y, F) : W \geq 0, Y > 0, F \geq 0\}$ such that

$$\tau_d = \inf\{t \geq 0 : (W_t, Y_t, F_t) \in \mathcal{D}\}, \tag{A.1}$$

and, for $t < \tau_d$,

$$H_t = \int_0^t \mathbf{1}_{\{(W_{s-}, Y_s, F_s) \in \mathcal{H}\}} dH_s, \tag{A.2}$$

$$U_t = \int_0^t \mathbf{1}_{\{(W_{s-}, Y_s, F_s) \in \mathcal{U}\}} dU_s. \tag{A.3}$$

The debt issuance rate G_t and the nonnegative jumps $\Delta H_t = H_t - H_{t-}$ and $\Delta U_t = U_t - U_{t-}$ are functions of (W_t, Y_t, F_t) .

Let $\mathcal{S}$ denote the set of these feasible Markov strategies.

B Verification Theorem

Theorem 1. Verification theorem. *Consider the baseline model with unrestricted payouts. Let $\Sigma^* \in \mathcal{S}$ be a Markov strategy and let $V(W, Y, F)$ be its associated equity value under the debt pricing rule induced by Σ^* . Σ^* is an MPE if (i) V_Y is bounded on the domain $\mathbb{Z}^{11}$ and (ii) the following variational inequality holds:*

$$\max\{\max_G \mathcal{L}^G V, 1 - V_W, \mathcal{M}V - V, -V\} = 0. \quad (\text{B.1})$$

In (B.1), $\mathcal{M}V(W, Y, F)$ is defined by

$$\mathcal{M}V(W, Y, F) = \sup_{H > 0} \{V(W + H, Y, F) - \phi F - (1 + \gamma)H\} \quad (\text{B.2})$$

and $\mathcal{L}^G V$ is given by

$$\begin{aligned} \mathcal{L}^G V = & [(1 - \pi)((r - \lambda)W + Y - cF) - \xi F + GP - \Psi(G, F)] V_W \\ & + (G - \xi F)V_F + \mu Y V_Y + \frac{1}{2} \sigma^2 Y^2 V_{YY} - rV, \end{aligned} \quad (\text{B.3})$$

where $P(W, Y, F)$ is the debt price induced by creditors' belief Σ^* .

Proof of Theorem 1. Let $\Sigma = (\tau_d, \{G_t, H_t, U_t\}_{t \in (0, \tau_d)}) \in \mathcal{S}$. Since U_t and H_t are nondecreasing processes, we have the following decomposition:

$$U_t = U_t^c + \sum_{s \in [0, t]} \Delta U_s, \quad H_t = H_t^c + \sum_{s \in [0, t]} \Delta H_s,$$

where U_t^c denotes the continuous component of U_t , $\Delta U_s := U_s - U_{s-}$ denotes the jump of U_s , H_t^c denotes the continuous component of H_t , and $\Delta H_s := H_s - H_{s-}$ denotes the jump of H_s . We normalize $U_{0-} = U_{0-}^c = H_{0-} = H_{0-}^c = 0$.

Define

$$T_n = (t + n) \wedge \inf\{\tau \in [t, \tau_d] : \int_t^\tau |e^{-r(s-t)} \sigma Y_s V_Y|^2 ds > n\} \quad (\text{B.4})$$

¹¹When (13) holds, we have $V_Y(W, Y, F) = v_y(w, y)$ which is asymptotically constant as $y \rightarrow \infty$. This is because $v(w, y)$ is asymptotically linear in y as $y \rightarrow \infty$.

with $\inf \emptyset = \tau_d$. The deterministic cap makes each T_n finite, including when the firm never defaults. Applying the generalized Itô formula to $e^{-rt}V(W_t, Y_t, F_t)$, and integrating from t to T_n yields

$$\begin{aligned}
& e^{-r(T_n-t)}V(W_{T_n}, Y_{T_n}, F_{T_n}) \\
&= V(W_t, Y_t, F_t) + \int_t^{T_n} e^{-r(s-t)} \mathcal{L}^G V(W_s, Y_s, F_s) ds \\
&+ \int_t^{T_n} e^{-r(s-t)} V_W (dH_s^c - dU_s^c) + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_{s-} + \Delta W_s, Y_s, F_s) - V(W_{s-}, Y_s, F_s)) \\
&+ \int_t^{T_n} e^{-r(s-t)} V_Y \sigma Y_s d\mathcal{B}_s,
\end{aligned} \tag{B.5}$$

where $\mathcal{L}^G V$ is given by (B.3).

Equation (B.1) implies $V_W \geq 1$. Combining this inequality with the identity $W_s = W_{s-} + \Delta H_s - \Delta U_s$, we obtain

$$\begin{aligned}
& \int_t^{T_n} e^{-r(s-t)} V_W dU_s^c - \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_s, Y_s, F_s) - V(W_{s-} + \Delta H_s, Y_s, F_s)) \\
& \geq \int_t^{T_n} e^{-r(s-t)} dU_s^c + \sum_{t \leq s \leq T_n} e^{-r(s-t)} \Delta U_s = \int_t^{T_n} e^{-r(s-t)} dU_s.
\end{aligned} \tag{B.6}$$

Using (B.1), we have $V \geq \mathcal{M}V$. For $\Delta H_s > 0$, this implies

$$V(W_{s-}, Y_s, F_s) \geq V(W_{s-} + \Delta H_s, Y_s, F_s) - \Delta H_s - \Delta X_s.$$

For $\Delta H_s = 0$, both the value increment and the issuance cost are zero. Hence,

$$V(W_{s-} + \Delta H_s, Y_s, F_s) - V(W_{s-}, Y_s, F_s) \leq \Delta H_s + \Delta X_s. \tag{B.7}$$

Under the fixed-cost convention, feasibility implies

$$\int_t^{T_n} \mathbf{1}_{\{\phi F_s > 0\}} dH_s^c = 0 \quad \text{almost surely;} \tag{B.8}$$

otherwise, cumulative issuance costs would be infinite, contradicting condition (iii) of Definition 2.

In the case $\phi F_s = 0$, we have $V(W_s, Y_s, F_s) \geq \mathcal{M}V(W_s, Y_s, F_s) \geq V(W_s + \epsilon, Y_s, F_s) - (1 + \gamma)\epsilon$ for

any small $\epsilon > 0$, which implies that $V_W(W_s, Y_s, F_s) \leq 1 + \gamma$. Hence, almost surely,

$$\begin{aligned}
& \int_t^{T_n} e^{-r(s-t)} V_W dH_s^c + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_{s-} + \Delta H_s, Y_s, F_s) - V(W_{s-}, Y_s, F_s)) \\
& \leq \int_t^{T_n} e^{-r(s-t)} (1 + \gamma) dH_s^c + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (\Delta H_s + \Delta X_s) \\
& = \int_t^{T_n} e^{-r(s-t)} (dH_s^c + dX_s^c) + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (\Delta H_s + \Delta X_s) \\
& = \int_t^{T_n} e^{-r(s-t)} (dH_s + dX_s), \tag{B.9}
\end{aligned}$$

where the inequality uses (B.7) and (B.8).

Using (B.6) and (B.9), we obtain

$$\begin{aligned}
& \int_t^{T_n} e^{-r(s-t)} V_W (dH_s^c - dU_s^c) + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_s, Y_s, F_s) - V(W_{s-}, Y_s, F_s)) \\
& = \int_t^{T_n} e^{-r(s-t)} V_W dH_s^c + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_{s-} + \Delta H_s, Y_s, F_s) - V(W_{s-}, Y_s, F_s)) \\
& \quad + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (V(W_s, Y_s, F_s) - V(W_{s-} + \Delta H_s, Y_s, F_s)) - \int_t^{T_n} e^{-r(s-t)} V_W dU_s^c \\
& \leq \int_t^{T_n} e^{-r(s-t)} (dH_s + dX_s) - \int_t^{T_n} e^{-r(s-t)} dU_s. \tag{B.10}
\end{aligned}$$

Equation (B.1) implies $\mathcal{L}^G V \leq 0$. Applying this inequality and (B.10) to (B.5), we obtain

$$\begin{aligned}
V(W_{t-}, Y_t, F_t) & \geq e^{-r(T_n-t)} V(W_{T_n}, Y_{T_n}, F_{T_n}) + \int_t^{T_n} e^{-r(s-t)} (dU_s - dH_s - dX_s) \\
& \quad - \int_t^{T_n} e^{-r(s-t)} V_Y \sigma Y_s d\mathcal{B}_s. \tag{B.11}
\end{aligned}$$

Note that the last term in (B.11), the integral with respect to $\mathcal{B}_s$, is a square-integrable martingale for $s \in [t, T_n]$. Taking conditional expectations in (B.11), we obtain:

$$V(W_{t-}, Y_t, F_t) \geq \mathbb{E}_t[e^{-r(T_n-t)} V(W_{T_n}, Y_{T_n}, F_{T_n})] + \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} (dU_s - dH_s - dX_s) \right]. \tag{B.12}$$

Since Y_t , as given by (1), follows a GBM with drift $\mu < r$, and since Brownian motion $\mathcal{B}_t$ satisfies $\lim_{t \rightarrow \infty} \frac{\mathcal{B}_t}{t} = 0$ almost surely, we have $\int_t^\infty |e^{-r(s-t)} Y_s|^2 ds < \infty$ almost surely. Since V_Y is

bounded, it follows from (B.4) that $\lim_{n \rightarrow \infty} T_n = \tau_d$. Because U_s , H_s , and X_s are nondecreasing in s and $\mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} dU_s \right] < \infty$, $\mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} (dH_s + dX_s) \right] < \infty$, the monotone convergence theorem gives

$$\begin{aligned} & \lim_{n \rightarrow \infty} \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} (dU_s - dH_s - dX_s) \right] \\ &= \lim_{n \rightarrow \infty} \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} dU_s \right] - \lim_{n \rightarrow \infty} \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} dH_s \right] - \lim_{n \rightarrow \infty} \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} dX_s \right] \\ &= \mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} (dU_s - dH_s - dX_s) \right]. \end{aligned} \quad (\text{B.13})$$

Since $V_W \geq 1$ and $V \geq 0$, we have

$$V(W, Y, F) \geq W + V(0, Y, F) \geq W \geq (W - F)^+.$$

On $\{\tau_d < \infty\}$, the accumulated quadratic variation in (B.4) is finite, so $T_n = \tau_d$ for all sufficiently large n , path by path. Consequently, the terminal value bounds the liquidation residual. On $\{\tau_d = \infty\}$, the prescribed liquidation payoff is zero and the discounted terminal value remains nonnegative. Fatou's lemma therefore gives

$$\begin{aligned} \liminf_{n \rightarrow \infty} \mathbb{E}_t [e^{-r(T_n-t)} V(W_{T_n}, Y_{T_n}, F_{T_n})] &\geq \liminf_{n \rightarrow \infty} \mathbb{E}_t [e^{-r(T_n-t)} (W_{T_n} - F_{T_n})^+] \\ &\geq \mathbb{E}_t [e^{-r(\tau_d-t)} (W_{\tau_d} - F_{\tau_d})^+ \mathbf{1}_{\{\tau_d < \infty\}}], \end{aligned}$$

where the terminal payoff is defined to be zero when $\tau_d = \infty$. Combining this bound with (B.12) and (B.13), we obtain

$$V(W_{t-}, Y_t, F_t) \geq \mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} (dU_s - dH_s - dX_s) + e^{-r(\tau_d-t)} (W_{\tau_d} - F_{\tau_d})^+ \mathbf{1}_{\{\tau_d < \infty\}} \right]. \quad (\text{B.14})$$

The right-hand side is the equity payoff under the arbitrary feasible strategy Σ , with creditors' pricing rule held fixed at that induced by Σ^* . Since V is the value associated with Σ^* itself, no feasible deviation has a higher value. Thus, Σ^* is a Markov perfect equilibrium. $\square$

Theorem 2. Verification theorem with payout covenants. *Consider the covenant model, let $\hat{\mathcal{S}} \subseteq \mathcal{S}$ denote the set of feasible Markov strategies whose payouts satisfy the covenant (30), and let $\hat{\Sigma}^* \in \hat{\mathcal{S}}$ be a Markov strategy. Denote the associated equity value and debt price by*

$\hat{V}(W, Y, F)$ and $\hat{P}(W, Y, F)$, respectively. Then $\hat{\Sigma}^*$ is an MPE if (i) $\hat{V}_Y$ is bounded on $\mathbb{Z}$ and (ii) the following variational inequalities hold:

$$\begin{cases} \max\left\{\sup_G \hat{\mathcal{L}}^G \hat{V}, 1 - \hat{V}_W, \mathcal{M}\hat{V} - \hat{V}, (W - F)^+ - \hat{V}\right\} = 0, & W + \eta Y \geq F, \\ \max\left\{\sup_G \hat{\mathcal{L}}^G \hat{V}, \mathcal{M}\hat{V} - \hat{V}, (W - F)^+ - \hat{V}\right\} = 0, & W + \eta Y < F. \end{cases} \quad (\text{B.15})$$

Here $\hat{\mathcal{L}}^G$ is the operator in (B.3) with P replaced by $\hat{P}$, and $\mathcal{M}\hat{V}$ is defined by (B.2) with V replaced by $\hat{V}$.

Proof of Theorem 2. Fix an arbitrary strategy $\Sigma \in \hat{\mathcal{S}}$. Throughout the proof, creditors' pricing rule is held fixed at $\hat{P}$. We use the control decomposition and localization from the proof of Theorem 1, replacing V with $\hat{V}$ throughout.

First, (B.15) implies

$$\hat{\mathcal{L}}^G \hat{V} \leq 0, \quad \mathcal{M}\hat{V} \leq \hat{V}, \quad \hat{V} \geq (W - F)^+ \geq 0. \quad (\text{B.16})$$

For continuous payouts, covenant feasibility gives

$$\int_t^{T_n} \mathbf{1}_{\{W_s + \eta Y_s < F_s\}} dU_s^c = 0 \quad \text{almost surely.}$$

Since $\hat{V}_W(W, Y, F) \geq 1$ when $W + \eta Y \geq F$,

$$\int_t^{T_n} e^{-r(s-t)} \hat{V}_W dU_s^c = \int_t^{T_n} e^{-r(s-t)} \mathbf{1}_{\{W_s + \eta Y_s \geq F_s\}} \hat{V}_W dU_s^c \geq \int_t^{T_n} e^{-r(s-t)} dU_s^c. \quad (\text{B.17})$$

The post-intervention cash balance is $W_s = W_{s-} + \Delta H_s - \Delta U_s \geq 0$. If $\Delta U_s > 0$, covenant feasibility requires $W_s + \eta Y_s \geq F_s$. It follows that $W + \eta Y_s \geq F_s$ for every $W \in [W_s, W_{s-} + \Delta H_s]$. Hence,

$$\hat{V}(W_s, Y_s, F_s) - \hat{V}(W_{s-} + \Delta H_s, Y_s, F_s) = - \int_{W_s}^{W_{s-} + \Delta H_s} \hat{V}_W(z, Y_s, F_s) dz \leq -\Delta U_s. \quad (\text{B.18})$$

For $\Delta U_s = 0$, the same inequality holds with both sides zero. Combining (B.17) and (B.18) yields

$$\sum_{t \leq s \leq T_n} e^{-r(s-t)} (\hat{V}(W_s, Y_s, F_s) - \hat{V}(W_{s-} + \Delta H_s, Y_s, F_s)) - \int_t^{T_n} e^{-r(s-t)} \hat{V}_W dU_s^c$$

$$\leq - \int_t^{T_n} e^{-r(s-t)} dU_s. \quad (\text{B.19})$$

The equity-issuance estimate in (B.9) remains valid with V replaced by $\hat{V}$, by exactly the same argument as in Theorem 1. Combining this estimate with (B.19) and proceeding as in (B.10), we obtain

$$\begin{aligned} & \int_t^{T_n} e^{-r(s-t)} \hat{V}_W(dH_s^c - dU_s^c) + \sum_{t \leq s \leq T_n} e^{-r(s-t)} (\hat{V}(W_s, Y_s, F_s) - \hat{V}(W_{s-}, Y_s, F_s)) \\ & \leq \int_t^{T_n} e^{-r(s-t)} (dH_s + dX_s) - \int_t^{T_n} e^{-r(s-t)} dU_s. \end{aligned} \quad (\text{B.20})$$

Applying the generalized Itô formula to $e^{-rt} \hat{V}(W_t, Y_t, F_t)$, and using (B.16) and (B.20), we obtain

$$\begin{aligned} \hat{V}(W_{t-}, Y_t, F_t) & \geq e^{-r(T_n-t)} \hat{V}(W_{T_n}, Y_{T_n}, F_{T_n}) + \int_t^{T_n} e^{-r(s-t)} (dU_s - dH_s - dX_s) \\ & \quad - \int_t^{T_n} e^{-r(s-t)} \hat{V}_Y \sigma Y_s d\mathcal{B}_s. \end{aligned} \quad (\text{B.21})$$

As in the proof of Theorem 1, taking conditional expectations in (B.21) yields

$$\hat{V}(W_{t-}, Y_t, F_t) \geq \mathbb{E}_t[e^{-r(T_n-t)} \hat{V}(W_{T_n}, Y_{T_n}, F_{T_n})] + \mathbb{E}_t \left[\int_t^{T_n} e^{-r(s-t)} (dU_s - dH_s - dX_s) \right]. \quad (\text{B.22})$$

The localization and monotone-convergence arguments in Theorem 1 apply unchanged: $T_n \uparrow \tau_d$, and (B.13) holds. For the terminal term, use the liquidation obstacle $\hat{V} \geq (W - F)^+$. On $\{\tau_d < \infty\}$, $T_n = \tau_d$ for all sufficiently large n , path by path; on $\{\tau_d = \infty\}$, the discounted candidate value is nonnegative. Fatou's lemma therefore yields

$$\liminf_{n \rightarrow \infty} \mathbb{E}_t[e^{-r(T_n-t)} \hat{V}(W_{T_n}, Y_{T_n}, F_{T_n})] \geq \mathbb{E}_t[e^{-r(\tau_d-t)} (W_{\tau_d} - F_{\tau_d})^+ \mathbf{1}_{\{\tau_d < \infty\}}]. \quad (\text{B.23})$$

Consequently, (B.13), (B.22), and (B.23) yield (B.14) with V replaced by $\hat{V}$. Since $\Sigma \in \hat{\mathcal{S}}$ is arbitrary and $\hat{V}$ is the value associated with $\hat{\Sigma}^*$, no covenant-feasible deviation yields a higher value. Thus $\hat{\Sigma}^*$ is an MPE. $\square$

C Valuation Details

This appendix collects supporting details for Section 3.

C.1 Debt Adjustment and Debt Pricing

The first-order condition (10) implies

$$G^* \Psi_G(G^*, F) - \Psi(G^*, F) = \sup_G \left\{ G \left(P + \frac{V_F}{V_W} \right) - \Psi(G, F) \right\}. \quad (\text{C.1})$$

Thus, the debt adjustment term in (11) is the maximized issuance surplus after valuing the additional debt burden in units of cash.

The derivatives underlying the scaling in Section 3 are

$$V_W = v_w, \quad V_Y = v_y, \quad V_F = v - wv_w - yv_y, \quad V_{YY} = v_{yy}/F.$$

Applying Itô's lemma to W/F and Y/F gives (14)–(16). Under the quadratic specification $\psi(g) = \theta g^2/2$, the FOC (17) implies

$$g^*(w, y) = \frac{1}{\theta} \left[p(w, y) + \frac{v(w, y) - yv_y(w, y)}{v_w(w, y)} - w \right]. \quad (\text{C.2})$$

The debt price is determined region by region. Let SR, PR, ER, and DR denote the regions in which the equilibrium strategy combines cash retention (saving) with debt issuance or repurchases, payouts, equity issuance, and default, respectively. With this convention, the debt price is characterized by

$$\begin{aligned} & [\mathcal{L}_{g^*} p(w, y) - g^*(w, y) p(w, y) + c + \xi] \mathbf{1}_{\text{SR}} - p_w(w, y) \mathbf{1}_{\text{PR}} \\ & - [p(w, y) - p(w + m^*, y)] \mathbf{1}_{\text{ER}} - p(w, y) \mathbf{1}_{\text{DR}} = 0. \end{aligned} \quad (\text{C.3})$$

Here $\mathbf{1}_{\text{SR}}$ is shorthand for $\mathbf{1}_{\{(w, y) \in \text{SR}\}}$, and similarly for the other regions. For equity issuance, m^* maximizes (21).

C.2 The High-EBIT Limit

We next characterize the limiting behavior as the EBIT-debt ratio y tends to infinity. As in [Greenwood, Hanson, and Jin \(2026\)](#), we consider the solution in which debt issuance approaches a finite rate g_∞^* and debt becomes asymptotically default-free. The limiting debt price is

$$\lim_{y \rightarrow \infty} p(w, y) = \frac{c + \xi}{r + \xi} \equiv p_\infty. \quad (\text{C.4})$$

This contrasts with [DeMarzo and He \(2021\)](#), where anticipated future borrowing depresses the debt price even as $y \rightarrow \infty$:

$$\lim_{y \rightarrow \infty} p^{DH}(y) = \frac{c(1 - \pi) + \xi}{r + \xi} = p_\infty - \frac{\pi c}{r + \xi},$$

so that the credit spread remains positive in the limit.

In this limit as $y \rightarrow \infty$, the firm can pay back its debt for every path (almost surely). As carrying cash yields no benefit but is costly, the firm optimally distributes all cash to its shareholders. Therefore, as $y \rightarrow \infty$,

$$v(w, y) \sim w + \underbrace{\frac{1 - \pi}{r - \mu} y}_{\text{unlevered asset value}} - \underbrace{\frac{c + \xi}{r + \xi}}_{\text{bond value}} + \underbrace{\frac{\pi c}{r + \xi}}_{\text{tax shield}} + \underbrace{\frac{g_\infty^* \psi'(g_\infty^*) - \psi(g_\infty^*)}{r + \xi}}_{\text{debt adjustment value}} \equiv v_\infty. \quad (\text{C.5})$$

The final term captures the net contribution of future debt issuance to equity value. Debt adjustment costs weaken the firm's incentive to adjust its debt level, affecting both the prices at which the firm can borrow and its equity value.

Substituting the resulting limiting equity value and debt price into [\(17\)](#) yields

$$\psi'(g_\infty^*) = \frac{\pi c}{r + \xi} + \frac{g_\infty^* \psi'(g_\infty^*) - \psi(g_\infty^*)}{r + \xi}. \quad (\text{C.6})$$

The limiting firm value is consequently

$$v_\infty(w, y) + p_\infty = w + \frac{1 - \pi}{r - \mu} y + \psi'(g_\infty^*). \quad (\text{C.7})$$

For the quadratic specification, the limiting issuance rate and equity value take the form

$$g_{\infty}^* = r + \xi - \sqrt{(r + \xi)^2 - \frac{2\pi c}{\theta}}, \quad (\text{C.8})$$

$$v_{\infty}(w, y) = w + \frac{1 - \pi}{r - \mu}y + A, \quad (\text{C.9})$$

where $A = -p_{\infty} + \theta g_{\infty}^*$. For the quadratic specification, with $\theta > 0$, a real solution to (C.6) requires $(r + \xi)^2 \geq 2\pi c/\theta$.

D Numerical Method

This appendix describes the numerical method underlying the quantitative analysis in Sections 4 and 5. The firm's problem combines continuous control of debt issuance and repurchases, singular control of payouts, impulse control of equity issuance, and the optimal timing of default. These decisions divide the state space into policy regions across which the debt price satisfies different equations. The computational challenge lies in the coupling between the equity-value equation (29) and the debt-pricing equation (C.3). To address it, we design a penalty-based iterative algorithm in the spirit of Dai, Jia, and Jin (2026).

D.1 Numerical Procedure for the Baseline Model

We approximate the coupled system (29)–(C.3) using a penalty method (see, e.g., Dai and Zhong (2010) and Davis (1993, Chapter 5)):

$$0 = \mathcal{L}_{g^*}v + \kappa(1 - v_w)^+ + \kappa(\mathcal{M}v - v)^+ + \kappa(-v)^+, \quad (\text{D.1})$$

$$0 = \mathcal{L}_{g^*}p - g^*p + c + \xi - \kappa \mathbf{1}_{\{v_w \leq 1\}}p_w - \kappa \mathbf{1}_{\{\mathcal{M}v \geq v\}}[p - p(w + m^*, y)] - \kappa \mathbf{1}_{\{v \leq 0\}}p. \quad (\text{D.2})$$

Here κ is a sufficiently large penalty parameter, $g^* = g^*(w, y)$ is given by (17), and $m^* = m^*(w, y)$ attains the maximum in the intervention operator $\mathcal{M}$ defined in (21).

Boundary condition at $w = 0$. Without cash reserves, lump-sum payouts are infeasible. The state-constraint boundary condition therefore reduces to

$$\max\{\mathcal{L}_{g^*}v, \mathcal{M}v - v, -v\} = 0 \quad \text{at } w = 0. \quad (\text{D.3})$$

Given the equilibrium debt policy g^* , continuation is feasible when the cash drift defined in (14) satisfies $\mu_w(0, y; g^*) \geq 0$. Accordingly, we impose (D.1) at these boundary points after removing the payout penalty. If $\mu_w(0, y; g^*) < 0$, the firm cannot remain solvent without external financing and must either issue equity or default: $v(0, y) = \max\{\mathcal{M}v(0, y), 0\}$. A similar boundary characterization appears in Bolton, Wang, and Yang (2019).

The corresponding boundary condition for the debt price at $w = 0$ is

$$0 = \mathcal{L}_{g^*}p - g^*p + c + \xi - \kappa \mathbf{1}_{\{\mathcal{M}v \geq v\}} [p - p(m^*, y)] - \kappa \mathbf{1}_{\{v \leq 0\}} p, \quad \text{if } \mu_w(0, y; g^*) \geq 0, \quad (\text{D.4})$$

$$p = p(m^*, y) \mathbf{1}_{\{\mathcal{M}v > 0\}}, \quad \text{if } \mu_w(0, y; g^*) < 0. \quad (\text{D.5})$$

Equation (D.5) sets the debt price equal to its post-issuance value following an equity issuance (when $v(0, y) = \mathcal{M}v(0, y) > 0$) and to zero upon default.

Iteration Algorithm. For the numerical solution, we truncate the state space to the finite computational domain $[0, w^{\max}] \times [0, y^{\max}]$, where $w^{\max}$ and $y^{\max}$ are chosen to be sufficiently large. We use uniform mesh sizes $\delta_w, \delta_y > 0$, and define the grid points by $w_i = (i - 1)\delta_w$, $i = 1, \dots, I$, and $y_j = (j - 1)\delta_y$, $j = 1, \dots, J$. Thus, $w_1 = y_1 = 0$, $w_I = w^{\max}$, and $y_J = y^{\max}$.

We solve the coupled nonlinear system (v, p) by the following scheme, which consists of an outer iteration over (v, p) , indexed by n , and, within each outer step, an inner iteration over v , indexed by ℓ . The maximum numbers of outer and inner iterations are $iter_{\text{out}}$ and $iter_v$, respectively, with corresponding tolerances ϵ_{out} and $\epsilon_v > 0$.

1. **Initialization.** Choose initial guesses (v^0, p^0) and set $n \leftarrow 0$, $\Delta_{\text{out}} \leftarrow +\infty$.

2. **Outer iteration.** Repeat Steps (a)–(d) until $\Delta_{\text{out}} < \epsilon_{\text{out}}$ or $n = iter_{\text{out}}$.

(a) Compute g^n from (C.2) using (v^n, p^n) .

(b) **Inner iteration.** Set $v^{n,0} \leftarrow v^n$, $\ell \leftarrow 0$, and $\Delta_v \leftarrow +\infty$. Solve for $v^{n,\ell+1}$ from

$$\begin{aligned} 0 = & \mathcal{L}^n v^{n,\ell+1} + \kappa \mathbf{1}_{\{v_w^{n,\ell} \leq 1\}} (1 - v_w^{n,\ell+1}) \\ & + \kappa \mathbf{1}_{\{\mathcal{M}v^{n,\ell} \geq v^{n,\ell}\}} (\mathcal{M}v^{n,\ell} - v^{n,\ell+1}) - \kappa \mathbf{1}_{\{v^{n,\ell} \leq 0\}} v^{n,\ell+1} \end{aligned} \quad (\text{D.6})$$

by the finite difference method, using upwind differencing for the first-order terms and subject to the boundary conditions given in Section 3.5. Here $\mathcal{L}^n$ is the operator $\mathcal{L}_g$ defined in (19) with p and g frozen at p^n and g^n , and

$$\mathcal{M}v^{n,\ell}(w_i, y_j) = \max_{\substack{m=w_{i'}-w_i \\ i < i' \leq I}} v^{n,\ell}(w_i + m, y_j) - \phi - (1 + \gamma)m. \quad (\text{D.7})$$

Set $\Delta_v \leftarrow \frac{\|v^{n,\ell+1} - v^{n,\ell}\|}{\max\{1, \|v^{n,\ell}\|\}}$, $\ell \leftarrow \ell + 1$, and repeat until $\Delta_v < \epsilon_v$ or $\ell = \text{iter}_v$.

Set $v^{n+1} \leftarrow v^{n,\ell}$ when the inner loop ends. Record the optimal proceeds $m_{i,j}^{n+1}$ and intervention value $\mathcal{M}v^{n+1}(w_i, y_j)$ at each grid point.

(c) Solve for p^{n+1} using the linearized version of (D.2):

$$\begin{aligned} 0 = & \mathcal{L}^n p^{n+1} - g^n p^{n+1} + c + \xi - \kappa \mathbf{1}_{\{v_w^{n+1} \leq 1\}} p_w^{n+1} \\ & - \kappa \mathbf{1}_{\{\mathcal{M}v^{n+1} \geq v^{n+1}\}} [p^{n+1} - p^n(w_i + m_{i,j}^{n+1}, y_j)] - \kappa \mathbf{1}_{\{v^{n+1} \leq 0\}} p^{n+1}, \end{aligned} \quad (\text{D.8})$$

subject to the boundary conditions given in Section 3.5.

(d) Set $\Delta_{\text{out}} = \max \left\{ \frac{\|v^{n+1} - v^n\|}{\max\{1, \|v^n\|\}}, \frac{\|p^{n+1} - p^n\|}{\max\{1, \|p^n\|\}} \right\}$ and $n \leftarrow n + 1$.

D.2 Extension to Payout Covenants

We adapt the procedure in Appendix D.1 by restricting the payout penalties to states where the covenant permits distributions:

$$0 = \mathcal{L}_{\hat{g}} \hat{v} + \kappa \mathbf{1}_{\{w + \eta y > 1\}} (1 - \hat{v}_w)^+ + \kappa (\mathcal{M}\hat{v} - \hat{v})^+, \quad (\text{D.9})$$

$$0 = \mathcal{L}_{\hat{g}} \hat{p} - \hat{g}\hat{p} + c + \xi - \kappa \mathbf{1}_{\{w + \eta y > 1\}} \mathbf{1}_{\{\hat{v}_w \leq 1\}} \hat{p}_w - \kappa \mathbf{1}_{\{\mathcal{M}\hat{v} \geq \hat{v}\}} [\hat{p} - \hat{p}(w + \hat{m}, y)]. \quad (\text{D.10})$$

The iteration scheme is otherwise unchanged. Boundary conditions are specified below.

At $w = 0$, we use the boundary conditions in Appendix D.1. At the right boundary, chosen sufficiently far into the payout region, we impose $\hat{v}_w(w^{\max}, y) = 1$ and $\hat{p}_w(w^{\max}, y) = 0$. At

$y = y^{\max}$, we use the asymptotic values in Appendix C.2. At $y = 0$, the coefficients of the y -derivative terms vanish. In the continuation region,

$$\max_g \mu_w(w, 0; g) \hat{v}_w(w, 0) - (r + \xi - g) \hat{v}(w, 0) = 0, \quad (\text{D.11})$$

$$\mu_w(w, 0; \hat{g}) \hat{p}_w(w, 0) - (r + \xi) \hat{p}(w, 0) + c + \xi = 0. \quad (\text{D.12})$$

For $w < 1$, payouts are prohibited and (31) governs the choice between continuation, equity issuance, and default. For $w > 1$, payouts are also permitted but must leave $w \geq 1$.

E Equilibrium with Contemporaneous Cash-Flow Shocks

This appendix presents the scaled formulation of the extension in Section 6. By the homogeneity property (13), the cash dynamics in (34) imply that, absent equity issuance and payouts, the cash-to-debt ratio evolves according to

$$dw_t = \mu_w(w_t, y_t; g_t) dt + \varepsilon y_t dZ_t. \quad (\text{E.1})$$

In the cash-retention region, the scaled equity value $v(w, y)$ and the debt price $p(w, y)$ satisfy

$$(r + \xi - g^*)v = \mu_w v_w + (\mu + \xi - g^*)y v_y + \frac{1}{2} \sigma^2 y^2 v_{yy} + \frac{1}{2} \varepsilon^2 y^2 v_{ww} + \rho \varepsilon \sigma y^2 v_{wy}, \quad (\text{E.2})$$

$$(r + \xi)p = c + \xi + \mu_w p_w + (\mu + \xi - g^*)y p_y + \frac{1}{2} \sigma^2 y^2 p_{yy} + \frac{1}{2} \varepsilon^2 y^2 p_{ww} + \rho \varepsilon \sigma y^2 p_{wy}, \quad (\text{E.3})$$

where $\mu_w = \mu_w(w, y; g^*)$ is given by (15), and g^* satisfies (17).

Figure 9 demonstrates six policy regions in the (w, y) space: (i) debt issuance (blue); (ii) debt repurchases (pink); (iii) payout with continuation (yellow), $dU_t > 0$; (iv) payout of all remaining cash followed by default (gray); and, on the zero-cash axis ($w = 0$), (v) equity issuance (purple segment), $dH_t > 0$, and (vi) default (red segment). As in Figure 1, in debt issuance and repurchase regions (i) and (ii), the firm holds cash and neither pays out nor issues equity: $dU_t = dH_t = 0$.

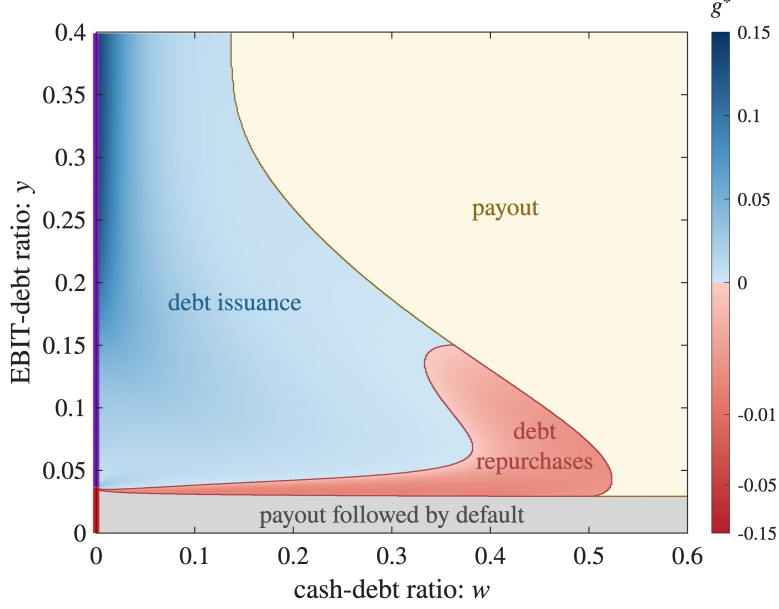


Figure 9: **Equilibrium solution regions with contemporaneous cash-flow shocks.** This figure illustrates the six-region solution with contemporaneous cash-flow risk ($\varepsilon = 0.20$, $\rho = 0$) and no payout covenant in the (w, y) space. Other parameters are given in Table 1.

F Debt Adjustment Costs and Finite Issuance Rates

This benchmark sets $\lambda = r$ and $\xi = 0$, prohibits equity issuance, and leaves payouts unrestricted. Cash earns no interest and debt is perpetual, bringing the financing environment closer to Lee and Ying (2026) while retaining our persistent EBIT process. The comparison isolates the role of convex issuance costs in determining financing policy and shows that the liquidity supplied by new debt can outweigh its dilution of existing creditors.

In the interior continuation region, the equity HJB equation (9) becomes

$$rV = (1 - \pi)(Y - cF)V_W + \mu Y V_Y + \frac{1}{2}\sigma^2 Y^2 V_{YY} + \sup_G \{G(PV_W + V_F) - \Psi(G, F)V_W\}. \quad (\text{F.1})$$

Rearranging the debt-pricing equation (12) gives

$$G^*(PV_W + V_F) - \Psi(G^*, F)V_W = rP - c - (1 - \pi)(Y - cF)V_W - \mu Y V_Y - \frac{1}{2}\sigma^2 Y^2 V_{YY}. \quad (\text{F.2})$$

When $\Psi \equiv 0$, the adjustment objective is linear in G . A finite interior optimum requires $PV_W + V_F = 0$, which determines $P = -V_F/V_W$ but leaves shareholders indifferent over the debt

adjustment rate. The debt-pricing equation (F.3) determines the rate as

$$G^* = \frac{rP - c - (1 - \pi)(Y - cF)P_W - \mu Y P_Y - \frac{1}{2}\sigma^2 Y^2 P_{YY}}{PP_W + P_F}, \quad (\text{F.3})$$

provided that $PP_W + P_F \neq 0$. The denominator measures the marginal price impact of costless debt issuance. In their model, [Lee and Ying \(2026\)](#) impose a parameter restriction ensuring that the corresponding quantity is strictly negative throughout the cash-retention region, so their debt policy is well-defined.

With $\Psi(G, F) = \theta G^2/(2F)$ and $\theta > 0$, the optimality condition instead gives

$$G^* = \frac{F}{\theta} \left(P + \frac{V_F}{V_W} \right). \quad (\text{F.4})$$

Substitution into (F.1) yields

$$rV = (1 - \pi)(Y - cF)V_W + \mu Y V_Y + \frac{1}{2}\sigma^2 Y^2 V_{YY} + \frac{\theta(G^*)^2}{2F}V_W. \quad (\text{F.5})$$

Debt policy now enters equity valuation, so equity value, debt prices, and financing policy must be determined jointly. The marginal effect of debt issuance on the debt price is

$$\mathcal{I}(W, Y, F) := \underbrace{[P - \Psi_G(G^*, F)]P_W}_{\text{liquidity effect}} + \underbrace{P_F}_{\text{dilution effect}}. \quad (\text{F.6})$$

Unlike (F.3), the optimality condition (F.4) does not divide by the marginal price impact. It can therefore yield a finite rate even where $\mathcal{I}$ is zero or positive.

Figure 10 illustrates states with $G^* > 0$ and $\mathcal{I} > 0$. Marginal net issuance proceeds replenish cash and support debt service, reducing default risk enough to outweigh the additional debt burden. Issuance also occurs where $\mathcal{I} < 0$, because shareholders may value the liquidity despite the losses imposed on existing creditors.

G Costless Equity with Costly Debt Adjustment

The rational benchmark in [Greenwood, Hanson, and Jin \(2026\)](#) extends [DeMarzo and He \(2021\)](#) by adding convex debt issuance costs; we refer to it as the modified DeMarzo–He model. Both

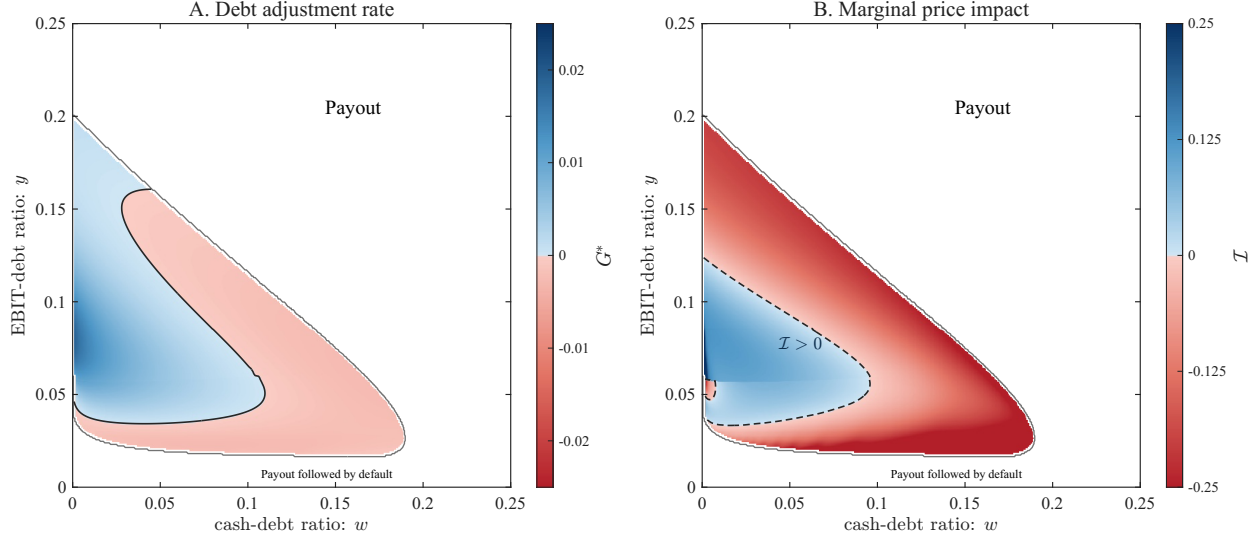


Figure 10: **Debt issuance and its marginal price impact.** Panel A plots G^* and Panel B plots $\mathcal{I}$ from (F.6), with $F = 1$ so that $G^* = g^*$. The black solid curve in Panel A marks $G^* = 0$; the black dashed curve in Panel B marks $\mathcal{I} = 0$. The parameters are $r = \lambda = 0.05$, $\pi = 0.20$, $c = 0.06$, $\mu = 0.02$, $\sigma = 0.35$, $\theta = 20$, and $\xi = 0$, with no equity issuance or payout covenant.

models assume costless equity issuance, so the firm never holds cash. Comparing the two isolates debt issuance costs as a commitment device: costly debt issuance curbs the leverage ratchet effect and restores part of the tax benefits of leverage to enterprise value. Comparing the modified DeMarzo–He model with ours, in turn, isolates the role of costly external equity. Section G.1 reviews the modified DeMarzo–He model and compares it with DeMarzo and He (2021); Section G.2 presents a penalty-based algorithm for solving the model.

G.1 Model and Quantitative Analysis

EBIT follows (1), outstanding debt evolves according to (2), and debt adjustment costs take the form (3) with $\psi(g) = \theta g^2/2$. All investors receive zero cash flows after default. Because the firm holds no cash in this benchmark, equity value $V(Y, F)$ and the unit debt price $P(Y, F)$ depend on (Y, F) only. The market value of equity is

$$V(Y_t, F_t) = \max_{\tau_d, G} \mathbb{E}_t \left[\int_t^{\tau_d} e^{-r(s-t)} [Y_s - \pi(Y_s - cF_s) - (c + \xi)F_s + G_s P_s - \Psi(G_s, F_s)] ds \right],$$

and the unit debt price is

$$P(Y_t, F_t) = \mathbb{E}_t \left[\int_t^{\tau_d^*} e^{-(r+\xi)(s-t)} (c + \xi) ds \right],$$

where τ_d^* is the firm's optimal default time.

By homogeneity, $V(Y, F) = v(y)F$ and $P(Y, F) = p(y)$ with $y \equiv Y/F$. Let y_d denote the endogenous default boundary. For $y > y_d$, the optimality condition (17) reduces to $p(y) = yv'(y) - v(y) + \psi'(g^*)$, so that

$$g^*(y) = \frac{1}{\theta} [p(y) + v(y) - yv'(y)]. \quad (\text{G.1})$$

The scaled equity value and debt price satisfy

$$(r + \xi)v(y) = \frac{1}{2}\sigma^2 y^2 v''(y) + (\mu + \xi)yv'(y) + (1 - \pi)(y - c) - \xi + \frac{\theta}{2}(g^*(y))^2, \quad (\text{G.2})$$

$$(r + \xi)p(y) = \frac{1}{2}\sigma^2 y^2 p''(y) + (\mu + \xi - g^*(y))yp'(y) + c + \xi, \quad (\text{G.3})$$

subject to the value-matching and smooth-pasting conditions at the default boundary,

$$v(y_d) = 0, \quad v'(y_d) = 0, \quad (\text{G.4})$$

the asymptotic condition $v(y) \sim v_\infty(0, y)$ as $y \rightarrow \infty$ with v_∞ given in (C.9), and, for the debt price, zero recovery in default and the risk-free value in the limit,

$$p(y_d) = 0, \quad \lim_{y \rightarrow \infty} p(y) = \frac{c + \xi}{r + \xi} \equiv p_\infty. \quad (\text{G.5})$$

We now show two properties of the equilibrium: the debt adjustment cost raises equity value and delays default, and the firm never repurchases debt.

Proposition 1 (Equity value and default). *Let V^{DH} and y_d^{DH} be the equity value and default boundary in DeMarzo and He (2021), that is, in the model with $\theta = 0$. In any smooth equilibrium of the modified DeMarzo–He model with $\theta > 0$, $V \geq V^{DH}$ and $y_d \leq y_d^{DH}$.*

Proof. Consider the no-trade problem: shareholders set $G \equiv 0$, so that $F_t = F_0 e^{-\xi t}$, and choose

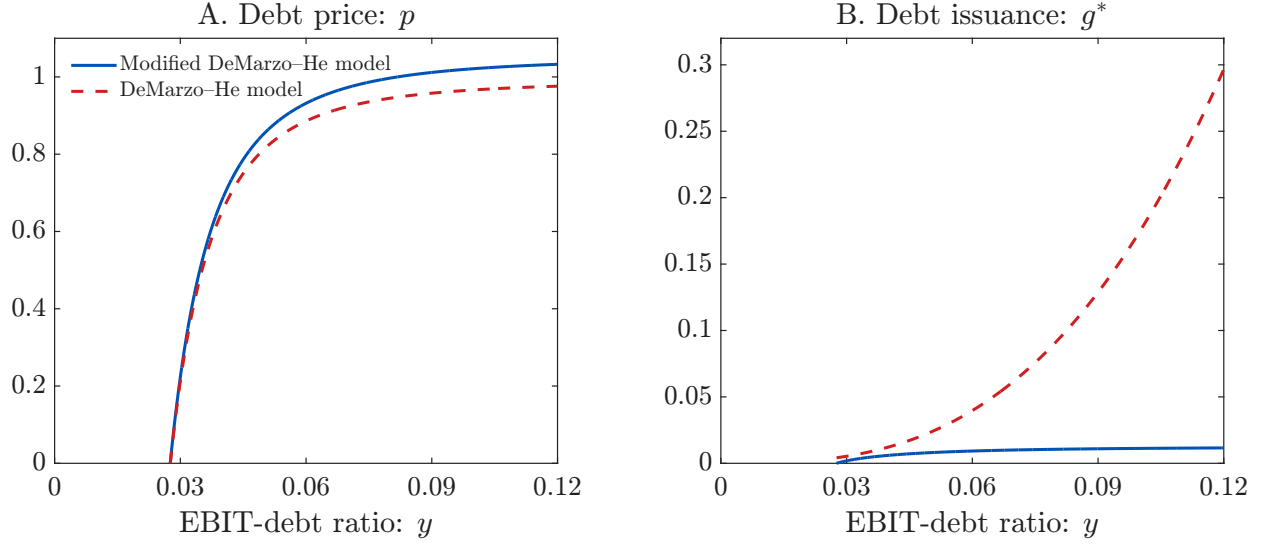


Figure 11: **The Effect of Debt Issuance Costs.** Parameters: $r = 0.05$, $c = 0.06$, $\pi = 0.20$, $\sigma = 0.35$, $\mu = 0.02$, $\xi = 0.15$, $\theta = 5$.

only the default time τ_d ,

$$V^0(Y_0, F_0) = \max_{\tau_d} \mathbb{E} \left[\int_0^{\tau_d} e^{-rs} [Y_s - \pi(Y_s - cF_s) - (c + \xi)F_s] ds \right].$$

In the modified model, shareholders can attain V^0 by setting $G \equiv 0$ and optimizing default, so $V \geq V^0$. In DeMarzo and He (2021), $V^{DH} = V^0$ by their Proposition 2. Hence $V \geq V^{DH}$. Wherever $V^{DH} > 0$, therefore, $V > 0$ and shareholders do not default, which gives $y_d \leq y_d^{DH}$. $\square$

Deferring a repurchase captures the tax shield on the deferred debt, as in DeMarzo and He (2021, Appendix C.2), and in addition saves the debt adjustment cost. Their argument therefore extends to the modified model and gives:

Proposition 2 (No repurchases). *Let $\theta > 0$. In any smooth equilibrium of the modified DeMarzo-He model, shareholders never repurchase debt: $g^*(y) \geq 0$ for every $y > y_d$.*

The result rests on costless equity, which absorbs the cash-flow mismatch that deferring a repurchase creates. In our model, retained cash earns less than the risk-free rate and future shortfalls require costly equity, so debt is a liquidity burden and a cash-rich firm may prefer retiring it to holding cash.

Figure 11 compares the modified DeMarzo-He model with DeMarzo and He (2021). For the parameters shown, debt issuance costs reduce issuance (Panel B) and raise debt prices (Panel A).

By making future borrowing costly, these costs provide partial commitment against the leverage ratchet, protecting existing creditors from dilution.

G.2 Numerical Procedure

We solve the coupled system (G.1), (G.2), and (G.3) using a penalty-based iterative scheme. Given the debt price p , scaled equity value satisfies $\max\{\mathcal{A}v, -v\} = 0$ on $(0, \infty)$, where

$$\mathcal{A}v(y) = \frac{1}{2}\sigma^2 y^2 v''(y) + (\mu + \xi)yv'(y) - (r + \xi)v(y) + (1 - \pi)(y - c) - \xi + \frac{\theta}{2}(g^*(y))^2. \quad (\text{G.6})$$

We approximate this variational inequality by a penalized equation, recover the default boundary $y_d = \inf\{y : v(y) > 0\}$, and update debt prices using (G.3). The detailed procedure is as follows.

1. On the grid $0 = y_0 < \dots < y_N = y^{\max}$, choose initial guesses (v^0, p^0, y_d^0) and set $n = 0$.
2. Given p^n , initialize $v^{n,0} = v^n$. At each inner iteration, linearize the discretized operator $\mathcal{A}$ around $v^{n,m}$, holding p^n fixed, and solve

$$\mathcal{A}^{n,m}v^{n,m+1} + \kappa(-v^{n,m+1})\mathbf{1}_{\{v^{n,m} < 0\}} = 0,$$

where $\kappa > 0$ is the penalty parameter, with boundary conditions $v_0^{n,m+1} = 0$ and $v_N^{n,m+1} = (1 - \pi)y_N/(r - \mu) + A$. Repeat until $\|v^{n,m+1} - v^{n,m}\| \leq \epsilon_{\text{in}}$, and set v^{n+1} equal to the final inner iterate.

3. Update the default boundary: $i_b = \min\{i : v_i^{n+1} > 0\} - 1$ and $y_d^{n+1} = y_{i_b}$. For $i > i_b$, compute $g_i^* = [p_i^n + v_i^{n+1} - y_i(v^{n+1})'_i]/\theta$ and use this policy to solve (G.3) for p^{n+1} , subject to $p_i^{n+1} = 0$ for $i \leq i_b$ and $p_N^{n+1} = p_\infty$.

4. Stop when

$$\|v^{n+1} - v^n\| + \|p^{n+1} - p^n\| + |y_d^{n+1} - y_d^n| \leq \epsilon_{\text{out}}.$$

Otherwise, set $n \leftarrow n + 1$ and repeat from step 2.