Dynamics of the Expectation and Risk Premium in the OIS Term Structure

Suresh Sundaresan, Zhenyu Wang, Wei Yang*

March 1, 2016

Abstract

The OIS yield curve alternated between positive and negative slopes during 2002–2015, as the economy went through the boom, recession, and recovery. We introduce a term structure model that incorporates the Fed open market operations and preserves the economic foundation and analytical tractability of the traditional CIR model. The model accounts for the dynamics of the market expectation of monetary policy while keeping the risk premium positive. The expectation explains the inversion of the yield curve before the financial crisis. The dynamics of the risk premium is consistent with the Fed's policy uncertainty before and after the crisis.

Keywords: Term structure, OIS, monetary policy, FOMC, Bayesian inference, MCMC
JEL classification codes: G12

*Sundaresan is with the Graduate School of Business at Columbia University; Wang and Yang are with the Kelley School of Business at Indiana University. Email: ms122@columbia.edu, zw25@indiana.edu, and weiyang1@indiana.edu. We are grateful for comments by the seminar participants at Indiana University and the Bank for International Settlements. The paper has benefited from the support by the High Performance Computing Facilities of Indiana University.

Contents

1 Introduction

An overnight index swap (OIS) is a contract that exchanges a fixed interest rate, which is referred to as an OIS rate, for a floating overnight rate. The market for OIS on the overnight federal funds rate has developed rapidly since 2002, and the OIS rates have gradually become new benchmarks for short-term interest rates.[1] During the 2007–2009 financial crisis, the spread between the three-month London interbank offered rate (Libor) and the three-month OIS rate was a widely watched index of credit risk in the interbank loan market. This spread, referred to as the Libor-OIS spread, was viewed as a barometer of the financial crisis. In the Term Auction Facility, which was the first emergency liquidity program initiated by the Federal Reserve (Fed) during the financial crisis, the one-month OIS rate served as the minimum bidding rate in the auctions of one-month loans.[2] In the post-crisis era, financial markets moved to adopt the OIS rates as the benchmark short-term risk-free discount rates for valuation of derivative securities and collateral.

The term structure of the OIS rates varied dramatically and alternated between positive and negative slopes several times during 2002–2015. The slope was positive in 2004–2005 and then became negative in late 2006 and stayed negative for more than a year. By the end of 2008, the slope of the OIS rates resumed to be positive. These movements occurred over a period roiled by a severe financial crisis and a tumultuous history of monetary policy over 110 meetings of the Federal Open Market Committee (FOMC). During this period, the economy went through the recovery in 2002–2004, the real estate boom in 2005–2006, the Great Recession in 2007–2009, and the near-zero interest rate monetary policy since December 2008. The variation of the overnight federal funds rate during this period is phenomenal: it dropped to 1% in 2003, climbed above 5% in 2006, and then plunged below 25 basis points (bps) in late 2008. This phenomenal movement was driven by the Fed's monetary policy, which changed from the gradual tightening in 2004 and 2005 to the multiyear extraordinary easing starting from late 2008.

Because of the important role of the OIS rates in modern financial markets and the dramatic variation of the OIS yield curve, we study the dynamics of the OIS term structure to better understand its link to monetary policy. For this study, we introduce a simple dynamic model of the OIS term structure. The model preserves the general equilibrium framework of the CIR model derived by Cox, Ingersoll, and Ross (1985), which aggregates all the information about production and consumption into the instantaneous interest rate that has a time-varying

[1] There are also OIS contracts that are denominated in other currencies. For example, an OIS denominated in Euros uses the Euro Overnight Index Average (EONIA) as the floating rate. An OIS denominated in British pounds uses the Sterling Overnight Index Average (SONIA) as the floating rate. We focus on the U.S. dollar OIS in this paper, but our model and econometric methodology can be applied to the OIS denominated in other currencies.

[2] See McAndrews, Sarkar, and Wang (2015) for more details about the Libor-OIS spread and the Term Auction Facility during the 2007–2009 financial crisis.

reversion center. However, our model adapts the reversion center to the Fed's open market operations, which keep the average overnight federal funds rate at the target set by the FOMC. More specifically, we assume that the reversion center of the instantaneous rate is a step function over time: the reversion center is constant during each FOMC intermeeting period (the time period between two consecutive FOMC meetings) but can take different values in separate intermeeting periods. As the result of adapting the reversion center to the Fed's open market operations, the CIR model is consistent with the Fed's implementation of monetary policy. For convenience, we refer to our model as *the Fed-CIR model*.

The restriction imposed on the time-varying reversion center brings two additional advantages — the Fed-CIR model is econometrically identifiable and admits a closed-form pricing formula. The original CIR model with a general time-varying reversion center does not admit a closed-form pricing formula and is generally not identified. Existing studies of the CIR model usually assume a constant reversion center because this assumption produces a closed-form pricing formula convenient for econometric estimation.[3] However, the CIR model with a constant reversion center fits the observed interest rates poorly, unless it is extended to have multiple factors.[4] Compared to existing implementations of the CIR model, the Fed-CIR model improves flexibility with the step function of the reversion center. Moreover, the closed-form pricing formula facilitates econometric implementation. We estimate the Fed-CIR model using daily data of OIS rates, and our econometric methodology is based on Bayesian inference and the Markov chain Monte Carlo (MCMC) sampling.

1.1 Summary of the Results

Although the Fed-CIR model is simple and has only a single factor, the step function of the reversion center makes it flexible in accounting for the dynamics of investors' expectation and the risk premium in the term structure of the OIS rates. The Fed-CIR model fits the term structure well, and importantly, suggests that the variation in the slope is mostly determined by investors' expectation of monetary policy. The model shows that the expectation of the Fed's tightening of monetary policy explains the positive slope of the OIS curve during 2004 and 2005. More notably, the expectation of the Fed's easing of monetary policy explains the negative slope of the OIS curve in 2006 and 2007. The estimated reversion center in the Fed-

[3]See Brown and Dybvig (1986), Gibbons and Ramaswamy (1993), Pearson and Sun (1994), Lamoureux and Witte (2002), among others.

[4]The literature reports that at least three factors are necessary for explaining the term structure of interest rates. Litterman and Scheinkman (1991) find that three principal components are necessary to capture the variation in the U.S. Treasury yields. Dai and Singleton (2002) compare the goodness-of-fit of various multifactor models and choose the three-factor Gaussian model with a flexible specification of the market price of risk. We also report in Section 5 that the single-factor CIR model with a constant reversion center performs poorly in explaining the dynamics of the OIS rates.

CIR model shows that the market expectation in 2006 and 2007 was ahead of (i.e., anticipates) the Fed's decisions. The market was however behind the Fed in the second half of 2008 when the Fed surprised the market by holding the federal funds rate unchanged in the August and September FOMC meetings and then by cutting the rate rapidly toward a range near zero by the December meeting. Finally, the model clearly shows the market expectation of the monetary policy normalization in 2015.

The risk premium estimated from the Fed-CIR model fluctuated over time but was always positive in the entire sample period. The positive risk premium is consistent with economic theories in which investors are risk averse. The risk premium as a fraction of the OIS rate peaked at the end of 2007 when the economy was entering into the recession and when the uncertainty over the Fed's action was arguably the highest. Once the Fed cut the target rate to the 0–25 bps range in December 2008 and clearly assured the market of the exceptionally low level of interest rates for an extended time period, the risk premium plunged. Taken together, the rise and fall of the risk premium estimated from the Fed-CIR model are consistent with the uncertainty of the Fed's monetary policy before and during the financial crisis.

In contrast to the results obtained from the Fed-CIR model, either the expectation or the risk premium, or both, estimated from other existing term structure models are counterintuitive. We have estimated a wide range of term structure models using the OIS data. These models have been reported in the literature to be successful in fitting the term structure of Treasury bond yields. All these models have multiple factors. Some of them have flexible market prices of risks, and others have stochastic volatilities or jumps. In this paper, we present the empirical results for a few representative models, including a two-factor CIR model derived by Cox et al. (1985), a three-factor Gaussian model recommended by Dai and Singleton (2002), and a four-factor jump model with stochastic volatility proposed by Piazzesi (2005).[5]

We find that multifactor CIR and Gaussian models explain the dynamics of the OIS term structure almost entirely through variations of the risk premium rather than through changes in the expectation. According to these models, the risk premiums in the OIS rates decreased sharply in 2007 when the economy approached the recession, and then became negative at the end of 2007 and the beginning of 2008 when the market was waiting for the Fed's further response to the still unfolding financial crisis. The negative risk premium is not only difficult to interpret given the uncertainty of future interest rates at the time, but is also inconsistent with equilibrium models in which investors are risk averse. In addition, there is hardly any

[5]All these models are affine term structure models. For the characterization, taxonomy, and empirical survey of affine term structure models, see Duffie and Kan (1996), Dai and Singleton (2000), and Piazzesi (2009). There are some deviations from affine models: Duarte (2004) proposes a semi-affine model, and Ahn, Dittmar, and Gallant (2002) present a quadratic Gaussian model. In another development, regime-switching term structure models [Bansal and Zhou (2002), Ang and Bekaert (2002), and Dai, Singleton, and Yang (2007), among others] suggest that factors following conventional dynamics but with different parameters across a small number of discrete regimes generate improved fit to the term structure.

relation between the expectations estimated from these models and the variation of the OIS term structure. This result is at odds with the economic intuition that short-term interest rates, such as the OIS rates in our sample, should be mainly determined by the expectation of future interest rates.

We also find it difficult to interpret the results of multifactor models that postulate jump processes for the target rate. These models, while far more complicated, are similar to the Fed-CIR model in assuming that the target rate may change on FOMC meeting dates. A well-known model of this kind is the four-factor jump model with stochastic volatility proposed by Piazzesi (2005). This model cannot be directly applied to the period in which the target rate is a range instead of a specific level. For this reason, we estimate the four-factor jump model using only the data before December 16, 2008. Similar to our Fed-CIR model, the risk premium estimated in the jump model is positive and the expectation component covaries with the OIS term structure. However, the risk premium stayed essentially constant over the sample period, independent of the state of the economy and monetary policy.

1.2 Relation to the Literature

Our study supports the view that the expectation of monetary policy is the major driver of the short-term OIS term structure. This view is economically compelling — since short-term interest rates have small duration risk, the expectation should be the major determinant and the risk premium should be relatively small.

Our study contributes to a growing line of studies that emphasize the role of monetary policy and more broadly, macroeconomic information, in explaining the term structure of interest rates. For example, Ang and Piazzesi (2003), Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2008), Joslin, Le, and Singleton (2013), Joslin, Priebsch and Singleton (2014) incorporate macroeconomic variables as observed factors into term structure models. Piazzesi (2005) models the target rate as a jump process. Heidari and Wu (2010) and Kim and Wright (2014) assume factors jump at macroeconomic events. Hördahl et al. (2015) argue that revisions in the expectation account for a significant part of the variations in the yield curve at macroeconomic news announcements.

Our results accentuate the difference between the term structures of long- and short-term interest rates. In the studies of long-term interest rates, risk premiums have been demonstrated to be important in term structure modeling. Duffee (2002) and Dai and Singleton (2002) emphasize that a model must accommodate flexible variations in risk premiums in order to fit the term structure of Treasury bonds. Piazzesi (2009) surveys the affine term structure models and suggests that negative risk premiums are probably necessary for explaining the Treasury yields. It is reasonable that the risk premium is an important component in long-term

Treasury yields because long-term rates have large duration risk. It is also possible that the risk premium in Treasury yields may become negative because of the demand for the special services provided by Treasury securities as liquid assets or money in financial markets and for financial institutions.

However, those arguments for the importance of the risk premium do not apply to the short-term OIS rates. Singleton (2006) recognizes the special challenge in fitting short-term interest rates. In particular, he conjectures that, besides three factors in most studies, a fourth factor may be necessary to capture the very short end of the yield curve. Our results further shed light on the different challenges in explaining short-term and long-term interest rates.

The rest of the paper is organized as follows. Section 2 introduces the Fed-CIR model of the OIS term structure. Section 3 describes the data and econometric method for the empirical study of the Fed-CIR model. Section 4 presents the main empirical results of the Fed-CIR model. Section 5 compares the Fed-CIR model with several representative multifactor models. Section 6 concludes and discusses potential future research.

2 A Simple Dynamic Model of the OIS Term Structure

2.1 The OIS Rates and the Fed's Open Market Operations

As described at the beginning of the paper, an overnight index swap is an interest rate swap contract that exchanges a fixed interest rate, referred to as the OIS rate, with a floating overnight interest rate, which is the effective federal funds rate. A fairly priced short-term OIS rate is equal to the yield on a discount bond of the same term. We formally show this equivalence in Appendix A.1.

The equivalence between OIS rates and discount bond yields makes short-term OIS rates ideal for the estimation of dynamic term structure models. These models specify pricing formulas of discount bond yields and thus would require extraction of the discount bond yield curve if the data are coupon bond rates. For example, because interest rate swaps on Libor have multiple payment periods, researchers have to bootstrap discount bond yield curves from swap curves.

Bootstrapping can be complicated and potentially imprecise, and the method used in bootstrapping may affect econometric inferences. Fama and Bliss (1987) use two different methods to bootstrap two sets of discount bond yields from the same set of coupon bond rates. Singleton (2006) compares the two sets of yields and show that they have considerably different statistical properties and lead to very different empirical results. By contrast, short-term OIS rates are discount bond yields and can be used directly in the estimation of dynamic term structure models. Our data are free from potential distortions of bootstrapping procedures.

The short-term OIS rates can also be viewed as yields on bonds that have negligible credit risk. OIS contracts carry little or even no credit risk for two major reasons: first, the two sides of the contract do not exchange the principal; second, the accrued interest loss is covered by cash collateral. The premium for illiquidity is also negligible in the OIS rates due to active trading. In fact, the financial industry has adopted the OIS rates as benchmark risk-free rates and for collateral valuation. In Appendix A.1, we provide more details about the OIS market and the industry practice.

Short-term interest rates are closely linked to the Fed's monetary policy and its open market operations. The FOMC schedules eight meetings per year, one about every six weeks. The exact meeting dates are unknown until the Fed announces the schedule through its press release. At the end of each meeting, the FOMC issues a policy statement proclaiming its decision on the target of the overnight federal funds rate and the forward guidance on the future path of the policy. Consequently, investors revise their view of the state of the economy and expectation about the future course of interest rates.

A short-term OIS is exposed to the risk of a potential change in the target rate if there are FOMC meetings before the maturity date. An FOMC meeting is not relevant to the OIS rate if the meeting is after the maturity date. Therefore, the precise information about the timing of potential changes in monetary policy is important in pricing short-term OIS rates. By contrast, exact meeting dates are perhaps less important in pricing long-term bonds, whose maturity dates are usually far beyond the scheduled FOMC meeting dates. Incorporating the schedule of FOMC meetings into modeling is a unique challenge to the analysis of the term structure of short-term OIS rates.

2.2 The CIR Model with Time-Varying Reversion Center

To set the stage for our model, we first review the classic *CIR model* introduced by Cox, Ingersoll and Ross (1985). The CIR model is a dynamic model of the term structure of interest rates with a sound economic foundation. It is derived from a general equilibrium framework with a production technology and risk averse agents. In this model, the instantaneous interest rate r_t at time t follows a mean-reverting, square-root process:

$$dr_t = \kappa(\vartheta_t - r_t)\,dt + \sigma\sqrt{r_t}\,dw_t \ , \tag{1}$$

where κ and σ are positive constants, and ϑ_t is a positive, deterministic function of time. The randomness of the interest rate is driven by a Wiener process w_t. The diffusion coefficient $\sigma\sqrt{r_t}$ is the local volatility. In the model, the instantaneous rate r_t is never negative. It reverts to ϑ_t, and the reversion is faster if κ is larger. Thus, ϑ_t is referred to as the reversion center of the instantaneous rate, and κ is the reversion speed.

In the CIR model, the time-t price of one dollar to be paid at time $t+s$ is

$$p_t(s) = \exp\big(c_t(s) - b(s)r_t\big), \tag{2}$$

$$b(s) = 2(\exp(\gamma s) - 1)/[\delta(\exp(\gamma s) - 1) + 2\gamma], \tag{3}$$

$$c_t(s) = -\kappa \int_0^s \vartheta_{t+u} b(u) du, \tag{4}$$

where $\gamma^2 = (\kappa+\lambda)^2 + 2\sigma^2$ and $\delta = \gamma + \kappa + \lambda$, respectively. The parameter λ is the market price of risk, and a negative λ implies a positive risk premium. In general, $c_t(s)$ in equation (4) does not guarantee a closed-form pricing formula for equation (2).[6]

If ϑ_t is a constant, denoted by θ, then stochastic process (1) is the well-known *single-factor CIR model*:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\, dw_t\ . \tag{5}$$

The literature often refers to the constant θ as the "long-run mean" of the instantaneous rate. With a constant θ, $c_t(s)$ in equation (4) is independent of t and can be obtained analytically, which leads to the closed-form pricing formula:

$$p_t(s) = \exp(a(s)\theta - b(s)r_t)\ , \tag{6}$$

$$a(s) = (2\kappa/\sigma^2)\big[\log(2\gamma) - \log\big(\delta(\exp(\gamma s) - 1) + 2\gamma\big) + \delta s/2\big]. \tag{7}$$

It follows that the yield at time t on a discount bond maturing at time $t+s$ is

$$y_t(s) = [b(s)r_t - a(s)\theta]/s, \tag{8}$$

which is an affine function of the instantaneous rate. The closed-form pricing formula is instrumental in the estimation, as demonstrated by Brown and Dybvig (1986), Gibbons and Ramaswamy (1993), Pearson and Sun (1994), Lamoureux and Witte (2002), and numerous others.

In the single-factor CIR model, the yield on a discount bond is also mean-reverting. To find the reversion center of the yield, from equations (8) and (5) and using Ito's formula, we have

$$dy_t(s) = \kappa[\xi(s) - y_t(s)]dt + s^{-1}b(s)\sigma\sqrt{r_t}dw_t, \tag{9}$$

$$\xi(s) = \theta[b(s) - a(s)]/s, \tag{10}$$

which suggests $\xi(s)$ as the reversion center of the yield. Let $s = h$, the term of the overnight federal funds rate, then $\xi(h)$ is the reversion center of the federal funds rate.[7] A constant θ in the single-factor CIR model results in a constant reversion center $\xi(h)$ for the federal funds rate. This is clearly inconsistent with the Fed's monetary policy that has moved the target of the overnight federal funds rate up and down substantially over time. Since the Fed's monetary policy and the resulting target may be different in separate intermeeting periods, the reversion center of the federal funds rate may be different. This, in view of equation (10), implies that the

[6]More details of the CIR model and its pricing formula can be found in Cox et al. (1985) starting on page 295.

[7]Note that money market interest rates, such as the overnight federal funds rate, are not the instantaneous rate, even if maturities are very short.

reversion center of the instantaneous rate may also be different. Motivated by this intuition, we present a simple model that incorporates the effect of the Fed's open market operations.

2.3 Adaptation to the Fed's Open Market Operations

Our model incorporates the Fed's open market operations into the original CIR model with the time-varying reversion center. We impose a structure on the time variation of the reversion center so as to be consistent with the economic environment in which the Fed adjusts the target only at the FOMC meetings. Specifically, in our model, the reversion center of the instantaneous rate is constant during each intermeeting period but can be different in separate intermeeting periods. As shown below, this simple structure not only accommodates the effect of the Fed's open market operations, but also delivers a convenient closed-form pricing formula.

More precisely, we assume that ϑ_t is a step function of time, where the steps are determined by the scheduled FOMC meetings. Let τ_j for $j = 0, \cdots, N$ be the scheduled FOMC meeting dates during the sample period. The Fed announces a monetary policy decision in meeting j at time τ_j. Let θ_j be the reversion center of the instantaneous rate during the intermeeting period $(\tau_{j-1}, \tau_j]$. The step function of ϑ_t is

$$\vartheta_t = \sum_{j=0}^{N} \theta_j I(\tau_j < t \leq \tau_{j+1}), \tag{11}$$

where function $I(\cdot)$ is an indicator that equals 1 if the condition holds, and 0 otherwise. A meeting time τ_j is a discontinuity point of ϑ_t if $\theta_j \neq \theta_{j+1}$, and a continuity point if $\theta_j = \theta_{j+1}$. Since the FOMC decision is announced after the meeting, the step function is right continuous. Taken together, we call equation (11), along with equation (1), *the Fed-CIR model*. It is a special CIR model that specifies ϑ_t as a step function adapted to the Fed's open market operations.

The pricing formula of the Fed-CIR model involves careful accounting of the scheduled FOMC meetings. Figure 2 illustrates the FOMC meeting dates and the steps of ϑ_t involved in pricing, at time t, a discount bond that matures at time $t+s$. Let τ_{j_t} be the last meeting date prior to time t. Suppose that there are n scheduled FOMC meeting dates $\tau_{j_t+1}, \cdots, \tau_{j_t+n}$, such that $t < \tau_{j_t+1} < \cdots < \tau_{j_t+n-1} < t+s \leq \tau_{j_t+n}$. These scheduled FOMC meetings divide the time interval $(t, t+s]$ into n subintervals. In each subinterval, the reversion center is constant. Denote the n values of ϑ_t by a vector: $\theta_t(s) = (\theta_{j_t+1}, \cdots, \theta_{j_t+n})'$.[8]

Assuming that the instantaneous rate follows stochastic process (1) and that the reversion center is step function (11), the time-t price of one dollar to be paid at time $t+s$ is

$$p_t(s) = \exp\left(a_t(s)'\theta_t(s) - b(s)r_t\right), \tag{12}$$

[8] The number of subintervals, n, depends on both t and s. For example, $n = 1$ if $t+s \leq \tau_{j_t+1}$. To be precise, we should use notation $n_{t,s}$ for the number of intermeeting periods in $(t, t+s]$, but we simply use n as it does not cause confusion.

where r_t is the instantaneous rate at time t. The n-dimensional vector $a_t(s)$ is given by

$$a_t(s) = (a_{t1}(s), \cdots, a_{tn}(s))', \tag{13}$$

$$\begin{aligned} a_{tj}(s) = (2\kappa/\sigma^2)\big\{ & \log\big[\delta(\exp(\gamma(s-s_j))-1)+2\gamma\big] \\ & -\log\big[\delta(\exp(\gamma(s-s_{j-1}))-1)+2\gamma\big]+\delta(s_j-s_{j-1})/2\big\}, \end{aligned} \tag{14}$$

where $s_0 = 0$, $s_j = \tau_{j_t+j} - t$ for $j = 1, \cdots, n-1$, and $s_n = s$. We can derive the pricing formula from equation (2) by showing $c_t(s) = a_t(s)'\theta_t(s)$. The latter equation can be obtained by substituting step function (11) into equation (4) and then integrating over each subinterval separately. A pricing formula very similar to equations (12)–(14) is originally derived by Wang (1996), in which he matches the step function with the swap maturity dates in order to better calibrate the CIR model for swaps. The new perspective in our application of the CIR model to the OIS is to link the step function of the reversion center to the Fed's open market operations.

If there is no scheduled FOMC meeting between t and the maturity time $t+s$, i.e., $t+s \le \tau_{j_t+1}$, then $\theta_t(s) = \theta_{j_t+1}$, and equations (12)–(14) reduce to equations (6) and (7), the pricing formula for the single-factor model with $\theta = \theta_{j_t+1}$ as the constant reversion center.[9] Thus, the Fed-CIR model is an extension of the single-factor CIR model, and it is much more flexible than the constant θ in the latter.

The Fed-CIR model is not an affine term structure model, although it may appear so if one imagines that ϑ_t is a latent factor in addition to r_t. Duffie and Kan (1996) emphasize that an important property of affine models is the "time-homogeneity:" both $a(s)$ and $b(s)$ depend only on the time to maturity s but not on the current time t. In the Fed-CIR model, however, $a_t(s)$ changes with t. Therefore, the Fed-CIR model presents an alternative to affine term structure models. In fact, as we will show later, the results of the Fed-CIR model are very different from an array of affine term structures models.

The pricing formula for the discount bond implies a closed-form formula for the yield:

$$y_t(s) = [b(s)r_t - a_t(s)'\theta_t(s)]/s. \tag{15}$$

As shown in the formula, the discount bond yield impounds information about future reversion centers. In other words, the yield reflects both the monetary policy set by the Fed for the current intermeeting period and the expectation on the monetary policy to be set in the scheduled future FOMC meetings up to time $t+s$. The coefficients $a_t(s)$ determine the loadings of the relevant reversion centers associated with future intermeeting periods.

If the market price of risk λ is nonzero, then investors demand a risk premium in the yield for the uncertainty about the future instantaneous rate. If λ is zero, then there is no risk premium, and the yield is solely determined by the expectation of the future path of the instantaneous rate. We can decompose the yield in equation (15) into two components: the

[9] Note that $b(s)$ in equation (12) is the same as in equation (3), but $a_t(s)$ in (14) is generally different from $a(s)$ in equation (7) if there are scheduled FOMC meetings during $(t,\ t+s)$.

expectation component and the risk premium component. We obtain the expectation component, $\bar{y}_t(s)$, by setting λ to zero in equation (15) while fixing all other parameters. The risk premium component is then $\tilde{y}_t(s) = y_t(s) - \bar{y}_t(s)$. The expectation component can go up or down with maturity, reflecting the market anticipation of rising or falling interest rates. The risk premium component is positive if $\lambda < 0$, or equivalently, investors are risk averse. The magnitudes of both components may depend on the level of the interest rate. We compute the proportions $\bar{y}(s)/y_t(s)$ and $\tilde{y}(s)/y_t(s)$ to examine the significance of the two components as parts of the interest rate.

The term spread between two yields can also be decomposed into two components. Let $s_1 < s_2$ be the terms of two yields. The expectation component of the term spread is $\bar{y}_t(s_2) - \bar{y}_t(s_1)$, and the risk premium component of the term spread is $\tilde{y}_t(s_2) - \tilde{y}_t(s_1)$. A positive or negative expectation component of the term spread implies the market anticipation of rising or falling interest rates. Since duration risk increases with maturity, economic intuition suggests that the risk premium component of the term spread should be positive.

In the Fed-CIR model, following the original CIR model, the reversion center ϑ_t is a deterministic function of time. An alternative is to assume that ϑ_t is a stochastic factor. This, as shown below, leads to models that are similar or equivalent to those we estimate to the OIS data and report in Section 5. Let $u_t = r_t - \vartheta_t$ be the deviation of the instantaneous rate from the reversion center. If ϑ_t and u_t are mean-reverting processes with stochastic volatilities:

$$d\vartheta_t = \kappa_\theta(\bar{\vartheta} - \vartheta_t) + \sigma_\vartheta \sqrt{\vartheta_t}\, dw_t^\vartheta, \tag{16}$$

$$du_t = \kappa_u(0 - u_t) + \sqrt{\alpha^2 + \beta^2 \vartheta_t}\, dw_t^u, \tag{17}$$

then the model belongs to the $A_1(2)$ class of two-factor affine models characterized in Dai and Singleton (2000). Our empirical analysis indicates that this model performs similarly to the two-factor CIR model derived by Cox et al. (1986). We can also model ϑ_t and u_t as Gaussian processes with constant volatilities:

$$d\vartheta_t = \kappa_\vartheta(\bar{\vartheta} - \vartheta_t) + \sigma_\vartheta dw_t^\vartheta, \tag{18}$$

$$du_t = \kappa_u(0 - u_t) + \sigma_u dw_t^u, \tag{19}$$

which results in a multifactor Gaussian model studied in Dai and Singleton (2002). If we model ϑ_t as a jump process along with other factors, then we obtain a multifactor jump model similar to that of Piazzesi (2005). In Section 5, we will show that the empirical results obtained using the Fed-CIR model are very different from those obtained using multifactor CIR, Gaussian, or jump models.

3 Data and Econometric Implementation

3.1 A Brief Description of the Data

The short-term OIS rates analyzed in this paper are the daily OIS indices obtained from Bloomberg. The data cover the period from May 6, 2002 to December 31, 2015.[10] Table 1 presents the summary statistics of the short-term OIS rates and the overnight federal funds rate in our data. The time-series averages show that the short-term yield curve has a positive slope on average. All the short-term rates have similar standard deviations of about 1.8%. The range of the rates is rather wide: from nearly 4 to over 500 bps.

Figure 1 presents the time series of selected interest rates. Panels A and B plot the federal funds rate and the one-week OIS rate. The plots show that the level of these interest rates varied dramatically during our sample period, rising from around 1% in 2003–2004 to around 5% in 2006–2007 and then plummeting all the way toward zero in 2008. Panel C plots the time series of the spread between the six-month OIS rate and the one-week OIS rate, and panel D plots the spread between the one-year OIS rate and the one-week OIS rate.

The plots show that the term structure of short-term OIS rates varied dramatically during the sample period. The term spreads were positive in 2004–2005 and then became negative in late 2006 and stayed negative for more than a year. After the federal fund rate dropped to near zero by the end of 2008, the term spreads resumed to be positive. The dramatic movements in the OIS rates displayed in Figure 1, in particular the unusual term structure prior to each of the sharp turns of monetary policy, pose a serious challenge to term structure modeling. For example, the inverted yield curve in late 2006 preceded the nosedive of the overnight rate that started in late 2007. This raises the question whether the market anticipated, a year ahead, the Great Recession and the ensuing change in monetary policy.

Using the press release by the Federal Reserve, we construct a database that tracks the public knowledge of the FOMC meeting schedule on each day. Typically, on a day in June, the Fed releases the tentative meeting schedule for the next year. Right before the press release, investors have the meeting schedule for only the next six months. Immediately after the press release, investors know the meeting schedule for the next 18 months. As each scheduled meeting day passes, the time horizon covered by the known meeting schedule shortens until the schedule for the next year is announced. On the day right before the following year's release of the meeting schedule, the known schedule is the shortest, which is slightly more than six months. As a result, on each day from June to the end of the year, at least a full year of future meeting dates are known. Starting from the beginning of the following year,

[10] May 6, 2002 is the first day on which Bloomberg began to report the OIS rates for all weekly and monthly maturities up to one year.

the known meeting schedule on each day falls below a year and shortens gradually to slightly more than six months in June.

The FOMC may also hold unscheduled meetings. In most unscheduled meetings, the FOMC reviews economic and financial developments without issuing policy statements. During our sample period, the FOMC cut the target rate at two unscheduled meetings, both of which were in 2008. Although the Fed's decisions at the unscheduled meetings affected the OIS rates, the market only became aware of the meetings after the Fed announced the meeting decisions. In other words, prior to the announcements, investors could not possibly incorporate unscheduled meetings in the pricing of the OIS rates. Only after the announcements were made, the economic content of the meeting decisions was incorporated in the interest rates.

The Fed occasionally alters some scheduled meeting dates slightly, and the rescheduling is also made public through its press release. Our database also tracks the rescheduling of the FOMC meeting dates. Thus, the database tracks precisely the public information about the meeting schedule and avoids any forward-looking bias.

There are structural changes in the marketplace since late 2008, especially after the financial crisis. Since our sample covers the period when the Fed resorted to unconventional monetary policies, we provide more detailed information in Appendix A.2 about the Fed's open market operations during this period.

Perhaps a significant change that affected the federal funds market was the initiation of the interest on excess reserves (IOER) announced on October 6, 2008. The effects of the Fed's operations on the OIS term structure may be different before and after the initiation of the IOER. To investigate the differential effects, we split the sample period into two subperiods in our analyses. The first subperiod is from May 6, 2002 to October 3, 2008. Following the weekend of October 4 and 5, 2008, the second subperiod starts on October 6, 2008 and ends on December 31, 2015. We will examine the potential differences between the empirical results in the two subperiods. We provide more details of the IOER in Appendix A.2.

3.2 An Outline of the Econometric Method

Our study follows the standard approach in econometrics, which assumes that investors know the true parameters of a model and econometricians estimate the parameters from the observed data.[11] In the Fed-CIR model, investors can price an OIS contract only if they know all the θ's up to the maturity of the contract. In order to avoid assuming that investors know all the future meeting dates, only the observed yields of maturities within the known meeting schedule are

[11]This is the standard assumption in the econometric analysis of economic models. It is far more challenging to consider an extension in which investors incorporate parameter uncertainty in pricing bonds. Lamoureux and Witte (2002) conduct Bayesian analysis of CIR models under the standard assumption. They point out that the analysis without parameter uncertainty is a necessary first step.

included in the estimation.

More specifically, on each day we use only the maturities up to but not beyond the end of the known meeting schedule. Because the time horizon of known meeting dates varies over time, the number of OIS rates used in the estimation also changes. In other words, on each day we use the federal funds rate and the OIS rates with maturities up to at least six months or more but exclude the OIS rates of maturities longer than a year or beyond the end of the known meeting schedule. Therefore, the data in our estimation are daily observations of the effective federal funds rate and the OIS rates of 15 maturities up to a year.

Let m_t be the total number of discount bond yields we use on date t in the estimation. Since the yields for the longest maturity used on each day ranges from six months to one year, m_t ranges from 10 to 16. Let $Y_t = (y_{t1}, \cdots, y_{tm_t})'$, where y_{ti} is the ith discount bond yield. Also let $S_t = (s_1, \cdots, s_{m_t})'$, where s_i is the time to maturity of the ith yield, and $s_i < s_j$ if $i < j$. Thus, we have $y_{ti} = Y_t(s_i)$. The discount bond yields on all T days used in the estimation are collected in $Y = (Y_1, \cdots, Y_T)'$. Let $\Theta = (\theta_1, \cdots, \theta_N)'$ be the vector of N reversion centers of the instantaneous rate in our sample period. Let $\Theta_t = (\theta_{j_t+1}, \cdots, \theta_{j_t+n_{t,s_{m_t}}})'$ be the vector of the steps appearing in the formula of the yields in Y_t. Furthermore, let

$$A_t = (A_{t1}, \cdots, A_{tm_t})' = \left(a_t(s_1)'\theta_t(s_1)/s_1, \cdots, a_t(s_{m_t})'\theta_t(s_{m_t})/s_{m_t}\right)', \tag{20}$$

$$B_t = (B_{t1}, \cdots, B_{tm_t})' = \left(b(s_1)/s_1, \cdots, b(s_{m_t})/s_{m_t}\right)'. \tag{21}$$

Both A_t and B_t are functions of unknown parameters $\phi = (\kappa, \sigma, \lambda)'$, and A_t also depends on Θ_t. It follows from equation (15) that the model-implied yields are $B_t r_t - A_t$. The realizations of the instantaneous rate on the T days, $R = (r_1, \cdots, r_T)'$, form the unobserved state variable.

Following Lamoureux and Witte (2002), we assume that all the observed discount bond yields are priced by the model with errors. This allows a large cross section of yields to be used in the estimation. The pricing errors follow independent Gaussian distributions and are heteroscedastic in the cross section — the variances of the pricing errors are different for the federal funds rates and the OIS rates of different maturities.[12] Under these assumptions, the ith observed yield on date t is

$$y_{ti} = B_{ti} r_t - A_{ti} + \epsilon_{ti}, \quad \epsilon_{ti} \sim N(0, \omega_i^2), \quad \text{for} \quad i = 1, \cdots, m_t, \tag{22}$$

where ω_i^2 is the variance of the pricing error. We denote the variance matrix of the pricing errors for the 16 maturities by $\Omega = \mathrm{diag}(\omega_1^2, \cdots, \omega_{16}^2)$. Since there are at most 15 terms of OIS rates in our data on each day, we only need to estimate $\omega(s)$ for at most 16 maturities. The submatrix of Ω relevant to the rates on date t used in the estimation is $\Omega_t = \mathrm{diag}(\omega_1^2, \cdots, \omega_{m_t}^2)$.

Given the model and the above assumptions, it is convenient to obtain the likelihood func-

[12] A popular alternative, e.g., often used with maximum likelihood, is to impute latent factors from some yields by assuming that these yields are precisely priced by the model. Then, the other yields are assumed to contain pricing errors. A disadvantage of this approach is that the empirical results may depend on the choice of the yields for the imputation.

tion of the observed yields conditional on the realizations of the instantaneous rate, R :

$$L(Y|R,\phi,\Theta,\Omega) = \prod_{t=1}^{T} L_t(Y_t|\,r_t,\phi,\Theta_t,\Omega_t), \tag{23}$$

$$L_t(Y_t|\,r_t,\phi,\Theta_t,\Omega_t) \propto |\Omega_t|^{-1/2} \exp\big[-(Y_t + A_t - B_t r_t)'\Omega_t^{-1}(Y_t + A_t - B_t r_t)\big]. \tag{24}$$

To obtain the marginal likelihood function of the observed yields only, we need to integrate out the unobserved instantaneous rate R:

$$L(Y|\,\phi,\Theta,\Omega) = \int_R L(Y|R,\phi,\Theta,\Omega)p(R|\,\phi,\Theta)dR, \tag{25}$$

where $p(R|\,\phi,\Theta)$ is the probability density of R implied by the Fed-CIR model.

Maximum likelihood is not a feasible method for the estimation of the Fed-CIR model due to the intractability of the marginal likelihood function in equation (25). First, the integration is not known analytically, which, in turn, precludes the analytical optimization of the likelihood function. Second, both numerical integration (such as the quadrature method) and numerical optimization suffer from the curse of dimensionality.[13] Not only is the computation intensive, but the numerical error also grows quickly with the dimension. In addition, among the unknown parameters, the likelihood function $L(Y|\,\phi,\Theta,\Omega)$ is optimized over Θ, which contains 118 steps for the period covered by our sample.

To estimate the Fed-CIR model, we undertake the Bayesian approach implemented with the Markov Chain Monte Carlo (MCMC) sampling. The details are provided in Appendix A.3. The Bayesian approach views model parameters as random variables characterized by the posterior distribution, while the MCMC is an iterative scheme to sample the posterior distribution. To accommodate the latent state variable, we also apply the data augmentation of Tanner and Wong (1987). As shown in Appendix A.3, the posterior distribution of the unknown parameters ϕ, Θ, and Ω and the latent R is

$$p(\phi,\Theta,\Omega,R|\,Y) \propto L(Y|R,\phi,\Theta,\Omega)\cdot p(R|\,\phi,\Theta)\cdot p(\phi,\Theta,\Omega), \tag{26}$$

where $L(Y\,|R,\phi,\Theta,\Omega)$ is the likelihood function, $p(R|\,\phi,\Theta)$ is the distribution of the instantaneous rates, and $p(\phi,\Theta,\Omega)$ denotes the joint prior distribution specified for the parameters.

The advantages of the Bayesian method have been demonstrated by Lamoureux and Witte (2002) in estimating the single-factor and multifactor CIR models. Johannes and Polson (2010) survey the applicability of this approach to a broad class of continuous-time models in finance. Overall, the Bayesian method is a powerful alternative to maximum likelihood.[14] The MCMC method is an effective way to overcome the curse of dimensionality [Smith and Gelfand (1992) and Casella and George (1992)] and obtain Bayesian inference for even highly nonlinear models. The combination of Bayesian inference and the MCMC method is particularly suitable for

[13]The dimension of the integration for our empirical estimation is $T = 3564$ because we need to integrate over r_t on 3564 trading days in our sample.

[14]For the classic treatise on the advantages of the Bayesian method, see Zellner (1971) and Zellner (1985). A comprehensive treatise on the advantages of combining the Bayesian method and the MCMC sampling can be found in Geweke (2005).

the Fed-CIR model.

To measure the goodness-of-fit, we focus on a model's performance in explaining the term spread. For conventional linear regressions, a widely-used measure is the R-squared, which is the ratio of the variation explained by a model to the total variation in the data. This idea can be extended to the Bayesian method with the MCMC sampling. Let $z_{ti} = y_{ti} - r_t$ be the term spread between the i^{th} term rate and the instantaneous rate. The total variation in the term spreads is $\Sigma_{t=1}^{T}\Sigma_{i=1}^{m_t}(z_{ti} - \bar{z})^2$, where $\bar{z}$ is the sample average of all z_{ti}. The term spread implied by the model is $\hat{z}_{ti} = \hat{y}_{ti} - r_t$. Thus, the sum of the unexplained variation in the term spreads is $\Sigma_{t=1}^{T}\Sigma_{i=1}^{m_t}(z_{ti} - \hat{z}_{ti})^2$. The proportion of the total variation explained by the model is

$$R^2 = 1 - \frac{\Sigma_{t=1}^{T}\Sigma_{i=1}^{m_t}(z_{ti} - \hat{z}_{ti})^2}{\Sigma_{t=1}^{T}\Sigma_{i=1}^{m_t}(z_{ti} - \bar{z})^2}\,. \tag{27}$$

The marginal posterior distribution of R^2 can be obtained from the MCMC samples. We refer to the posterior mean of R^2 as the Bayesian R-squared.[15]

4 Empirical Results

4.1 Estimated Reversion Center in the Fed-CIR Model

Since the adaptation of the reversion center to the FOMC meeting schedule is the key feature of the Fed-CIR model, we first present the empirical estimate of the step function Θ. Each step θ_j is the reversion center of the instantaneous rate during the intermeeting period $(\tau_j, \tau_{j+1}]$. In Figure 3, the thin solid line is the time series of the posterior mean of θ_j, and the gray line is the Fed's target rate. Note that the gray line is thicker after December 16, 2008, because the Fed's target is not a single rate but rather a range of 0–25 bps from December 16, 2008 to December 15, 2015, and a range of 25–50 bps since December 16, 2015. The light gray area from December 2007 to June 2009 indicates the only NBER-dated economic recession during our sample period.

Overall, Figure 3 reveals an interesting asynchronous relation between the reversion center and the Fed's monetary policy. The target rate was 1% at the beginning of the sample period. On June 30, 2004, the Fed started to tighten monetary policy, increasing the target rate by 25 bps in every FOMC meeting. The tightening continued until August 8, 2006, when the FOMC decided to maintain the target rate at 5.25%. The posterior mean of the reversion center climbed up ahead of the target rate during 2004 and the first half of 2005, showing that the market participants anticipated the Fed's tightening during this period. Starting late 2005, while the target was still rising, the reversion center began to decline. The reversion

[15] A similar Bayesian version of the R-squared has been introduced by Gelman and Pardoe (2006) for hierarchical models in statistics.

center continued to drop from fall 2006 through fall 2007, although the Fed kept the target rate constant at 5.25%. The Fed only began to lower the target in fall 2007. Thus, the decline in the reversion center was well ahead of that in the target rate, suggesting that the market expected the Fed's future easing of monetary policy.

From fall 2006 through fall 2007, when the Fed kept the target rate constant at 5.25%, the term spreads of the six-month and one-year OIS rates over the one-week OIS rate were both negative, as shown in panels C and D of Figure 1. It is well known that the long-term yield curve of Treasury bonds was also inverted from fall 2006 to fall 2007, but it is unclear or even controversial whether this inverted yield curve reflected investors' expectation of the Fed's future easing of monetary policy. Estrella and Trubin (2006) take the view that the inverted long-term yield curve of Treasury bonds in fall 2006 predicted that a recession would come about within a year and that the Fed would cut the target rate. On the other hand, Bernanke (2005) and Bernanke et al. (2011) take the alternative view that the inversion did not reflect the expected future cuts in the target rate, but was rather caused by a "global saving glut" — investments of some Asian countries' large surplus in the U.S. Treasury bonds pushed down both the long-term yields and the risk premium.

It is difficult to discriminate between these two views without explicitly modeling how the flow of global savings into the safe U.S. Treasury bonds affected the yield curve. However, the move of the reversion center during the period from fall 2006 to fall 2007 in Figure 3 appears more consistent with the view that the expectation of the forthcoming Fed's easing caused the inversion of the OIS yield curve. As already noted, the reversion center fell quickly during this period even though the target rate stayed constant. In addition, the yields of less than one-year maturities were probably not affected by the demand of long-term Treasury bonds. Based on a report by the U.S. Treasury and the Fed,[16] foreign holdings of long-term U.S. Treasury debt increased from $1,429 billion in June 2004 to $1,965 billion in June 2007. However, foreign holdings of short-term (maturities of one year or less) U.S. Treasury debt reduced from $317 billion in June 2004 to $229 billion in June 2007. Although the increase in foreign holdings of the long-term debt may be consistent with the inversion of the yield curve at the long end, it is difficult to relate the inversion at the short end to the reduction in foreign holdings of the short-term debt.

The Great Recession was indeed coming in fall 2007. On September 18, 2007, the FOMC cut the target rate from 5.25% to 4.75%. This was followed by subsequent cuts of the target rate down to 2% over the next seven meetings. During the same period, the reversion center fell nearly to zero. The drop of the reversion center is concurrent to the negative spreads in

[16] See "Report on Foreign Portfolio Holdings of U.S. Securities as of June 30, 2008" issued in April 2015 by the Department of Treasury, the Federal Reserve Bank of New York, and the Board of Governors of the Federal Reserve System.

Figure 1. This drop was quickly reversed when the FOMC surprised the market by its decisions to keep the target rate at 2% in the meetings on June 25, August 5, and especially September 16, 2008 immediately after the collapse of Lehman Brothers.[17] As soon as the reversion center moved back to 2%, the FOMC cut the target rate further by 50 bps at the October 8 meeting and by another 50 bps at the October 29 meeting. The Fed finally set the target into the range of 0–25 bps at the meeting on December 16, 2008. However, it took more than a year for the reversion center to drop below 25 bps.

The dramatic changes in monetary policy and the dynamics of the term structure of short-term interest rates in our sample period shed light on the question whether the Fed's decision leads the market or the market leads the Fed's decision. On Wall Street, this question is phrased as whether the Fed is ahead of or behind the curve. Some academic scholars [e.g., Fama (2012)] have also raised the question whether the Fed actually controls the interest rates. Figure 3 suggests that the Fed was behind the curve in all the days up to fall 2007 when the Great Recession started, as the change in the target rate always followed the change of the reversion center. However, the Fed was ahead of the curve in the second half of 2008. After dropping to nearly zero in early 2008, the reversion center came back to 2%, which was the level of the target rate at the time. When the Fed sharply cut the target toward zero in late 2008, the reversion center came down only slowly. Overall, the market and the Fed's monetary policy seem to be out of synchronization before and during the Great Recession of 2007–2009.

The reversion center stayed mostly inside the 0–25 bps range of the target during 2011–2014, except for a brief rise above the range in summer 2011, as shown in Figure 3. The nearly constant reversion center inside the target range suggests that the Fed successfully convinced the market that the interest rate would be kept near zero for an extended period. Indeed, in each meeting since June 2010, the FOMC statement included clear forward guidance that economic conditions warranted exceptionally low levels of the federal funds rate for an extended period. Woodford (2013) uses intraday OIS data to show that the OIS term spread dropped sharply at 2 p.m. on August 9, 2011 when the Fed announced clear forward guidance that the target rate would remain unchanged though mid-2013. He shows that the same happened to the OIS in Canada on April 21, 2009 when the Bank of Canada announced its conditional commitment to maintaining the target range of 0–25 bps through mid-2010.

The brief rise of the reversion center in summer 2011 indicates a short-lived episode of investors' expectation that the Fed might begin to normalize monetary policy. The FOMC statement on June 22, 2011 noted that inflation had picked up. On the same day, as part of its forward guidance, the FOMC released its economic projection of upward trends in both GDP growth and inflation in the forthcoming year. In the meantime, the Fed was also about to com-

[17] In the hindsight, Bernanke (2015) points out that keeping the interest rate at 2% in the FOMC meeting on September 16, 2008 was "certainly a mistake."

plete its program (QE2) of purchasing \$600 billion long-term Treasury bonds by June 2011, but the FOMC statement gave no hint on the additional large-scale asset purchase program (QE3), which was not announced until September 13, 2012. In fact, the meeting detail released on July 12, 2011 showed that the FOMC considered and adopted plans and principles for monetary normalization in the meeting of June 2011. The expectation to normalize monetary policy was quickly reversed in the subsequent meeting statements in August and September 2011, which noted that inflation had moderated. Consequently, the reversion center fell back into the near-zero range in late 2011, as shown in Figure 3.

The reversion center moved up steadily in 2015, clearly showing that the market anticipated the Fed's normalization of monetary policy. After the data for the last quarter of 2014 presented solid economic growth and strong job gains but slightly declined inflation, the FOMC said in its statement on January 28, 2015 that it "expects inflation to rise gradually toward 2 percent over the medium term as the labor market improves further." The committee held the target rate unchanged at its first meeting of 2015 and told the public that the timing of the target rate increase would depend on the information on employment and inflation. Since then, the term spreads in short-term OIS rates gradually moved higher, as shown in panels C and D of Figure 1. During the period of next six meetings, the labor market improvement was robust, but economic growth was moderate. The annualized inflation rate released in each month of January–October, 2015 was only 0.2% or lower,[18] although in each of the six meeting statements the FOMC expressed its confidence that inflation would "move back to its 2 percent objective over the medium term." Finally, as expected by the market, the FOMC raised the target rate to the range of 25–50 bps at its last meeting of 2015, despite the fact that the November inflation data continued to be weak.

4.2 Estimated Parameters in the Fed-CIR Model

Table 2 presents the summary statistics of the posterior distributions of σ, κ, and λ. The results in panel A are based on the estimation using the entire sample period. The parameters are quite precisely estimated, since the standard deviation of the marginal posterior distribution of each parameter is only a small fraction of the mean. The estimate of the local volatility parameter σ is particularly precise: the central 90 percentile has a narrow range of 0.0344$\sim$0.0360. The estimate of κ has a slightly wider band and falls into the range of 0.4913$\sim$0.7625 with a 90-percent probability. The market price of risk, λ, is estimated with a similar precision; the central 90-percentile interval is (-0.4146, -0.1445). The negative value of λ indicates a positive risk premium in the yield curve, consistent with the economic theory of risk-averse agents.

[18]The inflation rates quoted here are based on the Consumer Price Index (CPI) published by the Bureau of Labor Statistics.

We use the subperiods to check the stability of the parameter estimates. Panels B and C of Table 2 are the estimation using the data in the subperiods. The estimate of the volatility parameter σ is remarkably stable across the two subperiods, even though the level and the dynamics of the OIS rates are dramatically different. The posterior mean of σ as well as its standard deviation and the percentiles are almost identical in the three panels of Table 2.

The estimates of the reversion speed κ in the two subperiods are different. For the earlier subperiod, the posterior distribution of κ is similar to that for the whole period. For the later subperiod from October 6, 2008 to December 31, 2015, the posterior mean of κ is much higher. The high reversion speed is perhaps a reflection of the Fed's tight control of the interest rates during this period.

The estimates of the market price of risk parameter λ in the two subperiods are also different. The value of λ is much more negative for the later subperiod. These estimates appear to suggest that investors were more risk averse during and after the financial crisis. Note that a higher market price of risk since the financial crisis does not necessarily lead to a higher magnitude of the risk premium in the short-term OIS rates because the risk premium also depends on the uncertainty of future monetary policy, which was probably relatively low during the second period.

The Bayesian R-squared of the Fed-CIR model is 82.50% for the entire sample period. The high R-squared indicates that the simple Fed-CIR model is rather successful in fitting the 16 term spreads over more than 3000 days. The R-squared is 82.69% for the first subperiod and 78.57% for the second. In the second subperiod, during which the target rate is a narrow range and the interest rates are low, the variation of the spreads is perhaps more subject to market microstructure fluctuations that the model does not intend to capture, resulting in a slightly lower R-squared.

4.3 Estimated Expectation and Risk Premium Components

To better understand the dynamics of the OIS term structure, we compute the model-implied expectation and risk premium components of each OIS rate, as discussed in Section 2.3. As an illustration, we focus on the components in the term spread of the six-month and one-week OIS rates. Six month is the longest maturity that is always within the horizon of the announced FOMC meeting schedule. One week is the shortest maturity of the OIS rates in our data. Figure 4 presents the posterior means of the two components of the term spread between the six-month and one-week OIS rates, as well as the model-implied term spread. To facilitate comparison with the data, the model-implied term spread and the components are plotted against the observed term spread in the background. Consistent with the high R-squared of the Fed-CIR model, the model-implied term spread plotted in panel A tracks the actual spread rather closely over the entire sample period.

Panel B of Figure 4 shows that the expectation component moved up and down substantially during the sample period. More importantly, according to the Fed-CIR model, the movement in the term spread is in most part attributable to the dynamics of the expectation component. As discussed earlier, a positive or negative expectation component of the term spread indicates the market anticipation of rising or falling interest rates in the future. Thus, panel B indicates that the inverted yield curve during 2006–2007 is consistent with the view of Estrella and Trubin (2006) that the inversion was forecasting the subsequent cuts in the target rate. Panel B also reaffirms the asynchronous relation between the market expectation and the Fed's decision. The expectation component was rising before mid-2004 and then descending until the end of 2007. It was negative from the beginning of 2006 until the end of 2008. Compared to the path of the Fed's target rate during the same period, the market expectation was ahead of the Fed's decisions. In contrast, after the financial crisis, the expectation component stayed positive until late 2010, behind the Fed's decision to cut and hold the target to near zero starting December 2008.

Panel C of Figure 4 displays the risk premium component of the term spread between the six-month and one-week OIS rates. It is important to note that our estimate of the risk premium is always positive, even though neither the Fed-CIR model nor the econometric methodology imposes such a restriction. Obviously, a positive risk premium is consistent with the view that investors are risk averse. Even when the term spread is negative, the risk premium estimated from the Fed-CIR model is still positive. In other words, when the yield curve became inverted due to the expectation of the future drop in interest rates, investors still demand a positive premium for the uncertainty of future interest rates.

Not only the positivity of the risk premium makes economic sense, so do the magnitude and dynamics of the expectation and risk premium. Panel D of Figure 4 plots the expectation and risk premium components as proportions of the six-month OIS rate.[19] The risk premium proportion ranged between five and seven percent during the sample period. It increased from 2004 to 2007 and peaked at the end of 2007 when the economy just entered the Great Recession and when the Fed started to make several unusual changes in monetary policy. It then fell quickly in late 2008 when the Fed was clearly on the path of easing monetary policy by cutting the federal funds rate to a narrow range near zero. Right after the FOMC meeting on December 16, 2008, the proportion of the risk premium in the six-month OIS rate reached a low point while the proportion of the expectation component peaked. The expectation of monetary policy should be relatively unambiguous at the time because the FOMC stated in its

[19]For the one-week OIS rate, the expectation component was nearly 100 percent of the rate during the entire sample period, whereas the risk premium component was nearly zero percent. This is consistent with the intuition that duration risk should be negligible for a one-week loan. Therefore, the risk premium in the six-month rate is essentially the same as the risk premium in the term spread between the two rates.

forward guidance: "The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time."

5 Comparison with Multifactor Models

5.1 Multifactor CIR Models

A large number of models have been developed in the literature to fit the term structure of interest rates, and many studies recognize that multiple factors are necessary to capture the yield curve dynamics. Most models are affine dynamic term structure models as characterized by Duffie and Kan (1996) and Dai and Singleton (2000). To compare with our Fed-CIR model, we have applied a wide range of dynamic term structure models to the OIS data. In this section, we present the results of several representative multifactor models and examine the economic implications of these models as reflected in the expectation and risk premium components of the OIS rates.

The first type of models we investigate are the multifactor CIR models. These models are derived by Cox et al. (1985) as extensions to the single-factor CIR model and have been empirically implemented in Pearson and Sun (1994), Lamoureux and Witte (2002), among others. In a multifactor CIR model, the instantaneous rate r_t is an affine function of factors $x = (x_1, \cdots, x_k)'$, and the factors follow square-root processes driven by independent Brownian motions:

$$dx_{jt} = \kappa_j(\theta_j - x_{jt})\,dt + \sigma_j\sqrt{x_{jt}}\,dw_{jt}, \quad \text{for} \quad j = 1, \cdots, k. \tag{28}$$

The discount bond yield is an affine function of the factors and admits an analytical formula:

$$y_t(s) = \Sigma_{j=1}^{k}\big[b_j(s)x_{jt} - a_j(s)\theta_j\big]/s\,, \tag{29}$$

$$a_j(s) = (2\kappa_j/\sigma_j^2)\big[\delta_j s/2 + \log(2\gamma_j) - \log\big(\delta_j(\exp(\gamma_j s) - 1) + 2\gamma_j\big)\big], \tag{30}$$

$$b_j(s) = 2[\exp(\gamma_j s) - 1]/[\delta_j(\exp(\gamma_j s) - 1) + 2\gamma_j]\,, \tag{31}$$

where $\gamma_j^2 = (\kappa_j + \lambda_j)^2 + \sigma_j^2$ and $\delta_j = \gamma_j + \kappa_j + \lambda_j$. While multiple factors make the model flexible in fitting the yield curve, interpretation of the factors is a challenge. In particular, it is often unclear how the Fed's monetary policy is reflected in each factor.

Since we will discuss the empirical results of a three-factor Gaussian model and a four-factor jump model in the next two subsections, in this subsection we present only the single-factor and two-factor CIR models. The estimated posterior distribution of the parameters of these CIR models are reported in Table 3. Figures 5 and 6 plot the model-implied term spread between the six-month and one-week OIS rates and the decomposition of the spread into the expectation and risk premium components.

Figure 5 indicates that the single-factor CIR model almost entirely misses the variation of the term spread over time; it generates a positive spread that is almost constant as shown in Panel A. Not surprisingly, the Bayesian R-squared of the single-factor CIR model is only 10.31%, as reported in Table 3. The expectation component plotted in panel B is almost always negative from the beginning of 2002 to nearly the end of 2008. The expectation and risk premium components appear negatively correlated, as shown by panels B and C. The sum of them generates the nearly constant term spread shown in panel A. Panel D shows that the proportions of the expectation and risk premium components in the six-month OIS rate stayed roughly constant until late 2008. Since then, the proportion of the risk premium fluctuated at a lower level whereas the proportion of the expectation shifted to a higher level. Overall, the single-factor CIR model fits the data poorly, and in particular, fails to account for the variation of the term structure.

The two-factor CIR model fits the term spread much better than the single-factor model. Panel A of Figure 6 shows that an additional factor greatly helps account for the variation of the term spread. The term spread implied by the two-factor model broadly traces the observed term spread. The Bayesian R-squared is 86.20%, similar to the R-squared of the Fed-CIR model.

However, the additional factor has little effect on the expectation component. In panel B of Figure 6, the expectation component stayed negative until the last month of 2008, similar to what we have seen in the single-factor model. The negative expectation in 2004 and 2005 is particularly inconsistent with the Fed's actions and its clear public announcements of the plan to raise the target rate. During the second half of 2004 and most of 2005, the Fed announced a series of target rate hikes, and the yield curve was upward-sloping. The negative expectation, however, counterintuitively suggests that investors expected interest rates to fall during the period. Even more puzzling is that the expectation component became much less negative near the end of 2007, just when the term spread became more negative and the Fed started the process of cutting the target rate from 5.25% to nearly zero.

Panel C of Figure 6 reveals that the two-factor CIR model attributes the dynamics of the term spread largely to the risk premium component. To reconcile the positive term spread with the negative expectation component in 2004 and 2005, the two-factor CIR model produces a large and positive risk premium component. To account for the gap between the large negative term spread and the small negative expectation component in the period from late 2007 to mid 2008, the two-factor CIR model generates a large negative risk premium. Overall, the risk premium component in the model varies substantially and sometimes become negative.

Panel D of Figure 6 plots each of the expectation and risk premium components as a percentage of the six-month OIS rate. The risk premium proportion was negative in 2003 and during the period from fall 2007 through spring 2008. A negative risk premium cannot be justified in an economy with risk averse agents. It is also difficult to understand why the risk premium proportion decreased sharply during the period from mid-2006 to early 2008. During this period, a sweeping financial crisis was getting underway, the economy was entering into one of the worst recessions in history, and investors were uncertain and anxious about what the Fed would do.

5.2 Multifactor Gaussian Models

The second type of models we investigate are multifactor models with Gaussian factors, which have been extensively studied in the literature. These models allow for correlated factors and accommodate a flexible specification of time-varying market prices of risks. These features not only improve the model's ability to fit data [Duffee (2002)], but are also critical in accounting for time-varying risk premiums [Dai and Singleton (2002)]. We have estimated various multifactor Gaussian models using the OIS data; in this subsection, we focus on the three-factor Gaussian model recommended by Dai and Singleton (2002).

In the model, the instantaneous rate is an affine function of three factors:

$$r_t = \delta + x_{1t} + x_{2t} + x_{3t}\,. \tag{32}$$

The vector of the factors $x_t = (x_{1t}, x_{2t}, x_{3t})'$ follows a mean-reverting process:

$$dx_t = \kappa(\theta - x_t)dt + \sigma dw_t\,, \tag{33}$$

where the mean reversion parameter κ is a 3×3 matrix, the long-run mean θ is a 3×1 vector, the volatility σ is a 3×3 matrix, and w_t is a 3×1 vector of independent Wiener processes. To ensure that the parameters are empirically identifiable, we follow the normalization in Dai and Singleton (2002) to let κ be a lower-triangular matrix, θ be a zero vector, and σ be a diagonal matrix. The market price of risk is $\sigma^{-1}(\mu+\lambda x_t)$, where μ is a 3×1 vector and λ is a 3×3 matrix. The time-t yield of a discount bond that matures at time $t+s$ is:

$$y_t(s) = [b(s)'x_t - a(s)]/s, \tag{34}$$

where $a(s)$ and $b(s)$ depend on κ, θ, σ, δ, λ, and μ and are solved numerically from a set of ordinary differential equations [Duffie and Kan (1996)].

We estimate the three-factor Gaussian model using the OIS rates and report the parameter estimates in Table 4. In terms of the sample period, observation frequency, and maturities, there are substantial differences between our data and that of Dai and Singleton (2002). The data in Dai and Singleton (2002) are monthly observations of U.S. Treasury yields for the period of 1970–1995 and include maturities of 6 months and 2, 3, 5, 7, and 10 years. Our sample period of 2002–2014 does not overlap with theirs. In addition, our data are daily observations and the maturities range from overnight to at most one year. Hence, we should not expect the empirical results obtained using our data to be similar to those reported by Dai and Singleton (2002).

Figure 7 presents the term spread between the six-month and one-week OIS rates and its expectation and risk premium components implied by the three-factor Gaussian model. With an R-squared of 97.76% (reported in Table 4), the model fits the term structure of OIS rates far better than the two-factor CIR model and the Fed-CIR model. The excellent goodness of fit is also visible in panel A of Figure 7, in which the model-implied term spread tracks the detailed variation of the observed term spread most of the time.

The expectation and risk premium components obtained from the three-factor Gaussian model are however difficult to interpret. Panel B of Figure 7 shows that the expectation component is not only subdued but also moves mostly in the opposite direction to that of the observed term spread. For example, as the slope of the yield curve turned negative in mid-2006, the expectation component stayed positive and flat. Panel C suggests that the model almost entirely relies on the variation of the risk premium component to fit the term structure of OIS rates. The risk premium component moves closely with the observed term spread.

In panel D of Figure 7, the variations of the expectation and risk premium components as proportions of the six-month OIS rate are counterintuitive. It is difficult to reconcile why the proportion of the risk premium decreased sharply in late 2007 and early 2008 when the economy was entering the financial crisis and when market players were anxious and frequently surprised by the Fed's actions. It is also difficult to understand why the proportion of the expectation component was extremely low in December 2008 and in 2009 when the Fed's forward guidance committed to a policy of maintaining the near-zero interest rate for an extended time. In addition, the risk premium became negative several times during 2010–2013, inconsistent with the view that investors should be risk averse.

5.3 Multifactor Models with Jumps

The third type of models we investigate specify the reversion center ϑ_t as a stochastic process with jumps. A representative model of this type is the four-factor jump model introduced by Piazzesi (2005). This model specifies ϑ_t as the target rate, and jumps in ϑ_t occur only during the periods of the FOMC meetings.[20] More specifically, the target rate ϑ_t follows a process with jumps of 25 basis points:

$$d\vartheta_t = 0.0025(dN_t^+ - dN_t^-), \quad \text{Prob}\{dN_t^\pm = 1\} = \rho_t^\pm dt\,, \tag{35}$$

$$\rho_t^\pm = \bar{\rho} \pm \rho'(x_t - \bar{x})\,. \tag{36}$$

The above jump process of ϑ_t is a linear combination of two counting processes, N_t^+ and N_t^-, with jump intensities ρ_t^+ and ρ_t^-, respectively. The deviation between the instantaneous rate

[20]Heidari and Wu (2010) assume that a latent factor, instead of the target rate, jumps at the FOMC meetings. Some models do not align jumps with the FOMC meetings. For example, Kim and Wright (2014) assume that factors jump on the days of employment report announcements. Johannes (2004) assumes that the rates of Treasury bills can jump randomly at any time.

and the target, $u_t = r_t - \vartheta_t$, follows a mean-reverting process:

$$du_t = \kappa_u(0 - u_t)\,dt + \sqrt{v_t}\,dw_t^u, \tag{37}$$

where v_t is a stochastic volatility factor following a square-root process:

$$dv_t = \kappa_v(\bar{v} - v_t)\,dt + \sigma_v\sqrt{v_t}\,dw_t^v. \tag{38}$$

In the above two equations, κ_u, κ_v, and $\bar{v}$ are unknown parameters, and w_t^u and w_t^v are independent Wiener processes. The model also has a latent factor z_t that follows a Gaussian process:

$$dz_t = \kappa_z(0 - z_t)\,dt + dw_t^z, \tag{39}$$

where κ_z is an unknown parameter and w_t^z is a Wiener process independent of w_t^u and w_t^v. All together, there are four factors in the model: $x_t = (\vartheta_t, u_t, v_t, z_t)'$. The long-run mean of x_t is $\bar{x} = (\bar{\vartheta}\,,\,0\,,\,\bar{v}\,,\,0)'$, where $\bar{\vartheta}$ is calibrated to the sample average of the target rate. The market price of risks in the four-factor jump model is

$$\Lambda_t = (0\,,\ \lambda_u\sqrt{v_t}\,,\ (\lambda_v/\sigma_v)\sqrt{v_t}\,,\ \lambda_z)', \tag{40}$$

where λ_u and λ_v are unknown parameters. The market price of risks varies with the stochastic volatility v_t. The yield of a discount bond is an affine function of the factors but does not admit a closed-form solution. Piazzesi (2005) shows that the price of discount bonds can be solved numerically from five ordinary differential equations.

We apply the four-factor jump model to the daily OIS data before December 16, 2008. Since December 16, 2008, the Fed has set the target to be a range: 0–25 bps until December 15, 2015, and 25–50 bps afterwards. The jump process as specified in equation (40) requires the target rate to be a specific level, not a range. It is unclear to us how to use the range to estimate the jump process.[21] In the estimation, we confine the OIS data on each day to those covered by the meeting schedule known on the day, as we have done for the Fed-CIR model. The empirical results on the OIS rates should not be expected to be similar to those reported in Piazzesi (2005) because of notable differences in the sample period, observation frequency, and yield maturities. The data in Piazzesi (2005) are weekly observations of the target, the six-month Libor, and the two- and five-year swap rates for the period of 1994–1998. By contrast, we use daily series of the OIS rates and federal funds rates for the period from May 6, 2002 to December 15, 2008.

The estimation results of the parameters in the four-factor jump model are reported in Table 5. With an R-squared of 98.17% for the sample period before December 16, 2008, the jump model works as well as the three-factor Gaussian model in fitting the observed term spreads.[22]

[21] We have experimented with assuming that the target rate is the upper bound of the target range after December 16, 2008 and applied the jump model to the entire sample period. The model-implied term spreads before December 16, 2008 change little, but the estimates are systematically biased after December 16, 2008 because the average federal funds rate has been below the upper bound of the target range.

[22] The R-squared generated by the three-factor Gaussian model for the entire sample is 97.76% (reported in

Panel A of Figure 8 plots the posterior mean of the term spread between the six-month and one-week OIS rates estimated by the four-factor jump model. The model-implied spread practically coincides with the observed spread.

Panel B of Figure 8 shows that in the four-factor jump model, the expectation component tracks the fluctuations of the observed term spread. This contrasts with the three-factor Gaussian model, which explains the term spread almost entirely by the time-varying risk premium. Rather, the four-factor jump model is similar to the Fed-CIR model in attributing the dynamics of the term spread mostly to the change in the expectation. For example, like the Fed-CIR model, the jump model shows that the expectation component is negative in the year prior to the Great Recession, indicating the market expectation of future interest rate cuts.

The risk premium component of the term spread estimated by the four-factor jump model is however puzzling. Panel C of Figure 8 shows that except some fluctuations such as the small spikes at the end of 2007 and in December 2008, the risk premium component of the term spread was roughly constant for most of the sample period. This suggests that the stochastic volatility v_t played a minor role in the risk premium of short-term interest rates during this sample period. A possible reason is that v_t is the volatility of the deviation $u_t = r_t - \vartheta_t$, a variable less pertinent to monetary policy or the macroeconomy than the instantaneous rate r_t or the target ϑ_t.

For the six-month OIS rates, panel D of Figure 8 shows that the expectation and risk premium components as proportions of the OIS rate. In the four-factor jump model, because the risk premium component is roughly constant, its proportion behaves like the inverse of the level of the OIS rate. The risk premium proportion in the six-month OIS rate was as high as 20% in 2003 because the OIS rate was relatively low at the time. Toward the end of 2008, when the Fed cut the target rate to nearly zero, the expectation proportion in the six-month OIS rate goes to zero, leaving the risk premium proportion to shoot up toward nearly 100% of the OIS rate. Moreover, as the OIS rate climbed up during 2004–2007, the expectation proportion was rising, and the risk premium proportion was declining, both because the interest rate was going up.

Much of the variation in the risk premium proportion implied by the four-factor model is puzzling in light of the macroeconomy and monetary policy developments during the same period. It is hard to understand why the risk premium proportion should decline in the year prior to the Great Recession. It is perhaps even harder to understand why the risk premium proportion in early 2008 should be lower than that prevailed in 2003. It is also questionable whether the rapid drop of the expectation proportion toward zero in December 2008 is realistic when the Fed's forward guidance promised to keep the nearly-zero target rate for an extended

Table 4). If we estimate the three-factor Gaussian model to the sample before December 16, 2008, the R-squared is slightly higher.

period. As noted above, these counterintuitive results are largely due to the almost constant magnitude of the risk premium component implied by the jump model for this sample period.

6 Conclusion

The tumultuous history of the OIS rates in our sample poses challenges to a wide array of multifactor models. The empirical results produced by these multifactor models are counter-intuitive if we believe that investors were risk averse and that the uncertainty of the Fed's monetary policy was high in the early stage of the 2007–2008 financial crisis. The multifactor CIR and Gaussian models fit the OIS rates largely by time-varying risk premiums and attribute the inversion of the yield curve prior to the crisis to extremely low or negative risk premiums. Based on those models, the risk premiums in the OIS rates fell sharply as the financial markets approached the crisis, and the risk premium became negative at the time the U.S. economy was entering the Great Recession. The four-factor jump model with stochastic volatility cannot be applied to the data since December 16, 2008 when the target has become a range. For the earlier period, the risk premium implied by the model stays essentially constant, independent of the state of the economy and monetary policy.

Our study meets the challenges with the precise incorporation of the Fed's open market operations in the Fed-CIR model while still preserving the economic foundation and analytical tractability of the traditional CIR model. The Fed-CIR model has a time-varying reversion center of the instantaneous rate but imposes a restriction to align with the Fed's open market operations that keep the target rate constant during each intermeeting period. Besides fitting the OIS term structure well, our model accounts for the dynamics of the market expectation of monetary policy while keeping the risk premium positive. Using the Fed-CIR model, we show that the expectation of interest rate cuts caused the inversion of the OIS yield curve in the year right before the financial crisis. We also demonstrate that the risk premium in the OIS rates peaked at the beginning of the Great Recession and dropped to a very low level in December 2008 when the Fed started the extended period of the near-zero interest rate policy.

A potential extension of our analysis is to apply the Fed-CIR model to the term structure of Treasury bonds. A challenge to such application is how to account for investors' knowledge of future FOMC meeting dates. At any time, investors know the announced schedule of the FOMC meetings only up to at most one and half year. The exact date of an FOMC meeting is perhaps not as important for pricing long-term bonds. Since the Fed usually holds eight FOMC meetings each year, it is possible to estimate the approximate date for each FOMC meeting, as done by Piazzesi (2005). Another challenge is that inflation expectation and the associated risk premium are important in long-term Treasury bonds [e.g., Ang, Bekaert, and Wei (2008)]. Incorporating inflation in the Fed-CIR model and applying it to the analysis of the term structure of long-term Treasury bonds are interesting directions of future research.

While the flexible multifactor models exhibit somewhat higher in-sample goodness-of-fit than the Fed-CIR model, it remains an open question whether multifactor models deliver better out-of-sample forecasts of the change of the OIS term structure. This question is worth exploring in particular in light of the evidence that the Fed-CIR model provides an economically appealing account of the changes in the expectation and risk premium. The model that forecasts better is potentially more useful in practical applications. To compare the forecasting performance, we need to estimate the predictive posterior distributions. This is a separate research project we are currently pursuing.

A Appendix

A.1 The Market for Overnight Index Swaps

An overnight index swap is an interest rate swap contract that exchanges a fixed interest rate, referred to as the OIS rate, with a floating overnight interest rate. The floating interest rate in the U.S. dollar OIS contract is based on the effective federal funds rate published daily by the Federal Reserve Board (Table H.15). The effective federal funds rate, calculated daily by the Federal Reserve Bank of New York (NY Fed), is the weighted average of the rates on brokered federal funds. Like all other interest swaps, an OIS is settled by cash over each payment period, known as the Calculation Period. The terms of OIS contracts range from one week to several years. OIS contracts of terms longer than one year follow other swaps to use the three-month payment period in the settlement. By contrast, short-term OIS contracts with up to one-year maturity have only one payment period, and are settled by cash only on the maturity date. As a consequence of the single settlement period, a fairly priced short-term OIS rate is equal to the yield on a discount bond of the same term. We formally show this equivalence below.

The cash settlement of an OIS for a payment period reflects the profit or loss in daily interest payments accrued during the payment period. Let $s = n/360$ be the length of a payment period measured in years, where n is the number of calendar days during the period.[23] Let i denote a business day. Let $y_i(h_i)$ be the overnight interest rate on date i and h_i be the length of "overnight." The length of "overnight" (from one business day to the next business day) varies. For example, $h_i = 1/360$ if both date i and the next calendar day are business days, whereas $h_i = 3/360$ if date i is a Friday and the next Monday is a business day. If there is a bank holiday between date i and date $i + 1$, then the length of h_i should adjust accordingly.[24]

[23]Like other short-term money market instruments, short-term OIS contracts use the actual/360 day counting rule.

[24]If two business days are separated by a holiday, then $h_i = 2/360$. If Monday is a holiday, then the "overnight" from Friday to Tuesday is actually four-day long: $h_i = 4/360$.

The accrued floating interest is $L\Big[\Pi_{i=1}^{n}\big(1+y_i(h_i)h_i\big)-1\Big]$, where L is the notional amount of the loan in the OIS contract. Let R be the fixed rate in the OIS. The accrued fixed interest is simply LRs. The difference between the floating interest and the fixed interest is settled by cash at the end of the payment period.

Consider one side of the OIS as a borrower of L dollars who pays the floating overnight rate and the other side as a lender of L dollars who receives the fixed OIS rate. The value of the loan with the floating interest rate is simply L. Let $Y(s)$ be the annualized discount rate in the market for one-dollar payment in s years from now. The loan of L dollars that pays the fixed OIS rate and matures in s years should be valued by $L(1+Rs)/[1+Y(s)s]$. In a fairly priced OIS, the loans on the two sides should have the same value:

$$L = L(1+Rs)/[1+Y(s)s], \tag{41}$$

which implies $R = Y(s)$. That is, a short-term OIS rate is the same as the discount bond yield of the same term.

The short-term OIS rates can also be viewed as yields on bonds that have negligible credit risk. OIS contracts carry little or even no credit risk because the two sides of the contract do not exchange the principal and because the accrued interest loss is covered by cash collateral. OIS contracts are netted — two sides of an OIS contract only exchange the differences in accrued interests. There is no concern of losing the notional value of the principal. Duffie and Huang (1996) show that netting substantially reduces the impact of credit risk on swap rates.[25] The credit risk of an OIS contract is further mitigated by the required cash collateral against the negative market value of the contract. In the event of default, the cash collateral covers the loss accrued by the unfavorable interest rate moves during the payment period.[26] In the *Standard Credit Support Annex*, the International Swaps and Derivatives Association (ISDA) standardizes the calculation and requirement of cash collateral of the OIS. In order to minimize credit risk, the Annex eliminates the use of non-cash collateral for the OIS. One of the ISDA objectives is to support the adoption of the OIS rates as the benchmark risk-free rates for derivative security valuation.

The industry practice appears to be aligned with the ISDA's goal in adopting the OIS rates as risk-free rates. Since the summer of 2007, the three-month OIS rate has been subtracted from the Libor of the same term to create the widely watched Libor-OIS spread. This spread is viewed as a measure of the credit risk premium in the interbank loans. In December 2007, the Federal Reserve used the one-month OIS rate as the risk-free rate that floors the bidding rates for one-month loans in the Term Auction Facility. In June 2010, the LCH.Clearnet switched

[25] Their calculations suggest that for a typical swap, a fixed-rate counterparty with a 100 basis point higher credit spread raises the swap rate by only one basis point.

[26] Johannes and Sundaresan (2007) note that the rebates earned on the collateral of swaps affect the pricing of long-term swaps. Since we restrict our attention to the OIS rates of terms less than or equal to a year, the effect of the collateral rebates is unlikely to be empirically important.

to the OIS rates for discounting swaps. In July 2010, the International Derivatives Clearing Group moved to use the OIS rates for discounting derivative securities. In August 2011, the Chicago Mercantile Exchange began to use the OIS rates in settling collateral.

The premium for illiquidity is also negligible in the OIS rates due to active trading. Traders of OIS contracts are institutions, and the size of transactions is typically large. Based on the data provided by MarkitSERV to the regulators, Commodity Futures Trading Commission (2013) reports that there are 12,816 transactions on the OIS that amount to $16.878 trillion of notional value during the three months from June 1 to August 31, 2010. This is 37 percent of the total notional value of interest rate swaps transacted during the three months. The average notional value of an OIS transaction is $1.293 billion during the same period. Based on the data compiled from the Depository Trust & Clearing Corporation's (DTCC) real time Swap Data Repository (SDR), the ISDA (2013) reports that 66 percent of transactions on the dollar-denominated OIS are block trades, which are large trades with minimum size requirements.[27] It is unlikely for institutions that conduct such large transactions to surrender significant premiums to each other unless credit risk is at issue.

A.2 The Fed's Open Market Operations Since 2002

The Federal Open Market Committee (FOMC) makes all important decisions on monetary policy in their meetings. The FOMC schedules eight meetings per year, one about every six weeks. The exact meeting dates are unknown until the Fed announces the schedule through its press release. At the end of each meeting, the FOMC issues a policy statement proclaiming its decision on the target of the overnight federal funds rate and the forward guidance on the future path of the policy.

The FOMC may also hold unscheduled meetings. In some of the unscheduled meetings, the FOMC reviews economic and financial developments without issuing policy statements. During our sample period from May 6, 2002 to December 31, 2015, the FOMC held 14 such unscheduled meetings. Twelve of those meetings were unrelated to the target rate. At the other two unscheduled meetings, the FOMC issued policy statements and changed the target rate. At the meeting on January 22, 2008, the FOMC cut the target rate from 4.25% to 3.5%. At the meeting on October 8, 2008, it cut the target rate from 2% to 1.5%. Although the Fed's decisions at these two unscheduled meetings affected the OIS rates, the market only became aware of the meetings after the Fed announced the meeting decisions.

The FOMC delegates the implementation of monetary policy to the Federal Reserve Bank of New York (NY Fed). The NY Fed uses its System Open Market Account to adjust the money supply either permanently, or temporarily, or both. Traditionally, the NY Fed used repo (and re-

[27] For example, the minimum size for a block trade of the three-month OIS is $4 billion.

verse repo) transactions with primary dealers, who have accounts at clearing banks, to reduce temporary deviations of the federal funds rate from the target rate. These transactions modify reserves in the banking system and affect the federal funds rate. The Fed stopped these operations in November 2008 when the FOMC directed the NY Fed to add reserves to the banking system through large-scale purchases of mortgage-backed securities; the asset purchase program added so much liquidity to the banking system that it exceeded Fed's ability to offset by traditional operations. The suspension of repo operations lasted until December 16, 2015 when the FOMC raised the target rate to a range away from zero.

At about the same time when the Fed stopped repo operations, the Fed started to pay interest to depository institutions on both required and excess reserves. The interest on excess reserves (IOER) was originally hoped to be a tool to steer the federal funds rate toward the target. On October 6, 2008, the Fed announced that the IOER was effective on October 1, 2008. The initial IOER was set to be different from the target rate, but eventually it was set at the upper bound of the 0–25 bps target range on December 17, 2008 and stayed the same until December 16, 2015 when the IOER was raised to 0.50%. Theoretically, the IOER should serve as a floor of the federal funds rate: if the market rate of overnight funds is lower than the IOER, banks can arbitrage by borrowing funds from the market and holding the money in the reserve account at the Fed. However, the federal funds rate was traded regularly at rates well below 25 bps when the OIER was 25 bps.[28] Consequently, the federal funds rate was always within the target range during that period without the need for the Fed to conduct repo operations.[29]

Another important consequence of IOER is that the trading volume in the federal funds market fell substantially when banks held their excess reserves in their accounts at the Fed. According to the Federal Reserve Bank of New York (2015), the daily volume of overnight federal funds declined from more than \$200 billion before the crisis to about \$60 billion by the end of 2012. Since then, the daily volume has remained relatively stable around \$60 billion. Despite the decline in the trading volume, the Federal Reserve Bank of New York (2015) concludes that the overnight federal funds market has remained linked to other money market rates. Afonso, Kovner and Schoar (2011) reach a similar conclusion on the federal funds market of 2008.

[28]The Federal Reserve Bank of New York (2014) attributes the failure of arbitrage to large lenders such as Fannie Mae and Freddie Mac, which do not have reserve accounts to earn interest from the Fed, and to the change in regulations that eliminated banks' economic benefit of arbitrage.

[29]Even though there was no need for repo operations in implementing monetary policy from late 2008 to December 16, 2015, the NY Fed conducted small scale repo operations each year in order to keep the tool ready in case the change in market conditions calls for the use of the tool.

A.3 Details of MCMC Sampling for the Fed-CIR Model

The first step of Bayesian estimation is to specify prior distributions for unknown parameters. To extract the maximum information from the data, we use non-informative priors. Since κ, σ^2, and $\omega(s_j)$ are positive by definition, the prior distributions of $\log\kappa$, $\log\sigma^2$, and $\log\omega(s_j)$ are flat, following the standard approach in Bayesian statistics. The rationale of this type of priors can be found in Zelner (1971). Since λ may be either positive or negative, the prior distribution of λ is flat.

The steps θ_j should be positive, and the potential percentage change from θ_j to θ_{j+1} should be distributed around zero. Thus, $\log(\theta_1)$ has a flat distribution, and the rate of change in θ_j, with $j > 1$, is a random walk with a volatility of 100%. More precisely, $\log\theta_{j+1} - \log\theta_j$ has a normal distribution with mean −0.5 and variance 1.0. The normally distributed change of $\log\theta$ ensures that θ_j is always positive and that the potential change is smaller if current θ is lower. The mean of −0.5 implies $E[\theta_{j+1}|\,\theta_j] = \theta_j$ in the prior distribution. The variance of 1 makes the prior diffuse because two standard deviations of θ_{j+1} cover the range $(0.08\theta_j, 4.48\theta_j)$. We find that the estimation results do not change if we use a larger variance in the prior.

We also impose the Feller condition, $2\kappa\theta_j \geq \sigma^2$, in the prior distribution. This condition guarantees that the instantaneous interest rate is always positive. This constraint is easy to implement in the MCMC sampling because we can simply discard the proposed parameter draws that violate the condition.[30]

To accommodate the latent state variable, we apply the data augmentation of Tanner and Wong (1987). Essentially, besides the unknown parameters, we also obtain random draws of unobserved variable r_t in the MCMC sampling. In the CIR model, the transition probability distribution from r_{t-1} to r_t is:

$$cr_t \sim \chi^2(u,v), \quad c = 4\kappa/\{\sigma^2[1-\exp(-h\kappa)]\}, \tag{42}$$

$$u = 4\kappa\theta_{j_t}/\sigma^2, \quad v = cr_{t-1}\exp(-h\kappa), \tag{43}$$

where $\chi^2(u,v)$ is the non-central χ^2 distribution with u degrees of freedom and the non-centrality parameter v. As in early sections, h denotes the overnight term as described in subsection A.1. Let $f(r_t\,|\,r_{t-1})$ denote the transition probability density. The distribution of the instantaneous rates is

$$p(R\,|\,\phi,\Theta) = \prod_{t=1}^{T} f(r_t\,|\,r_{t-1}). \tag{44}$$

The above distribution depends on r_0, which is also treated as an unknown parameter. In other words, we expand ϕ to include r_0, so that $\phi = (\kappa,\sigma,\lambda,r_0)'$. We set the prior distribution of $\log(r_0)$ to be flat.

[30]The ease of imposing nonlinear constraints is another advantage of the MCMC Bayesian method. By contrast, a nonlinear constraint such as the Feller condition of the CIR model can be difficult to incorporate in maximum likelihood.

The posterior distribution of the unknown parameters ϕ, Θ, and Ω and the latent R is

$$p(\phi, \Theta, \Omega, R \,|\, Y) \propto L(Y|R, \phi, \Theta, \Omega) \cdot p(R \,|\, \phi, \Theta) \cdot p(\phi, \Theta, \Omega), \tag{45}$$

where $L(Y\,|R, \phi, \Theta, \Omega)$ is the likelihood function, and $p(\phi, \Theta, \Omega)$ denotes the joint prior distribution specified above for the parameters.

The posterior distribution in the equation above is not known analytically. We apply the MCMC method — we draw random samples of each unknown parameter and state variable (or a subset of them) while conditioning on the others, and iterate among them. The basic idea of the MCMC is that iterative sampling from conditional posterior distributions converges to the joint posterior distribution.

For almost all parameters and the state variable, the conditional posterior distribution is not one of the standard distributions. To sample these distributions, we apply the Metropolis-Hastings algorithm, which proposes a draw and then decides whether to accept or reject. An introduction to the Metropolis-Hastings method is Chib and Greenberg (1995). Following the recommendation from the literature, we use the random walk proposal distribution with a target average acceptance of about 30%. The only exception is the conditional distribution of the pricing error variance ω_i^2, which can be derived from equation (22) :

$$p(\omega_i^2 \,|\, r, \phi, \Theta, \Omega_{-i}, Y) \propto \omega_i^{-1} \exp\big[-\textstyle\sum_{t=1}^{T} I(i \le m_t)(y_{ti} + A_{ti} - B_{ti} r_t)^2/\omega_i^2\big], \tag{46}$$

where Ω_{-i} represents the subset of Ω obtained by excluding ω_i. The summation is over those dates with $i \le m_t$ because on each date t, only the yields with maturities $s_i \le s_{m_t}$ are included in the estimation. The above density function is an inverse gamma distribution, from which we draw samples of ω_i.

The MCMC Bayesian estimation of multifactor CIR models is similar. The priors of the parameters are similar to those in the Fed-CIR model. Lamoureux and Witte (2002) provide the detailed framework of the Bayesian estimation of multifactor CIR models. The implementation of the MCMC Bayesian estimation for Gaussian models and jump models follow the same idea.

B References

Ahn, D., and R. Dittmar, and A. Gallant, 2002, Quadratic term structure models: Theory and evidence, *Review of Financial Studies*, 15, 243–288.

Afonso, G., A. Kovner, and A. Schoar, 2011, Stressed, not frozen: The Fed funds market in the financial crisis, *Journal of Finance*, 16, 1109–1189.

Ang, A., and G. Bekaert, 2002, Regime switches in interest rates, *Journal of Business and Economic Statistics*, 20, 163–182.

Ang, A., G. Bekaert, and M. Wei, 2008, The term structure of real rates and expected inflation, *Journal of Finance*, 63, 797–849.

Ang, A., J. Boivin, S. Dong, and R. Loo-Kung, 2011, Monetary policy shifts and the term structure, *Review of Economic Studies*, 78, 429–457.

Ang, A., and M. Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics*, 50, 745–787.

Bansal, R., and H. Zhou, 2002, Term structure of interest rates with regime shifts, *Journal of Finance*, 57, 1997–2043.

Bernanke, B., 2005, The global saving glut and the U.S. current account deficit, speech at the Homer Jones Lecture, St. Louis, Missouri.

Bernanke, B., C. Bertaut, L. DeMarco, and S. Kamin, 2011, International capital flows and the returns to safe assets in the United States, 2003–2007, Working paper, Board of Governors of the Federal Reserve System.

Bernanke, B., 2013, *The Federal Reserve and the Financial Crisis*, Princeton University Press, New Jersey.

Bernanke, B., 2015, *The Courage to Act: A Memoir of a Crisis and its Aftermath*, W. W. Norton and Company, New York.

Brown, S., and P. Dybvig, 1986, The Empirical implications of the Cox, Ingersoll, Ross theory of the term structure of interest rates, *Journal of Finance*, 41, 617–630.

Casella, G., and George, E., 1992, Explaining the Gibbs sampler, *The American Statistician*, 46, 167–174.

Chib, S., and E. Greenberg, 1995, Understanding the Metropolis-Hastings algorithm, *The American Statistician*, 49, 327–335.

Commodity Futures Trading Commission, 2013, Procedures to establish appropriate minimum block sizes for large notional off-facility swaps and block trades, *Federal Register, Rules and Regulations*, Vol. 78, No. 105.

Cox, J., and J. Ingersoll and S. Ross, 1985, A theory of the term structure of interest rates, *Econometrica*, 53, 385–407.

Dai, Q., and K. Singleton, 2000, Specification analysis of affine term structure models, *Journal of Finance*, 50 (5), 1943–1978.

Dai, Q., and K. Singleton, 2002, Expectation puzzles, time-varying risk premia, and dynamic models of the term structure, *Journal of Financial Economics*, 63, 415–441.

Dai, Q., K. Singleton, and W. Yang, 2007, Regime shifts in a dynamic term structure model of U.S. Treasury bond yields, *Review of Financial Studies*, 20, 1669–1706.

Duarte, J., 2004, Evaluating an alternative risk preference in affine term structure models, *Review of Financial Studies*, 17, 379–404.

Duffee, G., 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance*, 57, 405–443.

Duffie, D., and M. Huang, 1996, Swap rates and credit quality, *Journal of Finance*, 51, 921–949.

Duffie, D., and R. Kan, 1996, A yield-factor model of interest rates, *Mathematical Finance*, 6, 379–406.

Estrella, A., and M. Trubin, 2006, The yield curve as a leading indicator: Some practical issues, *Current Issues In Economics And Finance*, 12(5), Federal Reserve Bank of New York.

Fama, E., 2012, Does the Fed control interest rates? Working paper, University of Chicago.

Fama, E., and R. Bliss, 1987, The information in long-maturity forward rates, *American Economic Review*, 77, 680–692.

Federal Reserve Bank of New York, 2014, *Domestic open market operations during 2013*.

Federal Reserve Bank of New York, 2015, *Domestic open market operations during 2014*.

Gelman, A., and I. Pardoe, 2006, Bayesian measures of explained variance and pooling in multilevel (hierarchical) models, *Technometrics*, 48, 241–251.

Geweke, J., 2005, *Contemporary Bayesian Econometrics and Statistics*, Wiley.

Gibbons, M., and K. Ramaswamy, 1993, A test of the Cox, Ingersoll, and Ross model of the term structure, *Review of Financial Studies*, 6, 620–658.

Heidari, M., and L. Wu, 2010, Market anticipation of Fed policy changes and the term structure of interest rates, *Review of Finance*, 14, 313–342.

Hördahl, P., O. Tristani, and D. Vestin, 2006, A joint econometric model of macroeconomic and term structure dynamics, *Journal of Econometrics*, 131, 405–444.

Hördahl, P., E. Remolona, and G. Vlente, 2015, Expectations and risk premia at 8:30 AM: What does the yield curve say about macroeconomic announcements?, BIS Working Paper.

International Swaps and Derivatives Association (ISDA), 2013, *The Annotated ISDA 2013 Standard Credit Support Annex*.

Johannes, M., 2004, The statistical and economic role of jumps in continuous-time interest rate models, *Journal of Finance*, 59, 227–260.

Joslin, S., A. Le, and K. Singleton, 2013, Why Gaussian macro-finance term structure models are (nearly) unconstrained factor-VARs, *Journal of Financial Economics*, 109, 604–622.

Joslin, S., M. Priebsch, and K. Singleton, 2014, Risk premiums in dynamic term structure models with unspanned macro risks, *Journal of Finance*, 69, 1197–1233.

Kim, D., and J. Wright, 2014, Jumps in bond yields at known times, Working paper, Federal Reserve Board.

Kuttner, K., 2001, Monetary policy surprises and interest rates: Evidence from the Fed funds futures market, *Journal of Monetary Economics*, 47(3), 523–544.

Lamoureux, C., and H. Witte, 2002, Empirical analysis of the yield curve: The information in the data viewed through the window of Cox, Ingersoll, and Ross, *Journal of Finance*, 57, 1479–1520.

Litterman, R., and J. Scheinkman, 1991, Common factors affecting bond returns, *Journal of Fixed Income*, 1, 54–61.

McAndrews, J., A. Sarkar, and Z. Wang, 2015, The Effect of term auction facility on the London interbank offered rate, Federal Reserve Bank of New York, Staff Report No. 335.

Pearson, N., and T. Sun, 1994, Exploiting the conditional density in estimating the term structure: An application to the Cox, Ingersoll, and Ross model, *Journal Of Finance*, 49, 1279–1303.

Piazzesi, M., 2009, Affine term structure models, in *Handbook of Financial Econometrics,* eds. Y. Aït-Sahalia and L. P. Hansen, Elsevier, North Holland.

Piazzesi, M., 2005, Bond yields and the Federal Reserve, Journal of Political Economy, 113, 311–344.

Rudebusch, G., and T. Wu, 2008, A macro-finance model of the term structure, monetary policy, and the economy, *Economic Journal*, 118, 906–926.

Singleton, K., 2006, *Empirical Dynamic Asset Pricing*, Princeton University Press, New Jersey.

Smith, A., and A. Gelfand, 1992, Bayesian statistics without tears: a sampling-resampling perspective, *The American Statistician*, 46, 84–88.

Tanner, M., and W. Wong, 1987, The calculation of posterior distributions by data augmentation, *Journal of the American Statistical Association,* 82, 528-550.

Wang, C., 1996, Valuation of interest rate swaps with term structure information, unpublished dissertation, Columbia University.

Woodford, M., 2013, Forward guidance by inflation-targeting central banks, Working paper, Columbia Unviersity.

Zellner, A., 1971, *An Introduction to Bayesian Inference in Econometrics,* Wiley, New York.

Zellner, A., 1985, Bayesian econometrics, *Econometrica*, 53, 253–269.

C Tables and Figures

Table 1: Summary Statistics of the Interest Rates

The sample mean, standard deviation, minimum, median, and maximum are calculated for daily observations of the effective federal funds rate and the OIS rates of 15 maturities for the period from May 6, 2002 to December 31, 2015. All numbers are in percentage points.

Maturity	Mean	Stdev	Min	Median	Max
Overnight	1.44	1.77	0.040	0.21	5.41
1 week	1.43	1.76	0.055	0.22	5.34
2 week	1.44	1.76	0.042	0.22	5.35
3 week	1.44	1.77	0.058	0.22	5.35
1 month	1.44	1.77	0.060	0.22	5.35
2 month	1.45	1.77	0.064	0.24	5.38
3 month	1.46	1.78	0.063	0.25	5.40
4 month	1.47	1.78	0.063	0.26	5.43
5 month	1.48	1.78	0.063	0.28	5.45
6 month	1.49	1.78	0.063	0.29	5.47
7 month	1.50	1.78	0.063	0.31	5.49
8 month	1.51	1.78	0.065	0.33	5.50
9 month	1.53	1.77	0.066	0.36	5.51
10 month	1.54	1.77	0.065	0.40	5.51
11 month	1.56	1.76	0.063	0.44	5.51
12 month	1.58	1.75	0.060	0.48	5.52

Table 2: Estimated Parameters of the Fed-CIR Model

The summary statistics of the posterior distributions of the parameters of the Fed-CIR model are obtained from 1000 random draws after 40,000 iterations in the MCMC sampling. The summary statistics are the mean, standard deviation, median, and the 5th and 95th percentiles. The statistics are reported in actual numbers. The header of each panel specifies the sample period for the estimation.

Panel A: From May 6, 2002 to December 31, 2015

$R^2 = 82.50\%$	Mean	Stdev	5%	Median	95%
σ	0.0352	0.0005	0.0344	0.0352	0.0360
κ	0.6270	0.0841	0.4913	0.6320	0.7625
λ	-0.2815	0.0841	-0.4146	-0.2879	-0.1445

Panel B: From May 6, 2002 to October 3, 2008

$R^2 = 82.69\%$	Mean	Stdev	5%	Median	95%
σ	0.0346	0.0007	0.0335	0.0347	0.0358
κ	0.4925	0.0775	0.3383	0.4943	0.6103
λ	-0.1558	0.0775	-0.2715	-0.1580	-0.0014

Panel C: From October 6, 2008 to December 31, 2015

$R^2 = 78.57\%$	Mean	Stdev	5%	Median	95%
σ	0.0339	0.0006	0.0328	0.0339	0.0349
κ	2.4369	0.3650	1.9305	2.3827	2.9355
λ	-2.2948	0.3633	-2.7834	-2.2440	-1.7824

Table 3: Estimated Parameters of Conventional CIR Models

The summary statistics of the posterior distributions of the parameters of the single-factor and two-factor CIR models are obtained from 1000 random draws after 40,000 iterations in the MCMC sampling. The summary statistics are the mean, standard deviation, median, and the 5th and 95th percentiles. The statistics are reported in actual numbers. The sample period is from May 6, 2002 to December 31, 2015.

Panel A: Single-Factor CIR Model

$R^2 = 10.31\%$	Mean	Stdev	5%	Median	95%
θ	0.0111	0.0039	0.0061	0.0105	0.0197
σ	0.0375	0.0045	0.0308	0.0376	0.0432
κ	0.2477	0.0779	0.1342	0.2369	0.3861
λ	-0.2439	0.0778	-0.3785	-0.2379	-0.1254

Panel B: Two-Factor CIR Model

$R^2 = 86.20\%$	Mean	Stdev	5%	Median	95%
θ_1	0.0062	0.0090	0.0018	0.0035	0.0182
θ_2	0.0123	0.0263	0.0030	0.0062	0.0326
σ_1	0.0470	0.0007	0.0459	0.0471	0.0483
σ_2	0.0445	0.0007	0.0434	0.0445	0.0457
κ_1	0.3016	0.1546	0.0567	0.2929	0.5691
κ_2	0.1612	0.0884	0.0296	0.1551	0.3214
λ_1	-0.9704	0.1545	-1.2384	-0.9586	-0.7259
λ_2	0.5057	0.0884	0.3456	0.5108	0.6395

Table 4: Estimated Parameters of the Three-Factor Gaussian Model

The summary statistics of the posterior distributions of the parameters of the three-factor Gaussian model are obtained from 1000 random draws after 40,000 iterations in the MCMC sampling. The summary statistics are the mean, standard deviation, median, and the 5th and 95th percentiles. The statistics are reported in actual numbers. The sample period is from May 6, 2002 to December 31, 2015.

$R^2 = 97.76\%$	Mean	Stdev	5%	Median	95%
δ	0.0150	0.0016	0.0122	0.0152	0.0176
κ_{11}	0.0331	0.0221	0.0024	0.0296	0.0733
κ_{22}	0.7572	0.4182	0.2209	0.7240	1.4705
κ_{33}	3.0114	0.9090	1.2254	3.1265	4.3013
κ_{21}	-0.0846	0.0525	-0.1738	-0.0834	0.0123
κ_{31}	-0.0285	0.0309	-0.0850	-0.0264	0.0175
κ_{32}	-0.5351	0.6572	-1.9309	-0.4210	0.1974
σ_{11}	0.0030	0.0001	0.0028	0.0031	0.0032
σ_{22}	0.0027	0.0002	0.0024	0.0027	0.0030
σ_{33}	0.0016	0.0003	0.0014	0.0016	0.0020
μ_1	-0.0015	0.0031	-0.0084	-0.0009	0.0020
μ_2	-0.0010	0.0029	-0.0052	-0.0006	0.0031
μ_3	-0.0008	0.0014	-0.0032	-0.0008	0.0005
λ_{11}	0.0037	0.0441	-0.0785	0.0088	0.0559
λ_{21}	-0.0556	0.0539	-0.1484	-0.0530	0.0279
λ_{31}	0.0370	0.0351	-0.0067	0.0357	0.0954
λ_{12}	3.6310	1.0722	2.1242	3.6155	5.2033
λ_{22}	1.2771	0.5960	0.2601	1.3513	2.1837
λ_{32}	0.5314	1.0867	-0.6210	0.3297	3.1725
λ_{13}	-3.2090	1.1949	-5.0368	-3.1829	-1.1666
λ_{23}	3.2484	1.9631	0.1616	3.5912	4.9992
λ_{33}	-3.4166	0.9796	-4.8132	-3.5306	-1.9195

Table 5: Estimated Parameters of the Four-Factor Jump Model

The summary statistics of the posterior distributions of the parameters of the four-factor jump model are obtained from 1000 random draws after 40,000 iterations in the MCMC sampling. The summary statistics are the mean, standard deviation, median, and the 5th and 95th percentiles. The statistics are reported in actual numbers. The sample period is from May 6, 2002 to December 16, 2008.

$R^2 = 98.17\%$	Mean	Stdev	5%	Median	95%
$\bar{v}$	0.0006	0.0003	0.0003	0.0005	0.0013
σ_v	0.0689	0.0123	0.0556	0.0663	0.0878
$\bar{\rho}$	65.53	34.29	18.67	62.12	135.06
ρ_θ	-5999	299	-6473	-5956	-5601
ρ_u	-22424	1922	-25108	-22744	-18860
ρ_v	69945	10630	54903	69110	89895
ρ_z	335.0	11.3	316.5	334.8	355.0
κ_u	3.4207	0.9609	2.3368	3.1388	5.1473
κ_v	4.1865	1.7440	1.7301	3.9174	7.3157
κ_z	1.6769	0.0561	1.5916	1.6830	1.7532
λ_u	-28.0944	4.8836	-37.4664	-27.7234	-21.5887
λ_v	-1.0571	1.9228	-4.7946	-0.8538	1.5940
λ_z	-1.5143	0.1198	-1.7122	-1.4982	-1.3772

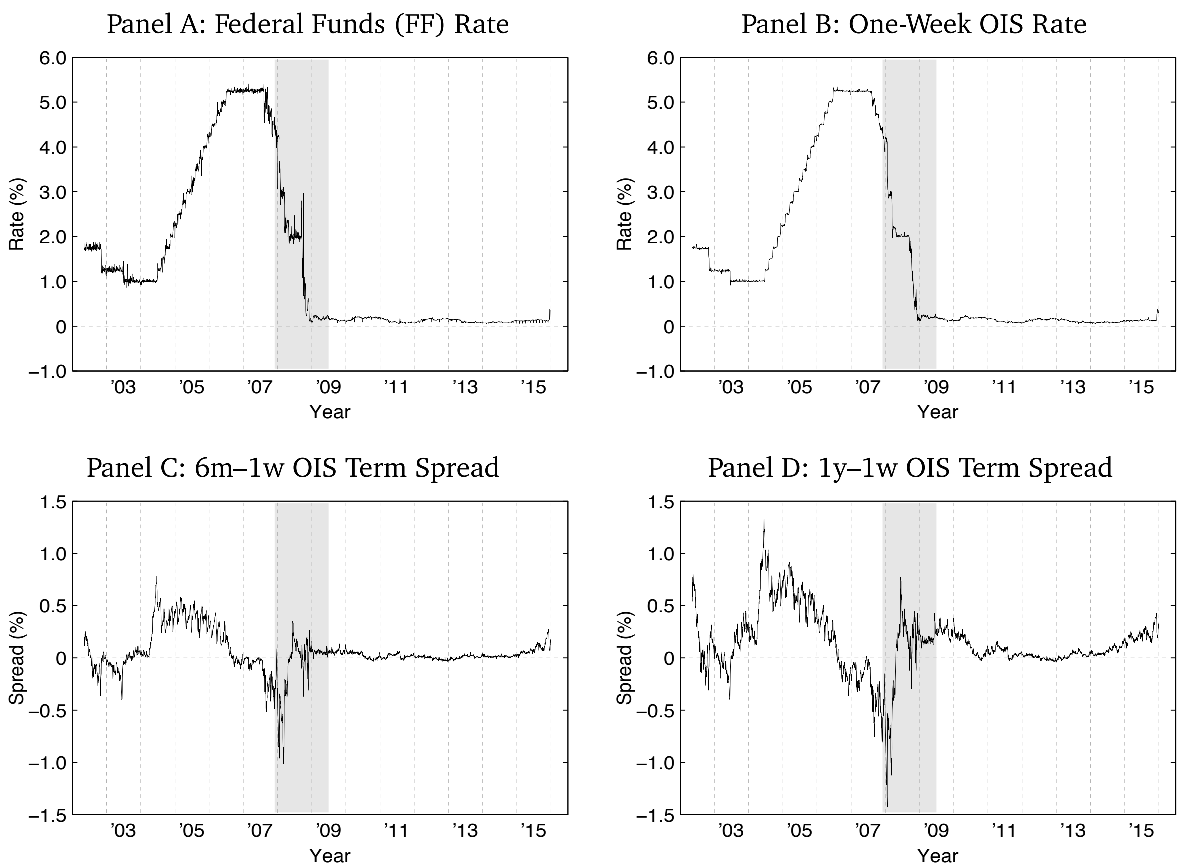

Figure 1: The Federal Funds and OIS Rates

Panel A plots the time series of the overnight federal funds rate. Panel B plots the one-week OIS rate. Panel C plots the spread between the six-month and one-week OIS rates. Panel D plots the spread between the one-year and one-week OIS rates. The sample period is from May 6, 2002 to December 31, 2015. The light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

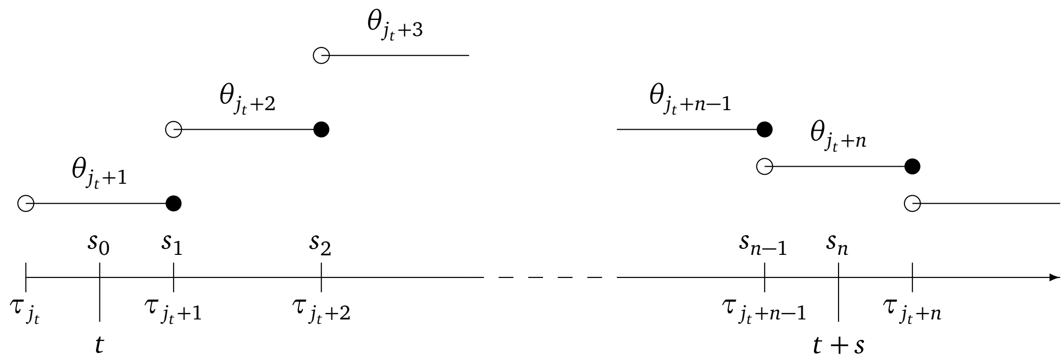

Figure 2: The Reversion Center in the Fed-CIR Model

This is an illustration of the step function of ϑ_t and the scheduled FOMC meeting dates involved in pricing a zero-coupon bond at time t. The time to maturity of the bond is s. The FOMC meeting dates are τ_{j_t+i} for $i = 0, \cdots, n$. The function values of ϑ_t are θ_{j_t+i} for $i = 1, \cdots, n$.

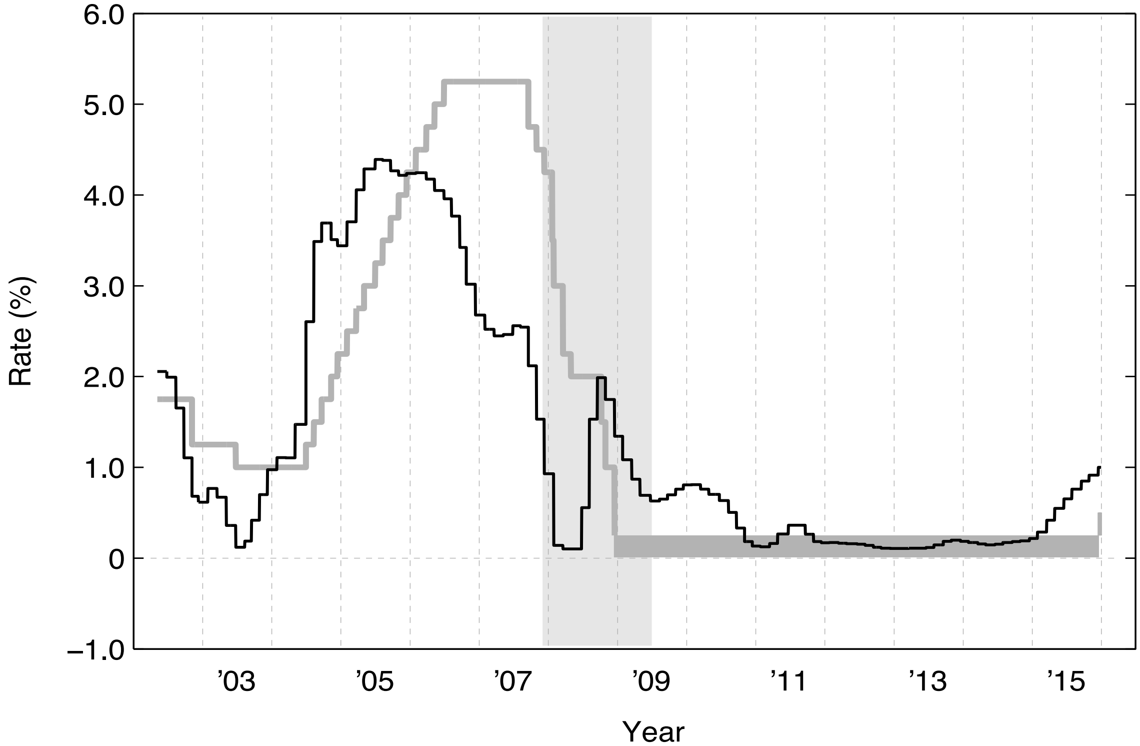

Figure 3: The Reversion Center of the Instantaneous Rate

The thin solid line is the posterior mean of θ_j, the reversion center of the instantaneous rate in the Fed-CIR model. The sample period is from May 6, 2002 to December 31, 2015. The gray line, which turns into a band since December 17, 2008, is the Fed's target rate. The light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

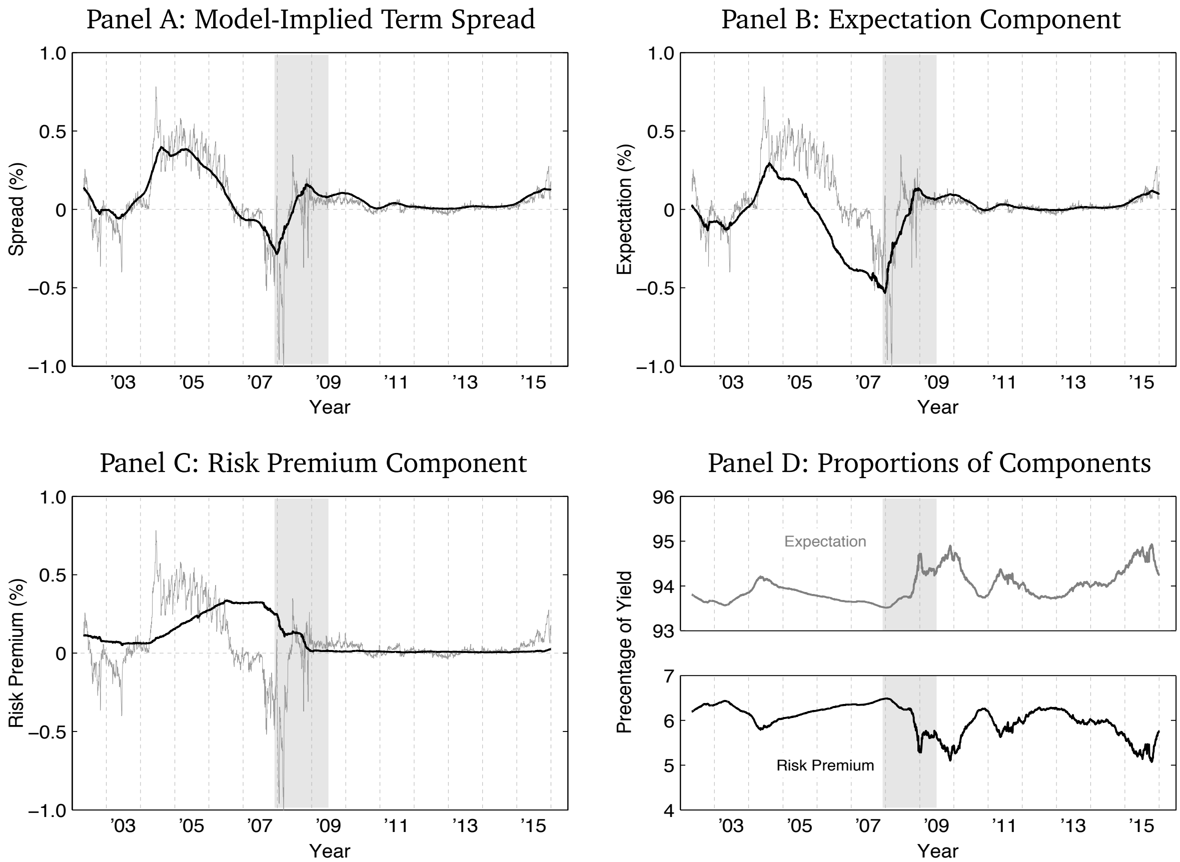

Figure 4: Expectation and Risk Premium Estimated by the Fed-CIR Model

In panels A, B, and C, the gray line is the observed term spread between the six-month and 1-week OIS rates. The solid line in panel A is the term spread between the six-month and 1-week OIS rates implied by the Fed-CIR model. The model-implied term spread is decomposed into the expectation and risk premium components. The solid line in panel B is the expectation component, and the solid line in panel C is the risk premium component. In panel D, the gray line is the percentage of the expectation component in the six-month OIS rate, while the solid line is the percentage of the risk premium component in the same rate. The sample period is from May 6, 2002 to December 31, 2015. In each panel, the light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

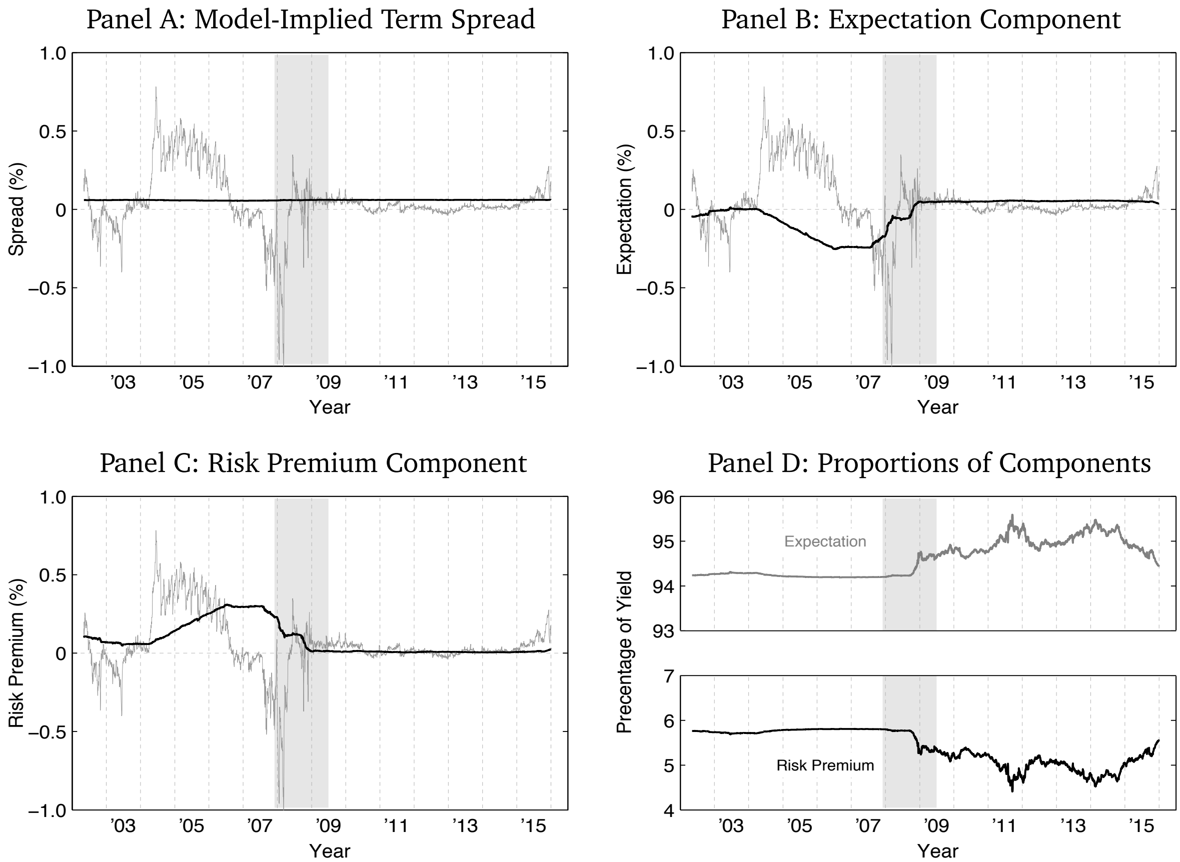

Figure 5: Expectation and Risk Premium Estimated by the Single-Factor CIR Model

In panels A, B, and C, the gray line is the observed term spread between the six-month and 1-week OIS rates. The solid line in panel A is the term spread between the six-month and 1-week OIS rates implied by the single-factor CIR model. The model-implied term spread is decomposed into the expectation and risk premium components. The solid line in panel B is the expectation component, and the solid line in panel C is the risk premium component. In panel D, the gray line is the percentage of the expectation component in the six-month OIS rate, while the solid line is the percentage of the risk premium component in the same rate. The sample period is from May 6, 2002 to December 31, 2015. In each panel, the light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

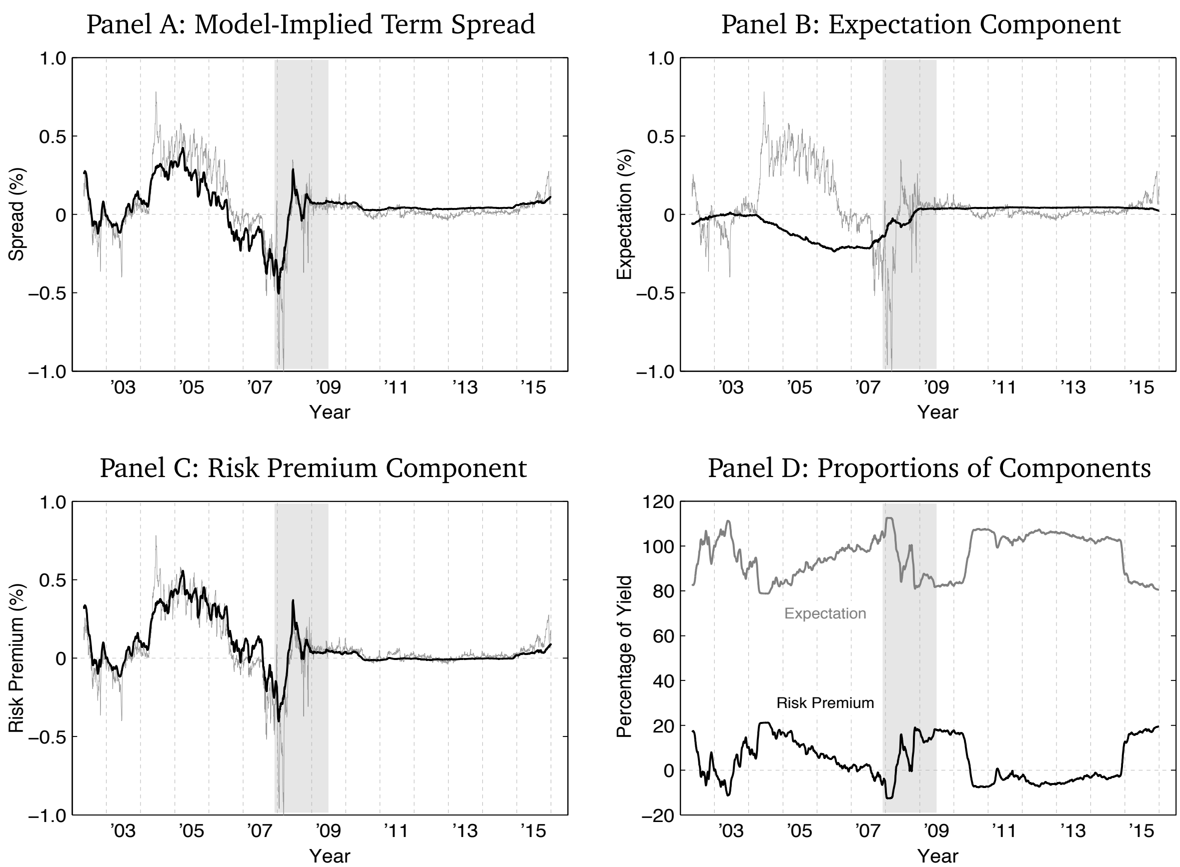

Figure 6: Expectation and Risk Premium Estimated by the Two-Factor CIR Model

In panels A, B, and C, the gray line is the observed term spread between the six-month and 1-week OIS rates. The solid line in panel A is the term spread between the six-month and 1-week OIS rates implied by the two-factor CIR model. The model-implied term spread is decomposed into the expectation and risk premium components. The solid line in panel B is the expectation component, and the solid line in panel C is the risk premium component. In panel D, the gray line is the percentage of the expectation component in the six-month OIS rate, while the solid line is the percentage of the risk premium component in the same rate. The sample period is from May 6, 2002 to December 31, 2015. In each panel, the light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

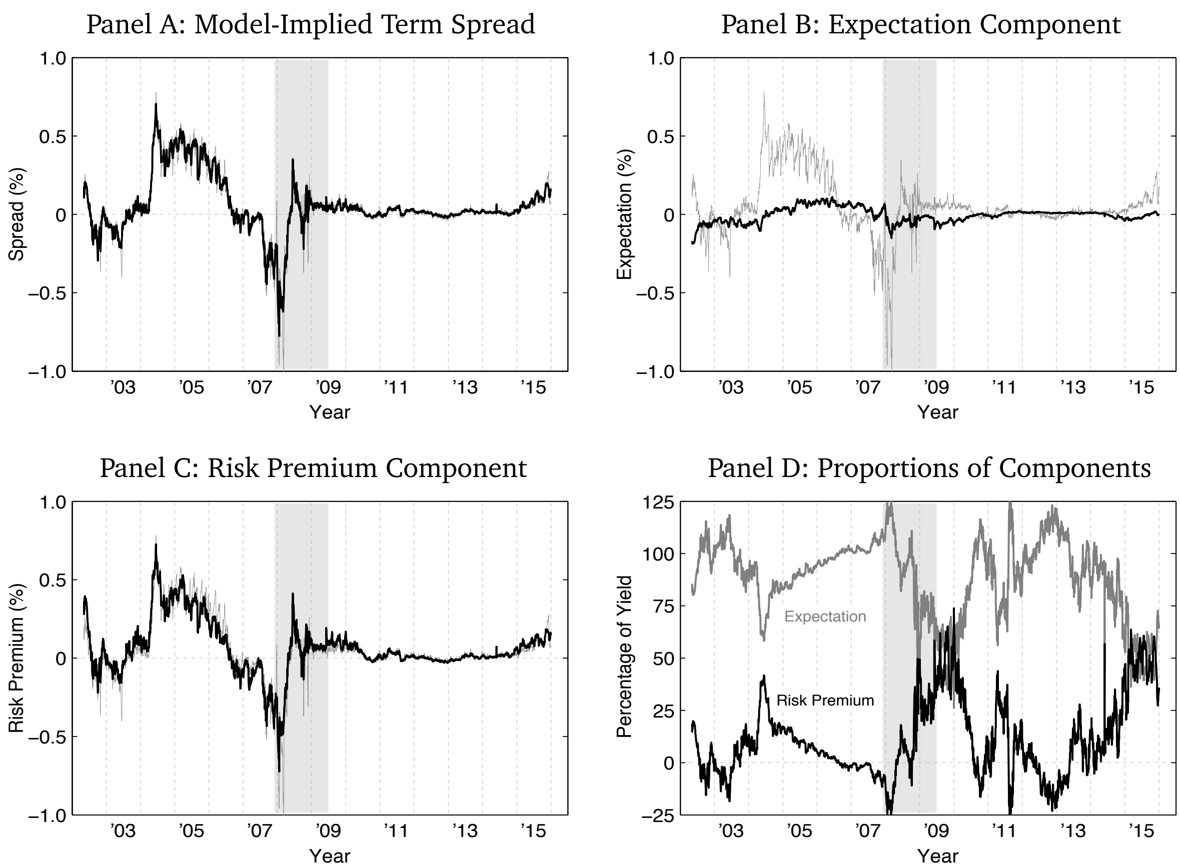

Figure 7: Expectation and Risk Premium Estimated by the Three-Factor Gaussian Model

In panels A, B, and C, the gray line is the observed term spread between the six-month and 1-week OIS rates. The solid line in panel A is the term spread between the six-month and 1-week OIS rates implied by the three-factor Gaussian model. The model-implied term spread is decomposed into the expectation and risk premium components. The solid line in panel B is the expectation component, and the solid line in panel C is the risk premium component. In panel D, the gray line is the percentage of the expectation component in the six-month OIS rate, while the solid line is the percentage of the risk premium component in the same rate. The sample period is from May 6, 2002 to December 31, 2015. In each panel, the light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.

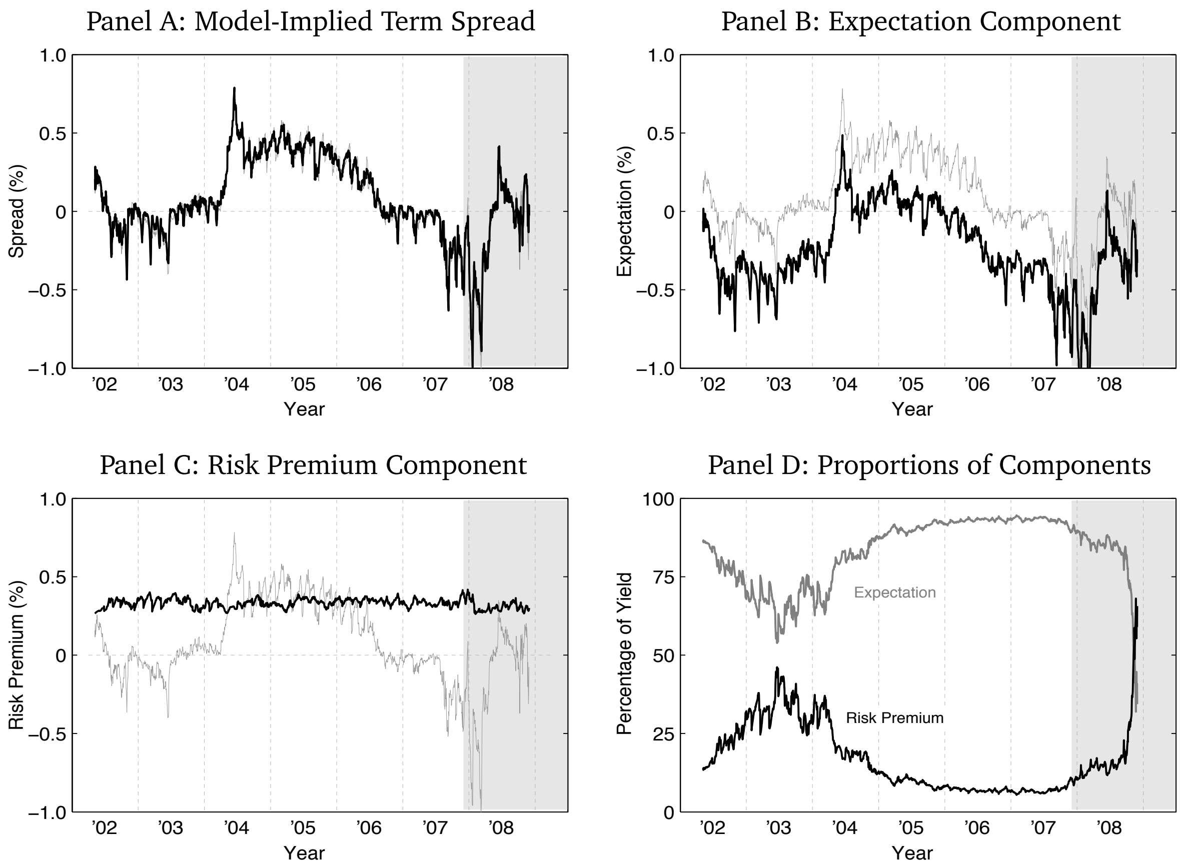

Figure 8: Expectation and Risk Premium Estimated by the Four-Factor Jump Model

In panels A, B, and C, the gray line is the observed term spread between the six-month and 1-week OIS rates. The solid line in panel A is the term spread between the six-month and 1-week OIS rates implied by the four-factor Jump model. The model-implied term spread is decomposed into the expectation and risk premium components. The solid line in panel B is the expectation component, and the solid line in panel C is the risk premium component. In panel D, the gray line is the percentage of the expectation component in the six-month OIS rate, while the solid line is the percentage of the risk premium component in the same rate. The sample period is from May 6, 2002 to December 16, 2008. In each panel, the light gray area from December 2007 to June 2009 indicates the NBER dating of the Great Recession related to the 2007–2008 financial crisis.