INFORMATION GOODS UPGRADES: THEORY AND EVIDENCE*

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Abstract

A substantial portion of information goods is sold through upgrades. I model a monopolist offering successive generations of an information good in a dynamic model. In each period, the monopolist offers up to two prices for each generation: a full price to those who have never purchased and a version upgrade price to consumers who own a previous generation. I employ an overlapping generations model with infinite-lived firms and consumers that reflects the effect of future profits on current decisions better than previous two-period models. The model's predictions accord well with data from the PC software industry. The model explains why: 1) firms issued version upgrades with every new generation, 2) firms provided a discount to those upgrading relative to first-time buyers and 3) late adopters commonly purchased the latest version at full price even though some earlier adopters with higher valuations did not upgrade to the latest version.

Keywords: durable goods, upgrades, software, price discrimination, information goods, product obsolescence.

JEL Classification Codes: D42, L11, D92, L86.

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In an upgrade offer, a firm sells a generation of its durable product to owners of an earlier generation product at a price different from the full price charged to those who have never purchased. Upgrades are an important marketing tool for information goods firms because the product's nearly perfect durability and low marginal cost make upgrade offers an appealing price discrimination tool.[1] Despite upgrades' importance in information goods and the attention these markets have attracted in antitrust cases, current theoretical models do not fully explain major empirical facts about their use. I develop a theoretical model that explains these regularities.

Previously, upgrades have been analyzed in two-period models. I employ an overlapping generations model with infinite-lived firms and consumers that explains observed upgrade use better than two-period models. The model predicts the effect of relevant factors on upgrade issuance and the relative prices of the product offerings. These predictions accord well with observed behavior in PC software, an information good for which significant upgrades data is available. The infinite-horizon feature of my model eliminates an "end-of-the-world" effect and leads to predictions that are more consistent with purchase patterns and observed upgrade discounts for PC software.

I model a monopolist under assumptions closely approximating an information goods market. The firm offers successive generations of a perfectly durable, zero marginal cost product. The firm offers a better product in each generation and no secondary market exists for the old product. The latter allows consumers, whose preferences for quality vary along a continuum, to credibly identify their purchase history. The model predicts when the firm will employ an upgrade and its price relative to the full price good.

The model's main predictions follow from the firm practicing a combination of static second-degree and inter-temporal price discrimination. Consumers with the highest taste for quality purchase earlier than those with lower tastes.[2] This divides old consumers into two cohorts: consumers attached to the firm and unattached consumers. Because consumers can credibly identify their previous purchases, the firm can practice second-degree price discrimination

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[1] Between March 1995 and March 1996, fifty-eight percent of the retail word processing products (sold through all major retail channels except OEM and direct) were upgrades according to PC Data, a market data firm. I thank Ann Stephens for providing this data.

[2] Coase (1972) argued that a durable-goods monopolist has an incentive to lower its price over time as increasingly lower-valuation consumers remain in the market. Coase argued and Stokey (1981) and Gul, Sonnenschein and Wilson (1986) formalized that a monopolist who is unable to commit to future prices will drop its price to marginal cost immediately because, otherwise, consumers anticipate and wait for the future price reduction. When firms can commit to prices within time periods as I assume, prices decline (assuming no innovation) in a step function.

between these groups. In the next section I summarize major empirical regularities from the PC software industry that my model explains but that previous models have only partially explained.

1. EXPLAINING UPGRADE USAGE IN PC SOFTWARE

While I focus on PC software because it is a major information goods market for which upgrades data is readily available, the results are applicable to information goods in general. A variety of information goods firms offer upgrades, including dial-up Internet service, web hosting, voice mailbox service, email service, web bulletin board listings and content databases.

I gathered upgrades data from the word processing and "C" language compiler software segments. The sixty-two word processing products in my sample include all the major word processors issued between 1985 and 1994 and the thirty-six "C" language compilers include all major packages issued between 1987 and 1998.[3] The number of major word processing product lines (11) greatly exceeded that for "C" compilers (5). Figure 1 lists the major product lines in my sample. For these products, I gathered quarterly full and version upgrade prices listed in mail order advertisements appearing in *PC Magazine*. These segments differ greatly in product variety, customer demographics and Microsoft's degree of dominance after the Windows operating system became widely accepted.

Primarily software developers use "C" compilers while word processors are more widely used. In contrast to the word processing segment, which Microsoft came to dominate after its introduction of a Windows-based product, in "C" language compilers Microsoft lagged behind Borland in releasing a Windows-compatible product, leading Borland to dominate the segment. Figure 2 contains summary statistics for the products in my sample. "C" compilers are, on average, more expensive than word processing packages and are issued somewhat more frequently, but have similar average upgrade discounts. I identified three major empirical regularities about PC software upgrades not fully explained by previous models: leapfrogging, universality of upgrades and upgrade prices strictly below full prices.

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[3] A list of the products is available upon request from the author. The word processing packages in my sample constitute over 65% of sales in 1986, 93% in 1987, 94% in 1988, 92% in 1989, 96% in 1990, 99% in 1991, 97% in 1992, 99% in 1993 and 99% in 1994 according to data provided by International Data Group, a market research firm. I thank Mary Wardley for providing this data. I do not have similar sales data for the "C" compiler products but I collected all products with prices listed in *PC Magazine*.

Figure 1 Word Processing and "C" Compiler Product Lines in Sample

Word Processing Product Lines

- Ami
- DisplayWrite
- Leading Edge
- Microsoft Word
- Multi-Mate
- OfficeWriter
- Professional Write
- Samna
- WordPerfect
- WordStar
- XyWrite

"C" Compiler Product Lines

- Quick C
- Microsoft C/C++
- Turbo C/C++
- Borland C++
- Watcom

Figure 2 Summary Statistics for Sample of PC Products

Variable	*Mean*	*St. Dev.*	*Minimum*	*Maximum*
Word Processing Products (n = 62)				
Full Price	235	65.6	67.9	349
Version Upgrade Price	80	41	0	199
Upgrade Discount	0.645	0.169	0.111	1.000
Time Between Upgrades	545	239	181	1402
"C" Compiler Products (n = 36)				
Full Price	446	198	80.0	895
Version Upgrade Price	165	89	25	300
Upgrade Discount	0.613	0.150	0.375	0.919
Time Between Upgrades	445	232	123	1216

Leapfrogging is common across all PC software segments. Leapfrogging occurs when pre-existing consumers with a lower valuation purchase the most recent version, while some previous purchasers eligible for an upgrade, who have a higher valuation, remain with the old version. Figure 3 provides estimates that owners of 63 million units of PC software products in four major segments chose not to upgrade in 1993 even though they were eligible. In the same year, 15.9 million full-price units were sold. All four major segments exhibited significant leapfrogging. Another source estimates that 92% of the installed base of word processor, spreadsheet, presentation manager and database manager PC software chose not to upgrade in 1994 even though the installed base expanded by 19% through the sale of full price units.[4]

Figure 3 Leapfrogging in PC Software Segments in 1993 (units in millions)

Software Segment	Beg-of-Year Installed Base	Upgrade Sales	Units[1] Leapfrogged	Full Price Sales	End-of-Year[2] Installed Base
Spreadsheets	24.75	4.25	20.50	4.25	29.00
Word Processing	28.80	5.10	23.70	5.10	33.90
Graphics Packages	12.05	1.35	10.70	3.15	15.20
Database Managers	9.94	1.44	8.50	3.36	13.30
Total	75.54	12.14	63.40	15.86	91.40

Source: Constructed from data in Exhibit 1 of Lotus Development Corp. estimates cited in *Lotus Development Corporation in 1994*, Anita M. McGahan, Harvard Business School Case #9-794-114.
[1]Equals Beg-of-Year Installed Base minus Upgrade Sales
[2]Equals Beg-of-Year Installed Base plus Full Price Sales

My model explains this behavior. Those who adopted early, and are eligible to upgrade, have more inelastic demand for the firm's product line than those who chose not to purchase. Given the relative demand elasticities, it is optimal for reasonable parameter values for the firm to set a price that restricts sales in the upgrade segment while simultaneously selling to old consumers who did not purchase the previous generation. This result, which arises from the infinite horizon assumption, sharply contrasts with the prediction from a two-period model that leapfrogging is not possible for information goods.

In my sample firms offered version upgrades with every product generation. Of the sixty-two word processing products and thirty-six "C"

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[4] Based on Exhibit 10 in Wintel (A): Cooperation or Conflict, David Yoffie, Harvard Business School Case #9-704-419.

compilers in my sample, firms offered version upgrades for all of them. My model predicts the universality of upgrades. Although consumers who upgrade only obtain value from the incremental improvement over the last generation product, with even minimal innovation the firm can charge a positive price for this improvement. Moreover, consumers with the highest valuation adopt a product when it is first issued and are therefore eligible to purchase a version upgrade. These consumers value this incremental improvement more than any other consumers. As a result, it is always optimal for the firm to offer a version upgrade. This is consistent with predictions from two-period models as well.

In my sample, firms always provided a discount off the full price to those upgrading from its previous version. For all sixty-two of the word processing programs and all thirty-six "C" compilers in my sample, firms offered a version upgrade discount. This seems counterintuitive because firms have an incentive to charge a premium to their attached consumers who have identified themselves as having more inelastic demand for the product line than those who have never purchased. This is true of those eligible to upgrade, which tends to push the upgrade price above the full price. However, those upgrading also face an opportunity cost from the loss of the value of their current asset, which tends to push the upgrade price below the full price.

Because PC software firms cannot force consumers to reveal previous purchases, upgrade prices cannot exceed full prices. Whether the upgrade price is strictly less than the full price depends on the relative strength of these two forces. In a two-period model upgrades do not receive a discount with zero marginal costs and high innovation, because the "end-of-the-world" effect reduces the second-period full price sufficiently that it binds against the upgrade price. In the infinite-horizon model, the elasticity effect is attenuated because the ongoing nature of the market lowers the demand elasticity of full price consumers. As a result, the firm discounts the upgrade relative to the full price in all but extraordinarily innovative or rapidly declining markets. This is consistent with my data, in which producers of word processors and "C" compilers offered upgrade discounts despite rapid innovation and market growth.

My paper adds to both the durable goods pricing and behavior-based price discrimination literatures. Conlisk, Gerstner, and Sobel (1984) and Sobel (1991) consider models in which a durable goods monopolist faces a new set of consumers in each period. They show that the monopolist can price discriminate by selling only to high valuation consumers immediately in each period and periodically running sales in which it discounts to the backlog of lower-valuation consumers. Their model differs from mine in that the firm does not offer successive generations of improved goods. The first comprehensive treatment of durable goods upgrades is Fudenberg and Tirole (FT) (1998), who develop a two-period model of upgrade pricing.[5] I will relate my results to this paper.

Upgrade models are a subset of price discrimination models, which FT (2000) term "behavior-based" price discrimination models. FT (2000) considers duopolists who can price discriminate between their own and rival's previous consumers. Villas-Boas (1999) extends their analysis to infinite-lived firms selling to overlapping generations of consumers and assumes firms can identify their own previous consumers but cannot distinguish new consumers from its rival's consumers. Taylor (1999) also considers an infinite-horizon model of firms tailoring prices to new versus previous consumers in the context of homogenous, non-durable goods. All three of these papers differ from mine in that they consider non-durable goods with no product innovation.

Durability in the presence of innovation alters results in two important ways. First, in the absence of a secondary market durability allows the firm to credibly verify and tailor its price to a customer's purchase history even if it is too costly for the firm to track such history. Moreover, in these cases, ownership of a previous product generation affects the incentives of customers to self-identify and potentially constrains the firm's pricing. Second, durability alters the opportunity cost and therefore the reservation price of those who repeat-purchase. By upgrading, consumers lose the value of their current asset to the extent the new purchase makes it obsolete and its value is irrecoverable on a secondary market.

I organize the remainder of the paper as follows. In the next section, I specify and solve the dynamic model of upgrades. Section 3 derives the model's implications and relates them to the empirical regularities. I conclude in Section 4. I relegate proofs of all results to Appendix 3.

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[5] Earlier two-period monopoly models of upgrade pricing include Lee and Lee (1998) who assume no secondary market and two types of consumers who may choose whether to reveal their previous purchases. van Ackere and Reyniers (1995) motivate upgrading by product depreciation rather than economic obsolescence and assume no secondary market but that firms can force consumers to reveal their previous purchases. Levinthal and Purohit (1989) consider the pricing of two generations of a durable good but do not allow for an upgrade price. Dhebar (1994) considers the existence of pure-strategy subgame-perfect equilibria in two-period models of overlapping generations of products with and without upgrades. Other authors have studied a monopolist's incentive to innovate when producing successive generations of durable goods. Waldman (1993) and Choi (1994) both analyze the monopolist's incentive to innovate in successive generations of durable goods with two types of consumers and allow for an upgrade price in the second period. Padmanabhan, et. al. (1995) show that firms may issue upgrades as a signal of potential product acceptance when consumers face uncertainty about the degree of network effects for the product.

2. AN INFINITE HORIZON MODEL

2.1. SPECIFICATION

I embed a standard vertical differentiation model in each period of an infinite horizon model with overlapping generations of consumers. The firm produces perfectly durable goods at zero marginal cost and chooses prices to maximize the present discounted (by factor $0 < \delta_f < 1$) value of lifetime profits.[6] Consumers live forever but face a positive probability, ρ, that they die after each period so that their expected lifetime is $1/\rho$. As I describe in more detail below, I assume that consumers participate in the full price market only in the first two periods of their lives and in the upgrades market only up to the third period of their lives (for a maximum of two periods in which to purchase an upgrade). This is necessary for tractability but, as I argue below, captures the essential tradeoffs in which I am interested. Consumers make their purchase decisions to maximize their discounted (by factor $0 < \delta_c < 1$) lifetime utility. A new cohort of consumers enters the market in each period.

To mitigate distortions introduced by the limited time consumers participate in the market, I assume that a consumer enjoys the use of any asset owned, full price or version upgrade, for the remainder of her life. If, for example, a consumer only enjoyed the use of a product for two periods, her lifetime value would drop significantly from waiting to purchase rather than purchasing in the current period, exacerbating the two-period assumption. If the consumer holds the asset until death, on the other hand, her lifetime value drops much less significantly from waiting to purchase.

The gain from the assumption of two periods in the market is tractability. Allowing consumers to participate in the market longer would multiply the number of state variables and incentive compatibility constraints. While obviously a simplification, two periods in the market captures the essential tradeoff between purchasing an inferior product sooner versus a superior product later. Additional periods in the market would allow consumers to wait longer to purchase a full price good or upgrade and allow for a menu of upgrade prices depending on which previous generation the consumer owned. My simplification relies on these being second-order effects.

Consumers' tastes for quality, x, are uniformly distributed over the $[0,1]$ interval. Quality may encompass costs such as the time to install, learn and

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[6] It is straightforward to introduce a depreciation rate for the product. This would simply multiply the consumer's discount factor in the results that follow.

customize the software product for their use as well as idiosyncratic tastes for product features. Consumers have constant marginal utility of income so that a type-x consumer obtains per-period utility of:

(1) $xq + I$

from the firm's product of quality q, where I is her net income.

The timing of the model is as follows. Within each time period t, the firm produces a perfectly durable good of quality q^t and sells up to two different products: a full price product to those who have never purchased at price $P^{F,t}$ and a version upgrade to owners of either of the previous two generations of the product at price $P^{V,t}$. The firm must sell rather than rent its products due to legal restrictions or transaction costs. I assume the firm can commit to prices within product generations but not between as in the previous literature, including FT. This appears to be a reasonable approximation for PC software. I reviewed street prices quoted by mail order companies in popular PC magazines and found that firms adjusted prices only slightly within versions except in cases where a firm's viability was uncertain. Firms often changed prices significantly between versions.[7]

Consumers choose from those products for which they qualify to maximize their expected lifetime utility. Since consumers are in the market for up to three periods, I will refer to a consumer in the first period of her life as "young," in the second as "middle-aged," and in the third as "old." A young consumer chooses whether to purchase a full price good or nothing. A middle-aged consumer who purchased when young may purchase the firm's version upgrade, the firm's full price good or nothing; while a middle-aged consumer who did not purchase before may purchase the full price good or nothing. An old consumer who owns the full price good from either of the previous two periods may purchase a version upgrade, the full price good or nothing, while an old consumer who upgraded in the previous period or has not purchased previously is out of the market. All three cohorts consider the effect that their current decisions have on their future choices, including the option to wait and purchase later if available. Figure 4 summarizes the options available to consumers.

To keep the model tractable, I make two assumptions about the upgrade process. First, as already noted above, a consumer can only upgrade once (i.e., a

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[7] This data is available from the author upon request. Commitment between generations is difficult because writing a contract on prices of future versions is encumbered by the difficulty in specifying future functionality.

young consumer who purchases the full price good can only upgrade when either middle-aged or old but not both). Second, I assume that the firm offers only a single price for upgrading to the current generation product (i.e., middle-aged consumers who own the previous generation product and old consumers who own the second-most recent generation product pay the same price for a version upgrade). In PC software, firms do not commonly price discriminate between a consumer who obtained the previous generation via an upgrade and a consumer who obtained it via the full price, implying that the revenue gains from discriminatory prices is often bounded by the administrative costs of offering bracketed upgrades.

Figure 4 Purchase Options Available to Consumer Cohorts in period t

Cohort	Purchase Options
Young	{Full Price, Nothing}
Middle-Aged	
Own Full Price Generation t-1	{Full Price, Version Upgrade, Nothing}
Never Purchased	{Full Price, Nothing}
Old	
Own Full Price Generation t-2	{Full Price, Version Upgrade, Nothing}
Own Full Price Generation t-1	{Full Price, Version Upgrade, Nothing}
Own Upgrade Generation t-1	{Nothing}
Never Purchased	{Nothing}

Consumers' taste for quality is fixed between time periods and consumers can dispose of goods at no cost. The firm's information structure is semi-anonymous so that consumers can reveal a previous purchase but firms cannot force this revelation.[8] This implies the firm's version upgrade cannot exceed its full price. Consumers cannot trade products in a secondary market consistent with the lack of an active resale market for most information goods, allowing consumers to credibly reveal their purchase history.

Between time periods t and $t+1$ four things happen. First, the firm enjoys exogenous technological innovation, which allows it to offer an improved

[8] Although some users register their software with the manufacturer, these lists are generally not available to retail software outlets allowing users to remain anonymous. The technology for verifying users' previous purchases has changed over time. Through the late 1980s, manufacturers required users to submit the program disks or the user manual cover. In the 1990s, manufacturers moved to a software check in the installation program.

version of its product: $q^{t+1} > q^t$.[9] The firm and consumers know the sequence of innovations for both firms, eliminating issues of private information. Second, a density, ρ, of consumers dies with equal probability for all consumers. Third, a uniform distribution of new consumers with density $m^{t+1} > 0$ enters the market. Fourth, each cohort of consumers ages by one period, old consumers exit the market and continue to hold any asset owned, middle-aged consumers who have never purchased exit the market and middle-aged consumers who upgraded to the current generation product exit the market and hold the upgrade.

I solve for a no-commitment, Markov-perfect equilibrium. This assumes the firm is unable to commit to future prices when setting current period prices and that only the previous period's history is relevant for today's decisions (see Maskin and Tirole, 1988). The state variables are the firm's installed bases of full price and version upgrade customers from the prior period. To simplify the analysis I assume that innovation and market size evolve at constant rates (specifically, $q^{t+1} = \theta q^t$ and $m^{t+1} = \varphi m^t$, $\forall t$ with $\theta > 1, \varphi > 0$). That is, the market is on a balanced growth path (bgp).

I solve for an equilibrium in which leapfrogging occurs and show that it is the most profitable equilibrium for a large range of parameter values. Leapfrogging occurs when a positive mass of consumers who purchased the full price in the previous period do not upgrade while a positive mass of consumers who did not purchase in the previous period and have a lower valuation for the product buy the full price good. Leapfrogging is impossible in previous two-period models but is extremely relevant for software markets as I demonstrated above.

2.2. SOLUTION

Because the firm can tailor prices to consumers to induce self-selection based on their purchase history, there are two segments of demand to consider in any period: a full price segment of consumers who have never purchased and an upgrade segment of consumers who own one of the firm's two previous product generations. I will denote the position of the marginal full price consumer in period t by f^t, position of the marginal middle-aged version upgrade consumer

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[9] Because I assume equal production costs in all periods it is never optimal for the firm to sell additional units of a previous generation product in any period. With sufficient cost differences it may be optimal to offer previous generations along with a current generation (see FT, 1998). This assumption combined with my assumption of no secondary market (see below) implies that there will be no previous generation products sold in any period.

by v_M^t, and the marginal old version upgrade consumer by v_O^t. Figure 5 displays the evolution of the market and positions of the marginal consumers in equilibrium.

I now solve for the no-commitment, Markov-perfect equilibrium subject to:

Assumption: $\delta_f \theta\varphi < 1$.

The Assumption is necessary to ensure finite discounted profits for the firm and places limits on the discounted rate of innovation and market growth. This constraint is not very restrictive given that the discount rate is measured over the life of a product generation (usually two or more years).

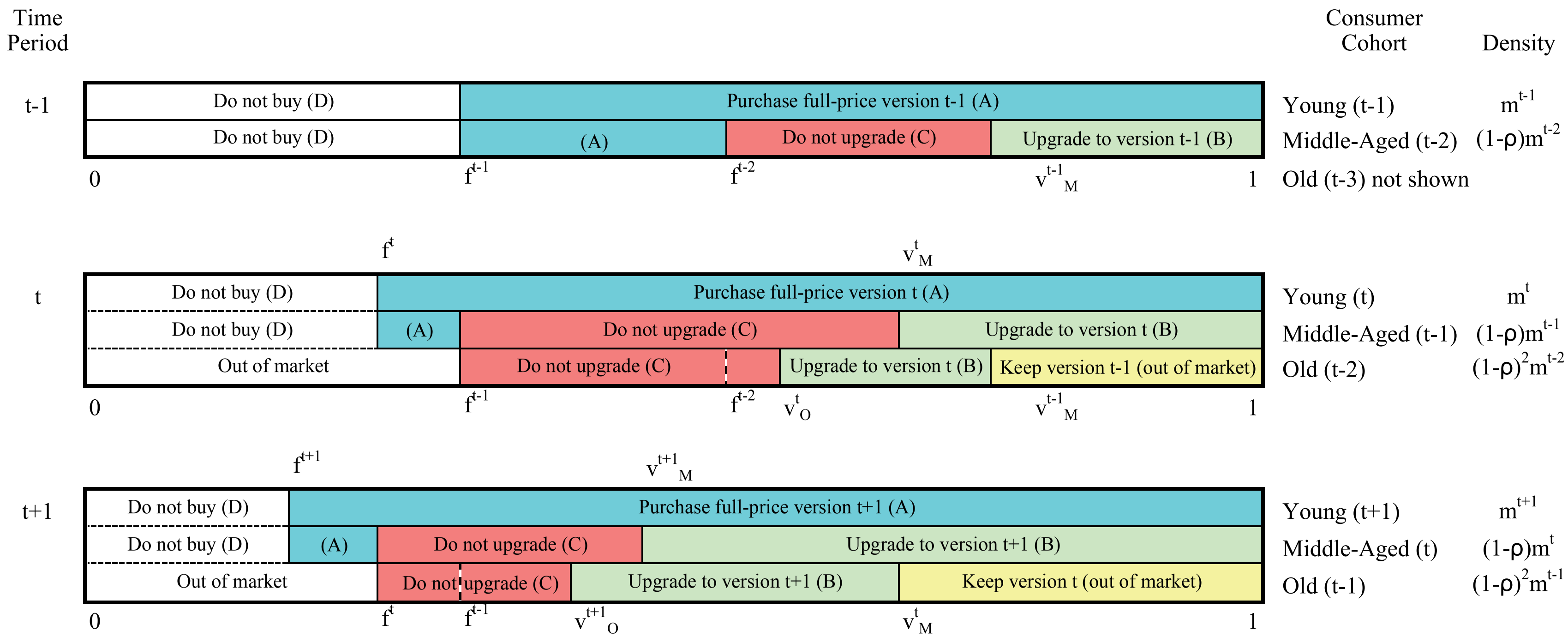
The market for the firm's full price good in period t includes young consumers with density m^t over the entire interval and middle-aged consumers who did not purchase last period and did not die with density $(1-\rho)m^{t-1}$ over the segment $\left[0, f^{t-1}\right]$ as shown in Figure 5. Recall that old consumers are out of the full price market.

Due to leapfrogging, there is a single margin for the young and middle aged cohorts because neither upgrade during their lifetime. Both cohorts obtain the same expected benefit from purchasing – they hold the period t asset until they die. Thus the firm faces young consumer demand of m^t over the segment $\left[f^t, 1\right]$ and middle-aged consumer demand of $(1-\rho)m^{t-1}$ over the segment $\left[f^t, f^{t-1}\right]$ assuming $f^t < f^{t-1}$, where f^t is the consumer who is indifferent between purchasing or not (margin between areas A and D for the young and middle-aged cohort in period t of figure 5). f^t is determined by: $f^t q^t / (1-\delta_c(1-\rho)) + I - P^{F,t} = I$. Solving I obtain:

$$(2)\ f^t = \frac{(1-\delta_c(1-\rho))P^{F,t}}{q^t}.$$

The market for the firm's version upgrade in period t consists of middle-aged consumers who purchased from the firm in period $t-1$ and did not die with density $(1-\rho)m^{t-1}$ over the segment $\left[f^{t-1}, 1\right]$ and old consumers who purchased from the firm in period $t-2$, have not died and who have not upgraded with density $(1-\rho)^2 m^{t-2}$ over the segment $\left[f^{t-2}, v_M^{t-1}\right]$, assuming $f^{t-2} < v_M^{t-1}$, as shown in Figure 5.

Figure 5: Market Structure in Equilibrium



The marginal middle-aged consumer is not eligible to upgrade when old so that she holds the period t asset until she dies. Thus, v_M^t, the marginal middle-aged consumer indifferent between upgrading and keeping their current asset is determined by: $v_M^t q^t / \left(1-\delta_c\left(1-\rho\right)\right) + I - P^{V,t} = = v_M^t q^{t-1} / \left(1-\delta_c\left(1-\rho\right)\right) + I$ (margin between areas B and C for the middle-aged cohort in period t of Figure 5). Solving I obtain:

$$(3)\ v_M^t = \frac{\left(1-\delta_c\left(1-\rho\right)\right) P^{V,t}}{q^t - q^{t-1}}.$$

It is useful to examine the inverse demand function, $P^{V,t} = v_M^t \left(q^t - q^{t-1}\right) / \left(1-\delta_c\left(1-\rho\right)\right)$, before solving the model. This price differs from the standard one-period, non-durable result: $v_M^t q^t$. The version upgrade price differs by an opportunity cost $-q^{t-1} / \left(1-\delta_c\left(1-\rho\right)\right)$, because upgrading renders the first-generation product obsolete as determined by innovation $\left(q^t - q^{t-1}\right)$. A better product makes the previous generation more obsolete, allowing the firm to charge a higher price. With negligible innovation, firms may provide an upgrade at little cost to their customer base. PC software firms often provide release upgrades (minor product issues within a version) at a low price. For example, in 1990 Microsoft offered version 1.1 of Word for Windows for \$7.50 to owners of version 1.0 even though the full price for version 1.1 was \$329. In addition, the price is adjusted by $1/1-\delta_c\left(1-\rho\right)$ because the consumer holds the asset until death.

The old marginal upgrade consumer will not participate in the market after period t and also holds the period t asset until she dies. Therefore, $v_O^t q^t / \left(1-\delta_c\left(1-\rho\right)\right) + I - P^{V,t} = \ = v_O^t q^{t-2} / \left(1-\delta_c\left(1-\rho\right)\right) + I$ determines v_O^t, the marginal old consumer who is indifferent between upgrading and keeping the $t-2$ generation product (margin between areas B and C for the old cohort in period t of Figure 5). Solving I obtain:

$$(4)\ v_O^t = \frac{\left(1-\delta_c\left(1-\rho\right)\right) P^{V,t}}{q^t - q^{t-2}}.$$

Thus, the firm faces upgrade demand of $(1-\rho)m^{t-1}$ over the segment $\left[v_M^t,1\right]$ assuming $f^{t-1}<v_M^t$ and demand of $(1-\rho)^2 m^{t-2}$ over the segment $\left[v_O^t,v_M^{t-1}\right]$ assuming $f^{t-2}<v_O^t<v_M^{t-1}$.

The firm maximizes the discounted value of profits at time t from sales of its full price good and version upgrade. The Bellman equation is:

$$(5)\ V\left(f^{t-1},v_M^{t-1},q^{t-1},m^{t-1}\right)=$$
$$\max_{P^{F,t},P^{V,t}} \left\{\left[\left(1-f^t\right)m^t+\left(\left(1-f^t\right)-\left(1-f^{t-1}\right)\right)(1-\rho)m^{t-1}\right]P^{F,t}+(1-\rho)\right.$$
$$\left[\left(1-v_M^t\right)m^{t-1}+(1-\rho)\left(\left(1-v_O^t\right)-\left(1-v_M^{t-1}\right)\right)m^{t-2}\right]P^{V,t}$$
$$\left.+\delta_f V\left(f^t,v_M^t,q^t,m^t\right)\right\}.$$

This assumes the firm cannot commit to future prices and a Markov-perfect equilibrium where f^{t-1}, v_M^{t-1}, q^t and m^t are the state variables. Incentive compatibility (IC), individual rationality (IR) and non-negativity (NN) constraints are necessary for the equilibrium. The IC constraints require that the marginal consumer of each product weakly prefers her choice to all other choices for which she is eligible (including waiting to purchase next period). This includes the semi-anonymous constraint that ensures the upgrade price does not exceed the full price. The IR constraints ensure that the marginal consumer of each good weakly prefers purchasing to not. The NN constraints ensure that prices and market shares are positive (including the leapfrogging constraint). Appendix 1 details these restrictions.

I solve along a bgp $\left(q^{t+1}=\theta q^t,m^{t+1}=\varphi m^t\right)$. This allows me to express v_O^t in terms of v_M^t:

$$(6)\ v_O^t=\frac{\theta^2\left(1-\delta_c(1-\rho)\right)P^{V,t}}{q^t\left(\theta^2-1\right)}=\frac{\theta}{(\theta+1)}v_M^t\ .$$

I transform the Bellman equation to make it stationary and reduce the dimensionality of the state space: $\tilde{V}\left(f^{t-1}, v_M^{t-1}\right) = V\left(f^{t-1}, v_M^{t-1}, m^t, q^t\right) \Big/ \left(m^t q^t\right)$. Applying this transformation and imposing the bgp on the continuation value I obtain:

$$
(7)\ \tilde{V}\left(f^{t-1}, v_M^{t-1}\right) = \max_{P^{F,t}, P^{V,t}} \Big\{ \left[\left(1-f^t\right) + \left((1-\rho)/\varphi\right)\left(\left(1-f^t\right) - \left(1-f^{t-1}\right)\right)\right] P^{F,t}/q^t + \frac{(1-\rho)}{\varphi} \left[\left(1-v_M^t\right) + \left((1-\rho)/\varphi\right)\left(\left(1-\theta v_M^t/(\theta+1)\right) - \left(1-v_M^{t-1}\right)\right)\right] P^{V,t}/q^t + {} + \delta_f \theta \varphi \tilde{V}\left(f^t, v_M^t\right) \Big\}.
$$

The Bellman reflects the fact that the firm's optimal prices in each period depend on the previous and next period's states. Last period's cutoffs determine what fraction of middle-aged and old consumers are still in each market, while next period's products act as substitutes for current young and middle-aged consumers as reflected in the continuation value. The Assumption ensures that the net present value of the firm's profits is bounded.

Because of the constraints $f^{t-1} < v_M^t$ and $f^{t-2} < v_O^t$ (see period t of Figure 5), the full price problem is separable from that for upgrades. The marginal young full price consumer will be outside the extensive margin of version upgrade consumers in both of the following two periods and the marginal middle-aged full price consumer will be outside the extensive margin of version upgrade consumers when old. So we can define $\tilde{V}\left(f^{t-1}, v_M^{t-1}\right) = \tilde{V}_F\left(f^{t-1}\right) + \tilde{V}_V\left(v_M^{t-1}\right)$ where:

$$
(8)\ \tilde{V}_F\left(f^{t-1}\right) = \max_{P^{F,t}} \Big\{ \left[\left(1-f^t\right) + \left((1-\rho)/\varphi\right)\left(\left(1-f^t\right) - \left(1-f^{t-1}\right)\right) P^{F,t}/q^t\right] + \delta_f \theta \varphi \tilde{V}_F\left(f^t\right) \Big\}
$$

and
$$
\tilde{V}_V\left(v_M^{t-1}\right) = \max_{P^{V,t}} \frac{(1-\rho)}{\varphi} \Big\{ \left[\left(1-v_M^t\right) + \left((1-\rho)/\varphi\right)\left(\left(1-\theta v_M^t/(\theta+1)\right) - \left(1-v_M^{t-1}\right)\right) P^{V,t}/q^t\right] + \delta_f \theta \varphi \tilde{V}_V\left(v_M^t\right) \Big\}.
$$

The method of solution is constructive. I first posit the firms' value (profit), policy (price), and transition (marginal consumer) functions. I then solve the firm's profit maximization problem by optimizing the Bellman equation. The resulting equation allows me to solve for the unknown constants in the firm's pricing and profit functions. To solve for the full price, suppose that the firm's value, pricing and transition functions are (a, b, c, d, e, g and h are unknown constants):

$$(9)\ \tilde{V}_F\left(f^{t-1}\right) = a + bf^{t-1} + c\left(f^{t-1}\right)^2$$

$$(10)\ P^{F,t}\left(f^{t-1}\right) = \left(d + ef^{t-1}\right)q^t$$

$$(11)\ f^t\left(f^{t-1}\right) = g + hf^{t-1}.$$

The first-order condition for the full price good simplifies to:

$$(12)\ 2P^{F,t}\left(1-\delta_c\left(1-\rho\right)\right)\left(\delta_f\varphi^2 c\left(1-\delta_c\left(1-\rho\right)\right)\theta - \left(1+\varphi-\rho\right)\right) + q^t\left(\delta_f\varphi^2 b\left(1-\delta_c\left(1-\rho\right)\right)\theta + \varphi + f^{t-1}\left(1-\rho\right)\right) = 0.$$

This can be solved for $P^{F,t}$, which is linear in f^{t-1}, and then substituted back into the Bellman equation. The resulting Bellman equation is quadratic in f^{t-1} so that I can equate coefficients to solve for a, b and c. Plugging the results for these three coefficients back into the solution for $P^{F,t}$ and equation (2) allows me to solve for d, e, g and h:

$$(13)\ d = \frac{g}{1-\delta_c\left(1-\rho\right)}$$

$$(14)\ e = \frac{h}{1-\delta_c\left(1-\rho\right)}$$

$$(15)\ g = \frac{\varphi}{r - \delta_f\varphi\theta\left(1-\rho\right)}$$

$$(16)\ h = \frac{\left(1-\rho\right)}{r},$$

where $r = 1+\varphi-\rho+\sqrt{\left(1+\varphi-\rho\right)^2 - \delta_f\varphi\theta\left(1-\rho\right)^2}$. This leads directly to the first technical result:

Technical Result 1: The parameters of the full price transition function are real and between zero and one (i.e., $0 < g, h < 1$*). The* *parameters of the full price policy function are real and positive (i.e.,* $0 < d, e$*).*

This implies that the firm's full price market share increases in each successive period (i.e., $f^t < f^{t-1}$), approaching $1 - g/(1-h)$ if it begins with a share below this. That is, the monopolist follows the Coasian path, selling to more consumers with lower valuations in each period. The second-order condition simplifies to:

$$(17)\ -\frac{\left(1-\delta_c\left(1-\rho\right)\right)}{\left(\varphi\left(q^t\right)^2\right)r},$$

which is met for all values. The transformed Bellman is stationary as $P^{F,t}$ is proportional to q^t while f^t is unrelated to q^t. The firm's untransformed value function scales up proportionally with increases in market size and product quality. If all consumers have a one-period life-span (i.e., $\rho = 1$), the firm prices according to a standard one-period Hotelling model (i.e., $P^{F,t} = f^t = 1/2$). If there are no new consumers entering the market (i.e., $\varphi = 0$), the firm prices according to the standard one-period Hotelling result for a durable good with a pre-existing market share (i.e., $P^{F,t} = f^{t-1}q^t / 2\left(1-\delta_c\left(1-\rho\right)\right)$ and $f^t = f^{t-1}/2$).

To solve for the version upgrade price, suppose that the firm's value, policy and transition functions are ($a_V, b_V, c_V, d_V, e_V, g_V$ and h_V are unknown constants):

$$(18)\ \tilde{V}_V\left(v_M^{t-1}\right) = a_V + b_V v_M^{t-1} + c_V\left(v_M^{t-1}\right)^2$$

$$(19)\ P^{V,t}\left(v_M^{t-1}\right) = \left(d_V + e_V v_M^{t-1}\right)q^t$$

$$(20)\ v_M^t\left(v_M^{t-1}\right) = g_V + h_V v_M^{t-1}.$$

The first-order condition for the version upgrade simplifies to:

$$\begin{aligned}(21)\ & 2P^{V,t}\left(1-\delta_c\left(1-\rho\right)\right)\left(\theta\delta_f\varphi^3 c_V\left(1-\delta_c\left(1-\rho\right)\right)\left(\theta+1\right)\theta^2\right.\\ & \left.-\left(\theta-1\right)\left(1-\rho\right)\left(\theta\left(1+\varphi-\rho\right)+\varphi\right)\right) + q^t\left(\theta-1\right)\left(\delta_f\varphi^3 b_V\left(1-\delta_c\left(1-\rho\right)\right)\left(\theta+1\right)\theta^2\right.\\ & \left.+\left(\theta^2-1\right)\left(1-\rho\right)\left(\varphi + v_M^{t-1}\left(1-\rho\right)\right)\right) = 0.\end{aligned}$$

This can be solved for $P^{V,t}$, which is linear in v_M^{t-1}, and then substituted back into the Bellman equation. The resulting Bellman equation is quadratic in v_M^{t-1} so that I can equate coefficients to solve for a_V, b_V and c_V. Plugging the results for these three coefficients back into the solution for $P^{V,t}$ and equation (3) allows me to solve for d_V, e_V, g_V and h_V:

(22) $d_V = \dfrac{(\theta - 1) g_V}{\theta\left(1 - \delta_c (1-\rho)\right)}$

(23) $e_V = \dfrac{(\theta - 1) h_V}{\theta\left(1 - \delta_c (1-\rho)\right)}$

(24) $g_V = \dfrac{\varphi(\theta + 1)}{r_V - \delta_f \varphi\theta(1-\rho)(\theta+1)}$

(25) $h_V = \dfrac{(1-\rho)(\theta+1)}{r_V}$,

with $r_V = \theta(1+\varphi-\rho)+\varphi+\sqrt{\left(\theta(1+\varphi-\rho)+\varphi\right)^2 - \delta_f \varphi\theta(1-\rho)^2(\theta+1)^2}$. This leads directly to the second technical result:

> *Technical Result 2: The parameters of the version upgrade transition function are real and between zero and one (i.e., $0 < g_V, h_V < 1$). The parameters of the version upgrade policy function are real and positive (i.e., $0 < d_V, e_V$).*

As with the full price market, the firm increases its market share in each successive period (i.e., $v_M^t < f^t$) approaching $1 - g_V / (1 - h_V)$ if it begins with a share below this. The monopolist's upgrades also follow the Coasian path. The second-order condition simplifies to:

(26) $-\dfrac{\left(1-\delta_c(1-\rho)\right)(1-\rho)\theta}{\left(\varphi^2 \left(q^t\right)^2 \left(\theta^2 - 1\right)\right) r_V}$,

which is met for all values. The transformed Bellman is stationary as $P^{V,t}$ is proportional to q^t while v_M^t is unrelated to q^t. The firm's untransformed value function scales up proportionally with increases in market size and product quality.

The following technical result is useful for deriving the main results:

Technical Result 3: Parameters of the version upgrade transition function are greater than those of the full price transition function (i.e., $g_V > g$ and $h_V > h$).

This implies that if the firm begins with a full price share below $1-g/(1-h)$ and the initial full price share is weakly greater than the upgrade share, the version upgrade share is always below that of the full price good. Moreover, the version upgrade share converges to a value less than that of the full price market share (i.e., $1-g_V/(1-h_V) < 1-g/(1-h)$). If all consumers have a one-period life-span (i.e., $\rho = 1$), the firm prices according to a standard one-period Hotelling model with the consumer holding a pre-existing asset (i.e., $v_M^t = 1/2$ and $P^{V,t} = (q^t - q^{t-1})/2$).

Appendix 1 shows that all constraints on the equilibrium are met for a wide range of parameter values. To show that this is an equilibrium it is necessary to consider whether the firm can do better by changing its strategy. There are three possible deviations to consider: 1) the firm prices such that no middle-aged consumers buy the full price good (i.e., $f^t > f^{t-1}$), 2) the firm prices such that all previous middle-aged purchasers upgrade (i.e., $v_M^t = f^{t-1}$), 3) the firm prices such that the semi-anonymous constraint binds (i.e., $P^{F,t} = P^{V,t}$), and 4) the firm prices such that no old consumers upgrade (i.e., $v_O^t = v_M^{t-1}$). Appendix 2 describes simulation results that demonstrate the equilibrium is robust to these deviations for a wide range of parameter values.

3. Implications of the Model

I now use the results of the model to explain the empirical regularities identified in Section 1. Unlike a two-period model, my infinite horizon model allows for leapfrogging:

Result 1: When the firm's market shares are below the bgp shares and the firm begins with a weakly greater market share for the full *price good than the version upgrade, leapfrogging occurs. This contrasts with a two-period model, in which leapfrogging is impossible with zero marginal costs. Introducing a declining market in the second period of a two-period model does not reverse the impossibility of leapfrogging with a uniform distribution of consumer types.*

In the FT (1998) two-period model, the "end-of-the-world" effect leads to a low second-period full price. First-period consumers anticipate the low second-period price so that all but very high-valuation consumers wait to purchase. Due to their high valuation, all of the first-period purchasers upgrade in the second period. Introducing a declining market in the second period leads to greater sales in the first period, because the firm is less able to commit to high second-period prices. However, declining growth is not sufficient to overcome the "end-of-the-world" effect and make leapfrogging possible. My model eliminates the firm's myopia so the firm faces a trade-off between current and future profits. As a result, sales are more evenly distributed across time periods, making leapfrogging possible for any market share below the bgp. On the bgp, leapfrogging does not occur because sales of the full price good equal those of the earlier period even though the market share for the version upgrade is below that of the full price good.

The model predicts that version upgrades should always be issued:

Result 2: The firm offers a version upgrade with every generation.

High-valuation consumers purchase in the first period of their life and are eligible to purchase a version upgrade in the second period of their life, making it possible for the firm to charge a positive version upgrade price with even a slightly improved product (assuming zero fixed costs of issuance). This regularity observed in the data is also consistent with a two-period model such as FT (1998). While the semi-anonymous constraint can bind off the bgp if the full price share is sufficiently greater than the version upgrade share, in a growing market the firm always discounts the upgrade when it is sufficiently close to the bgp:

Result 3: Under costless production and in a growing market, the semi-anonymous constraint (the version upgrade price is below the full-price) is always met sufficiently close to the bgp. This contrasts with a two-period model, which predicts the constraint binds for high innovation rates.

Whether the semi-anonymous constraint binds depends on the relative strength of an opportunity cost and an elasticity effect. Consumers who upgrade bear an opportunity cost because they own a non-tradable asset that provides benefit, pushing the upgrade price below the full price. On the other hand, those eligible to upgrade have the lowest demand elasticity for the firm's product, pushing the upgrade price above the full price. In FT's (1998) two-period model with a uniform distribution of consumers such as I employ, the semi-anonymous constraint binds for innovation rates greater than one hundred percent $(\theta > 2)$. Because the second period is the firm's last chance to sell its full price good and demand for it is composed only of low-valuation consumers, the firm sets a low full price. This incentive is powerful enough that the semi-anonymous constraint binds for high innovation rates. In my model, the opportunity to sell to future consumers and the presence of new higher-valuation consumers elevates the full price and the firm discounts the version upgrade as long as the market is growing and the firm is near the bgp. This is consistent with my PC software data, in which upgrade discounts are universal, the markets for these products grew rapidly and firms are highly innovative.

My model also offers theoretical predictions for the level of prices. The model predicts:

> *Result 4: The full price is: a) increasing in the innovation rate, b) increasing in the firm's discount factor, c) increasing in the consumer's discount factor and d) decreasing in the previous full price market share.*

> *Result 5: The version upgrade price is: a) increasing in the innovation rate, b) increasing in the firm's discount factor, c) increasing in the consumer's discount factor and d) decreasing in the previous version upgrade market share.*

The full price is increasing in the innovation rate, which follows simply from the increased value of the product to consumers. The full price is also increasing in the firm discount factor. This follows from the diminution of Coasian dynamics. A higher firm discount factor increases the firm's incentive to charge higher future prices which consumers anticipate, decreasing their elasticity for current purchases.[10] The full price is also increasing in the consumer discount

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[10] Many papers focus on methods for reducing the commitment problem of the durable goods producer. These include Bond and Samuelson (1984) who consider depreciation, Bulow (1982) who considers product durability and Bond and Samuelson (1987) who focus on innovation.

factor because consumers keep their purchase in perpetuity, so that a higher discount factor increases the present value of utility from purchasing. Finally, the full price is decreasing in the firm's previous market share, consistent with Coasian dynamics.

The model also allows for either decreasing or increasing prices over time. Declining prices are possible even if the absolute value of the software is increasing over time. Declining prices are more likely with lower rates of innovation, lower firm and consumer discount factors and more mature markets.

For the version upgrade price, a higher innovation rate makes the upgrade more valuable because it diminishes the opportunity cost effect. The marginal upgrade consumer benefits only from the incremental improvement between generations – a higher innovation rate increases this. A higher firm discount factor allows the firm to commit to higher future prices allowing it to charge higher current prices. Since consumers hold onto any upgrade owned for the remainder of their lives, the upgrade is more valuable the higher their discount factor. Finally, the upgrade price is decreasing in the firm's previous market share consistent with Coasian dynamics.

4. CONCLUSION

This paper provides a model of upgrade pricing for information goods firms and uses it to explain several empirical regularities in PC software. The model expands on previous work by making firms and consumers infinite-lived and providing empirical regularities to judge the model's implications. An infinite-horizon model more realistically captures firm and consumer incentives to balance current and future sales making leapfrogging possible and upgrade discounts universal, consistent with the PC software market. This highlights the importance of allowing for the full dynamics of consumer and firm decision-making in intertemporal decisions.

Endogenizing both time and rate of innovation would clarify the trade-off between innovating longer versus faster. Some work on this issue has been done. Rob and Fishman (1996) compare the optimal frequency of durable good product introductions for a social planner, monopolist and duopolist, but consider homogenous consumers and do not allow for upgrades. Ellison and Fudenberg (2000) show that firms may have an incentive to introduce more upgrades than is socially optimal when software generations are backward- but not forward-compatible.

Appendix 1
Constraints on Equilibrium

a) IR constraints: consumers earn non-negative utility from purchasing:
   i. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{F,t} \geq 0,\ \forall x \in \left[f^t, 1\right]$
   ii. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{V,t} \geq xq^{t-1} \big/ \left(1-\delta_c\left(1-\rho\right)\right), \forall x \in \left[v_M^t, 1\right]$
   iii. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{V,t} \geq xq^{t-2} \big/ \left(1-\delta_c\left(1-\rho\right)\right), \forall x \in \left[v_O^t, v_M^{t-1}\right]$

b) NN constraints: shares and prices are positive:
   i. $0 < f^t$, ii. $f^t < f^{t-1}$, iii. $f^{t-1} < v_M^t$, iv. $f^{t-2} < v_O^t$, v. $v_O^t < v_M^{t-1}$, vi. $v_M^t \leq 1$, vii. $v_O^t < 1$, viii. $0 < P^{F,t}$, ix. $0 < P^{V,t}$

c) IC - semi-anonymous constraint: upgrade consumers prefer it over the full price good:
   i. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{V,t} > xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{F,t}, \forall x \in \left[v_M^t, 1\right]$
   ii. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{V,t} > xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{F,t}, \forall x \in \left[v_O^t, v_M^{t-1}\right]$

d) IC – intertemporal constraints: young consumers of the full price good prefer purchasing in the current period rather than waiting until the next period and middle-aged consumers of the version upgrade prefer purchasing in the current period rather than waiting until the next period:
   i. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{F,t} > \delta_c\, xq^{t+1} \big/ \left(1-\delta_c\left(1-\rho\right)\right) - \delta_c P^{F,t+1}, \forall x \in \left[f^t, 1\right]$
   ii. $xq^t \big/ \left(1-\delta_c\left(1-\rho\right)\right) - P^{V,t} > xq^{t-1} + \delta_c\, xq^{t+1} \big/ \left(1-\delta_c\left(1-\rho\right)\right) - \delta_c P^{V,t+1}, \forall x \in \left[v_M^t, 1\right]$

Fulfillment of Constraints

Assume that the firm's market shares in both segments are below the bgp market shares and that the firm begins with a weakly greater market share for the full price good than the version upgrade (i.e., $f^0 \leq v_M^1$).

a) IR constraints:
   i. , ii. and iii. met by construction.
b) NN constraints:
   i. $0 < f^t$ since $g, h > 0$.
   ii. $g \big/ \left(1-h\right) < f^{t-1}$ since share is below the bgp share and this implies $g + hf^{t-1} = f^t < f^{t-1}$ ($1-h$ is positive by Technical Result 1).
   iii. $f^{t-1} < v_M^t$ because $f^{t-1} = \dfrac{g\left(1-h^{t-1}\right)}{1-h} + h^{t-1} f^0 < \dfrac{g_V\left(1-h_V^{t-1}\right)}{1-h_V} + h_V^{t-1} v_M^1 = v_M^t$ since $g < g_V$, $h < h_V$ by Technical Result 3 and $f^0 \leq v_M^1$.
   iv. Using Excel Solver it was verified that $f^{t-2} < v_O^t$ for a large range of parameter values.

v. $v_O^t < v_M^{t-1}$ since $v_O^t = \theta v_M^t \big/ (\theta+1) < v_M^t < v_M^{t-1}$ where the last step follows from Technical Result 2.

vi. $v_M^t \le 1$ since $h_V < 1$ by Technical Result 1.

vii. $v_O^t < 1$ since $v_O^t = \theta v_M^t \big/ (\theta+1) < v_M^t < v_M^{t-1}$ and $v_M^t \le 1$ by vi.

viii. $0 < P^{F,t}$ follows from $d, e > 0$ by Technical Result 1.

ix. $0 < P^{V,t}$ follows from $d_V, e_V > 0$ by Technical Result 2.

c) IC – semi-anonymous constraints:

i. This requires $P^{V,t} < P^{F,t}$. This is met for a large range of parameter values. The constraint binds only if innovation is very rapid or the market is declining. For example, with $\rho = 0.5$ and $\varphi \ge 1$ the innovation rate can be as high as $\theta \approx 4.63$ without the constraint binding, while if $\theta = 2$ the market must decline at a rate of $\rho \approx 0.22$ for the constraint to bind.

ii. See i).

d) IC – intertemporal constraints:

i. The inequality simplifies to $xq^t \left(1-\delta_c\theta\right) \big/ \left(1-\delta_c\left(1-\rho\right)\right) > P^{F,t} - \delta_c P^{F,t+1}$.

Substituting the policy function for price: $x\left(1-\delta_c\theta\right) > g + hf^{t-1} - \delta_c\theta\left(g + hf^t\right)$.

Using the policy function for the marginal full price consumer: $x\left(1-\delta_c\theta\right) > f^t - \delta_c\theta\left(g + hf^t\right)$. This simplifies to $x > \left(1-\delta_c\theta h\right) f^t \big/ \left(1-\delta_c\theta\right) - \delta_c\theta g \big/ \left(1-\delta_c\theta\right)$ since $\delta_c\theta < 1$ by the Assumption. Using Excel Solver it was confirmed that this constraint is met for a wide range of parameter values subject to the Assumption.

ii. The inequality simplifies to $xq^t \left(\theta - \delta_c\theta^2 - \left(1-\delta_c\left(1-\rho\right)\right)\right) \big/ \left(\theta\left(1-\delta_c\left(1-\rho\right)\right)\right) > P^{V,t} - \delta_c P^{V,t+1}$. Substituting the policy function for price: $x\left(\theta - \delta_c\theta^2 - \left(1-\delta_c\left(1-\rho\right)\right)\right) >$ $\left(\theta-1\right)\left(g_V + h_V v_M^{t-1}\right) - \delta_c\theta\left(\theta-1\right)\left(g_V + h_V v_M^t\right)$. Using the policy function for the marginal middle-aged version upgrade consumer: $x\left(\theta - \delta_c\theta^2 - \left(1-\delta_c\left(1-\rho\right)\right)\right) > \left(\theta-1\right)v_M^t - \delta_c\theta\left(\theta-1\right)\left(g_V + h_V v_M^t\right)$. If this condition is met for $x = v_M^t$, it is met for $x > v_M^t$, therefore I substitute v_M^t for x. Then this simplifies to $v_M^t > \dfrac{g_V}{\theta/\left(\theta-1\right) - h_V + \left(1-\rho\right)/\left(\theta\left(\theta-1\right)\right)}$. Using Excel Solver it was confirmed that this constraint is met for a wide range of parameter values subject to the Assumption.

Appendix 2
Deviations

To determine whether each of the possible deviations was profitable for any values of the state variable I used Matlab to solve the Bellmans numerically for step sizes of 0.005 for f^{t-1}.[1] I solved each Bellman for all feasible combinations of $\delta_c, \delta_f \in \{0.1, 0.2, ...0.9\}$, $\varphi, \theta \in \{1.1, 1.3, ...1.9\}$ and $\rho = 0.5$ (set consumer's expected lifetime equal to time that she participates in each market).

1) The firm prices such that no middle-aged consumers buy the full price good (i.e., $f^t > f^{t-1}$). To accommodate this possible deviation the full price Bellman simplifies to:

(A1)

$$\tilde{V}_F^{D1}\left(f^{t-1}\right) = \max \left\{ \max_{P^{F,t}} \left[\left(1-f^t\right) P^{F,t} / q^t + \delta_f \theta \varphi \tilde{V}_F^{D1}\left(f^t\right) \right], \max_{P^{F,t}} \left[\Pi_F\left(f^{t-1}\right) + \delta_f \theta \varphi \tilde{V}_F^{D1}\left(f^{t-1}\right) \right] \right\}$$

,

where $\Pi_F\left(f^{t-1}\right)$ is the current full price profits in the Bellman of Equation 8) of the text. Although it was optimal for the firm to deviate for some parameter values, it was optimal only very close to the bgp share. Across all combinations of the parameters, the firm never deviated when its share was more than 5.1% above the bgp share. Moreover, it is optimal for the firm to set $f^t < f^{t-1}$ for all $0.505 < f^{t-1} \le 1.0$ regardless of parameter values. The intuition is that the monopolist does better by expanding output in each successive period as long as a sufficient number of high-valuation consumers remain in the market. When the market is substantially penetrated only low-valuation consumers remain and it is optimal for the firm to restrict output in each successive period.

2) The firm prices such that all previous middle-aged purchasers upgrade (i.e., $v_M^t = f^{t-1}$). In this case the Bellman is not separable in the full price good and version upgrade so to accommodate the possible deviation the Bellman is:

(A2)

$$\begin{aligned}\tilde{V}^{D2}\left(f^{t-1}, v_M^{t-1}\right) = \max \Bigg\{ & \max_{P^{F,t}, P^{V,t}} \left[\left[\left(1-f^t\right) + \left((1-\rho)/\varphi\right)\left(\left(1-f^t\right) - \left(1-f^{t-1}\right)\right) \right] P^{F,t}/q^t + \right. \\ & \left. \frac{(1-\rho)}{\varphi} \left[\left(1-f^{t-1}\right) + \left((1-\rho)/\varphi\right)\left(\left(1-v_O^t\right) - \left(1-v_M^{t-1}\right)\right) \right] P^{V,t}/q^t + \delta_f \theta \varphi \tilde{V}^{D2}\left(f^t, f^{t-1}\right) \right], \\ & \max_{P^{F,t}, P^{V,t}} \left[\Pi_F\left(f^{t-1}\right) + \Pi_V\left(v_M^{t-1}\right) + \delta_f \theta \varphi \tilde{V}^{D2}\left(f^t, v_M^t\right) \right] \Bigg\},\end{aligned}$$

[1] The Matlab code and output is available upon request from the author.

where $\Pi_V\left(v_M^{t-1}\right)$ is the current upgrade profits in the Bellman of Equation 8) of the text, $P^{V,t}=\max\left\{0,\frac{q^t-q^{t-1}f^{t-1}}{\left(1-\delta_c\left(1-\rho\right)\right)}\right\}$ and $v_O^t=\frac{\theta^2\left(1-\delta_c\left(1-\rho\right)\right)P^{V,t}}{q^t\left(\theta^2-1\right)}$. It was not profitable for the firm to deviate for any combination of parameter values. The intuition is that the monopolist does better by restricting sales of upgrades to only higher valuation consumers. Selling an upgrade to all middle-aged owners of the full price good requires an unprofitable reduction in the version upgrade price.

3) The firm prices such that the semi-anonymous constraint binds (i.e., $P^{F,t}=P^{V,t}$). In this case the Bellman is not separable in the full price good and version upgrade so to accommodate the possible deviation the Bellman is:

(A3) $$\tilde{V}^{D3}\left(f^{t-1},v_M^{t-1}\right)=\max\left\{\begin{array}{l}\max\limits_{P^{F,t}}\Bigg[\left[\left(1-f^t\right)+\left(\left(1-\rho\right)/\varphi\right)\left(\left(1-f^t\right)-\left(1-f^{t-1}\right)\right)\right]P^{F,t}/q^t+ \\ \quad\frac{\left(1-\rho\right)}{\varphi}\left[\left(1-v_M^t\right)+\left(\left(1-\rho\right)/\varphi\right)\left(\left(1-\theta v_M^t/\left(\theta+1\right)\right)-\left(1-v_M^{t-1}\right)\right)\right]P^{F,t}/q^t+\delta_f\theta\varphi\tilde{V}^{D3}\left(f^t,v_M^t\right)\Bigg], \\ \max\limits_{P^{F,t},P^{V,t}}\left[\Pi_F\left(f^{t-1}\right)+\Pi_V\left(v_M^{t-1}\right)+\delta_f\theta\varphi\tilde{V}^{D3}\left(f^t,v_M^t\right)\right]\end{array}\right\},$$

where $v_M^t=\min\left\{\frac{\left(1-\delta_c\left(1-\rho\right)\right)P^{F,t}}{q^t-q^{t-1}},1\right\}$. It was not profitable for the firm to deviate for any combination of parameter values. The intuition is that increasing the version upgrade price to equal the full price restricts output unprofitably.

4) The firm prices such that no old consumers upgrade (i.e., $v_O^t>v_M^{t-1}$). To accommodate this possible deviation the version upgrade Bellman simplifies to:

(A4)
$$\tilde{V}_V^{D4}\left(v_M^{t-1}\right)=\max\left\{\max_{P^{V,t}}\frac{\left(1-\rho\right)}{\varphi}\left[\left(1-v_M^t\right)P^{V,t}/q^t+\delta_f\theta\varphi\tilde{V}_V^{D4}\left(v_M^t\right)\right],\max_{P^{V,t}}\left[\Pi_V\left(v_M^{t-1}\right)+\delta_f\theta\varphi V_V^{D4}\left(v_M^t\right)\right]\right\}$$
.

Although it was optimal for the firm to deviate for some parameter values, it was optimal only very close to the bgp share. Across all combinations of the parameters, the firm never deviated when its share was more than 4.7% above the bgp share. Moreover, it is optimal for the firm to set $v_O^t<v_M^{t-1}$ for all $0.555<v_M^{t-1}\le 1.0$ regardless of parameter values. The intuition is that the monopolist does better by expanding output in each successive period as long as a sufficient number of high-valuation consumers remain in the market. When the market is substantially

penetrated only low-valuation consumers remain and it is optimal for the firm to restrict output in each successive period.

Appendix 3
Proofs of Results

Technical Result 1: The Assumption ensures that the radicals of g and h are real and that the denominator of g is positive (the denominator of h is positive even without the Assumption). This implies $0 < g, h$. $0 < d, e$ immediately follows by inspection. The Assumption combined with the fact that the radicals are positive implies that $g, h < 1$.

Technical Result 2: The radical is real when $\left(\theta(1-\rho)+\varphi(\theta+1)\right)^2 > \delta_f \varphi\theta(1-\rho)^2(\theta+1)^2$. Since $\delta_f \varphi\theta < 1$ by the Assumption and $0 < 1-\rho < 1$, this is met when the market is growing (i.e., $\varphi > 1$) by inspection. Using Excel Solver it was confirmed that this is also met when the market is shrinking (i.e., $\varphi < 1$) for all parameter values subject to the Assumption. To show that the denominator of g_V is positive (the denominator of h_V is positive since the radical is positive), take the square root of both sides of the above expression:
$\theta(1-\rho)+\varphi(\theta+1) > \sqrt{\delta_f \varphi\theta}(1-\rho)(\theta+1)$. Since $\delta_f \varphi\theta < 1$ by the Assumption and rearranging this implies: $\theta(1-\rho)+\varphi(\theta+1) > \delta_f \varphi\theta(1-\rho)(\theta+1)$ .Thus, $0 < g_V, h_V$. $0 < d_V, e_V$ immediately follows by inspection. Using Excel Solver it was confirmed that $g_V, h_V < 1$ for all parameter values.

Technical Result 3: First note that: $(1-\rho+\varphi)^2 > \varphi(1-\rho+\varphi) - \varphi(1-\rho)/(2\theta+1)$. Multiplying by $(2\theta+1)$ gives:
$(2\theta+1)(1-\rho+\varphi)^2 > (2\theta+1)\varphi(1-\rho+\varphi) - \varphi(1-\rho)$. Adding $\theta^2(1-\rho+\varphi)^2$ and doing some minor rearranging yields: $\left(\theta^2+2\theta+1\right)(1-\rho+\varphi)^2 > \theta^2(1-\rho+\varphi)^2 + 2\theta\varphi(1-\rho+\varphi) + \varphi^2$.
Factoring this we obtain:
$(\theta+1)^2(1-\rho+\varphi)^2 > \left(\theta(1-\rho+\varphi)+\varphi\right)^2$. Subtracting $\delta_f \varphi\theta(1-\rho)^2$ from both sides and since both sides are positive by Technical Results 1 and 2: $(\theta+1)\sqrt{(1-\rho+\varphi)^2 - \delta_f \varphi\theta(1-\rho)^2} >$
$\sqrt{\left(\theta(1-\rho+\varphi)+\varphi\right)^2 - \delta_f \varphi\theta(1-\rho)^2(\theta+1)^2}$. Adding $(1-\rho+\varphi)$ to the left hand side and φ to the right hand side we get an intermediate result to prove both parts of the result:
$(1-\rho+\varphi)+(\theta+1)\sqrt{(1-\rho+\varphi)^2 - \delta_f \varphi\theta(1-\rho)^2} >$
$\varphi + \sqrt{\left(\theta(1-\rho+\varphi)+\varphi\right)^2 - \delta_f \varphi\theta(1-\rho)^2(\theta+1)^2}$. For $h_V > h$, add $\theta(1-\rho+\varphi)$ to both sides:
$(\theta+1)(1-\rho+\varphi)+(\theta+1)\sqrt{(1-\rho+\varphi)^2 - \delta_f \varphi\theta(1-\rho)^2} >$

$\theta(1-\rho+\varphi)+\varphi+\sqrt{(\theta(1-\rho+\varphi)+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2(\theta+1)^2}$. Rearranging (the denominators are positive by Technical Results 1 and 2) and multiplying both sides by $(1-\rho)$:

$$\frac{(\theta+1)(1-\rho)}{\theta(1-\rho+\varphi)+\varphi+\sqrt{(\theta(1-\rho+\varphi)+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2(\theta+1)^2}}>$$

$$\frac{(1-\rho)}{(1-\rho+\varphi)+\sqrt{(1-\rho+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2}}$$ which proves the result.

For $g_V > g$, add $\theta(1-\rho+\varphi)$ to both sides of the intermediate result and subtract $(\theta+1)\delta_f\varphi\theta(1-\rho)$ from both sides:

$$(\theta+1)(1-\rho+\varphi)-(\theta+1)\delta_f\varphi\theta(1-\rho)+(\theta+1)\sqrt{(1-\rho+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2}>$$

$\theta(1-\rho+\varphi)+\varphi-(\theta+1)\delta_f\varphi\theta(1-\rho)+\sqrt{(\theta(1-\rho+\varphi)+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2(\theta+1)^2}$. Rearranging (the denominators are positive by Technical Results 1 and 2) and multiplying both sides by φ:

$$\frac{(\theta+1)\varphi}{\theta(1-\rho+\varphi)-\delta_f\varphi\theta(1-\rho)(\theta+1)+\varphi+\sqrt{(\theta(1-\rho+\varphi)+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2(\theta+1)^2}}>$$

$$\frac{\varphi}{(1-\rho+\varphi)-\delta_f\varphi\theta(1-\rho)+\sqrt{(1-\rho+\varphi)^2-\delta_f\varphi\theta(1-\rho)^2}}$$ which proves the result.

Result 1: Leapfrogging occurs when some old consumers eligible to purchase the version upgrade do not, while some old consumers with lower valuations purchase (i.e., $f^{t-1} < v_M^t$ and $f^t < f^{t-1}$). Parts b) ii) and iii) of Appendix 1 shows that leapfrogging occurs subject to the Assumption, the firm's market shares in both segments being below the bgp market shares, and the firm beginning with a weakly greater market share for the full price good than the version upgrade. FT (1998) show that leapfrogging is not possible with zero marginal cost in a two-period model (their proposition 3, p. 253). Part e) of the "Constraints on Equilibrium" discusses the range of parameter values for which a leapfrogging equilibrium exists. The final part of the theorem is shown by modifying FT's Lemma 5 (page 251) by introducing new consumers in the second period in proportion ρ to the original uniform distribution of consumers $\theta\in[0,1]$. Using FT's notation, $\theta^*=\frac{1}{2}$ (the unconstrained optimal cutoff for version upgrades), $\theta^m(\theta_1)=\frac{\theta_1+\rho}{2(1+\rho)}$ (cutoff for sales of the full price good in the second period) and

$$\theta_1=\frac{(1+\delta)(1+\rho)^2V_L-\delta\left((1+\rho)^2-1/2\right)V_H}{2(1+\delta)(1+\rho)^2V_L-\delta\left(2(1+\rho)^2-1/2\right)V_H}$$ (cutoff for sales of the full price good in the first period). $\theta_1>\theta^*$ for all values of ρ so that leapfrogging is not possible.

Result 2: Part b) vi) of Appendix 1 shows that sales of version upgrades exceed zero subject to the Assumption.

Result 3: On the bgp, $P^{V,t}=\left[d_V+e_V\left(g_V/\left(1-h_V\right)\right)\right]q^t$ and $P^{F,t}=\left[d+e\left(g/\left(1-h\right)\right)\right]q^t$. Using Excel Solver it was confirmed that the first expression is less than the second for all parameter values subject to the Assumption and $\varphi\geq 1$. FT (1998) provide the range of innovation rates for which the semi-anonymous constraint binds with zero marginal production cost in a two-period model of upgrade pricing (equation following equation 18, page 252). For a uniform distribution of consumers such as I employ, the constraint binds for $\theta\geq 2$.

Result 4: a) Follows directly from the fact that $\partial d/\partial\theta>0$ and $\partial e/\partial\theta>0$, which are true by inspection. b) Follows directly from the fact that $\partial d/\partial\delta_f>0$ and $\partial e/\partial\delta_f>0$, which are true by inspection. c) Follows directly from the fact that $\partial d/\partial\delta_c>0$ and $\partial e/\partial\delta_c>0$, which are true by inspection. d) Follows directly from $e>0$ as shown in Technical Result 1.

Result 5: a) d_V can be rewritten as:

$$\frac{\varphi^2\left(1-1/\theta^2\right)}{\left(1+\varphi-\rho+\varphi/\theta-\delta_f\varphi(1-\rho)(\theta+1)+\sqrt{\left(1+\varphi+\rho+\varphi/\theta\right)^2-\delta_f\varphi(1-\rho)^2\left(\theta+2+1/\theta\right)}\right)}.$$

The numerator is increasing and the denominator is decreasing in θ. Similarly, e_V can be rewritten as:

$$\frac{(1-\rho)\left(1-1/\theta^2\right)}{\left(1+\varphi-\rho+\varphi/\theta+\sqrt{\left(1+\varphi+\rho+\varphi/\theta\right)^2-\delta_f\varphi(1-\rho)^2\left(\theta+2+1/\theta\right)}\right)}.$$

The numerator is increasing and the denominator is decreasing in θ. b) Follows directly from the fact that $\partial d_V/\partial\delta_f>0$ and $\partial e_V/\partial\delta_f>0$, which are true by inspection. c) Follows directly from the fact that $\partial d_V/\partial\delta_c>0$ and $\partial e_V/\partial\delta_c>0$, which are true by inspection. d) Follows directly from $e_V>0$ as shown in Technical Result 2.

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