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Incentives and performance in the presence of wealth effects and endogenous risk

Ming Guo[a], Hui Ou-Yang[b,*]

[a]*Department of Economics, Duke University, Durham, NC 27708, USA*
[b]*Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA*

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Abstract

Two of the most widely tested predictions of agency theory are that there exists a negative association between an agent's pay-performance sensitivity (PPS) and the risk of output, and that PPS enhances performance. Empirical evidence has been mixed. This paper proposes a new utility function and develops a model that introduces a "wealth effect" and also allows the agent to control the (idiosyncratic) risk of output. When risk is endogenous, the paper shows that the two predictions may not hold.

JEL classification: J33; G30; M40

Keywords: Agency theory; Incentives and performance; Endogenous risk; Wealth effects

1. Introduction

Two of the most widely tested results of agency theory are that there is a negative tradeoff between risk and incentives and that managerial incentives enhance firm performance. Here incentives or pay-performance sensitivities (PPS) often represent the fraction of the total output that an agent receives. Under general utility functions as well as general output processes, it is usually not possible to characterize explicitly optimal contracts and other properties that can be tested empirically. Instead, to generate testable implications, the

* Corresponding author. Fax: +1 919 660 7971.
E-mail addresses: mg22@duke.edu (M. Guo), huiou@duke.edu (H. Ou-Yang).

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literature typically adopts specific assumptions regarding both an agent's preference and the output process under the agent's control. For example, following Holmström and Milgrom [19,20], a large number of papers assume that the agent is risk averse with a negative exponential utility function and that his effort or action influences only the mean of a normally distributed output. Consequently, the agent's initial wealth has no impact on the provision of incentives, and the risk of the output is exogenously given. Because the agent is averse to risk, the higher the risk, the lower the PPS. Because risk is beyond the agent's control, the higher the PPS, the higher the mean of the output, thus the output is higher on average. [1] In addition, following Leland and Pyle [29], it is generally believed that higher proportional ownership of a firm by its managers, which serves as a signaling device in this model, leads to a higher value of the firm. [2]

The two results of agency theory have been subject to extensive empirical testing; however, the studies have yielded mixed results. As Prendergast [39] points out, the empirical work testing for a negative tradeoff between risk and incentives has not had much success. In various corporate settings that involve executives of publicly traded firms, Lambert and Larker [26], Aggarwal and Samwick [1], Jin [24], and Garvey and Milbourn [12] find a negative relation; Demsetz and Lehn [10] and Core and Guay [8] find a positive relation; while, Garen [11], Yermack [44], Bushman et al. [6], and Ittner et al. [21] find no significant relation. In other settings that involve sharecroppers and decisions to franchise, positive or no significant relations have been most likely to arise. [3] In addition, the empirical evidence for the relation between incentives and performance has been mixed. Morck et al. [33], McConnell and Servaes [31], and Lazear [27] find a positive relation; whereas, Himmelberg et al. [17] and Palia [37] find little evidence that managerial incentives enhance performance.

Given that exponential-normal assumptions are so prevalent in the agency literature and that the predictions based on them have been taken almost literally, it is of great importance to examine their robustness. The objective of this paper is to provide such a study in a framework that incorporates wealth effects as well as endogenizes risk. One way to incorporate wealth effect is to assume a power utility function for the agent. We will explain later that with the power utility function, one can neither obtain the agent's expected utility function explicitly nor verify the sufficiency of the agent's solutions. To introduce wealth effects in a tractable manner, we develop a new utility function, whose absolute risk aversion decreases with the level of the agent's consumption. More specifically, this utility function is a linear combination of multiple negative exponential functions; we term it as an *extended negative exponential utility function* or the *ENE utility function* for brevity. With the ENE utility function, one can both solve the agent's maximization problem in closed form and verify the sufficiency of the agent's first-order conditions.

In our model, the agent can affect separately both the mean and the variance of output at a personal cost. In other words, the agent takes two separate actions to manage the mean and the risk of the output. [4] The agent's cost increases when he chooses a higher mean or a

[1] See, for example, [41] for an extension of the Holmström–Milgrom [19] model, and [13,34,38] for comprehensive reviews.

[2] See also [9,25,30].

[3] See, for example, [39] for a comprehensive summary of the empirical results.

[4] Since the output is normally distributed, we shall use *variance* and *risk* interchangeably.

lower risk, where choosing a higher mean or a lower risk should be understood as the agent exerting effort to increase the mean or decrease the risk of the firm's output.

The potential conflicts between principal and agent arise because (1) the agent's actions are unobservable to the principal and (2) the agent is more risk averse toward firm-specific risks than the principal, so that he may prefer a different risk level. For example, the manager (agent) of a firm may not expend sufficient amount of effort in increasing the level of the firm's cash flow, but instead may expend excessive amount of effort in managing certain firm-specific risks. The shareholders (principals) of the firm prefer a high level of the firm's cash flow but may be unconcerned about firm-specific risks because they can diversify away firm-specific risks by holding a fully diversified portfolio. Since shareholders cannot perfectly monitor the manager's activities, they must provide appropriate incentives to induce the manager to take appropriate actions with regard to the level and riskiness of the firm's output.

When risk is endogenously determined, we show that managerial incentives proxied by PPS or managerial ownership of the output may be positively or negatively related to risk. The agent maximizes his expected utility. Given a higher PPS, the agent may choose a higher or lower level of risk, depending on the relative cost of choosing risk and mean. He may be better off increasing or decreasing simultaneously both the mean and the risk of the output. [5] Suppose that the agent's cost of managing risk is relatively high compared to that of managing mean. For a high PPS the agent may choose a high level of risk to save cost while choosing a high level of mean. Hence, PPS would be positively related to risk in this case. [6] On the other hand, given a high PPS, the agent may choose a low risk level if his cost of managing risk is low, leading to a negative relation between PPS and risk. If cross-sectional regressions are performed, then positive, negative, or insignificant results may arise, depending on the sample and the control variables employed.

Under an ENE utility function, changes in the agent's initial wealth can affect the PPS-risk tradeoff in two competing ways. An increase in the agent's initial wealth makes the agent less averse to risk, leading to a potentially higher PPS. When PPS is higher, the agent may increase or decrease risk, depending on his cost of managing mean and risk. On the one hand, given a higher PPS, the agent may want to both increase the mean and decrease the risk, if his cost of managing mean (relative to that of managing risk) is low. A high mean benefits both principal and agent, and a low risk means a low risk premium in the agent's compensation. In this case, a negative relation between PPS and risk arises. On the other hand, if the agent's cost of managing mean is relatively high, given a higher PPS, he will still try to increase the mean, but he may also increase risk to save cost because the agent can now tolerate more risk due to a higher initial wealth. A positive relation between PPS and risk thus follows.

Regarding the relation between incentives and performance, we show that a higher PPS does not necessarily lead to better performance measured by the expected value of the output

[5] Economically, this statement means that the agent exerts high (low) effort in managing mean (risk).

[6] Notice that when risk is exogenously given, a higher PPS means that the agent must bear more risk. Since the agent is risk averse, he demands a risk premium for bearing risk, which represents a cost to the principal. The marginal increase in the agent's risk premium increases with the level of risk. Therefore, PPS decreases with risk in equilibrium.

less the agent's compensation. When the principal is risk neutral, this measure represents a proxy for the market price of the project. Again, given a higher PPS, a risk-averse agent does not necessarily choose a higher mean. He may choose both a lower mean and a lower risk to maximize his expected utility, depending on the costs between choosing mean and choosing risk. Since the principal is less averse to risk than the agent, she may not value a lower risk as much as the agent does. Consequently, the value of a project may be lower with a higher PPS. In contrast, in the absence of both the wealth effect and the endogenous control of risk, PPS serves as a unique predictor for incentives. The higher the PPS, the higher the principal's net amount of the output. We show that once a wealth effect and the control of risk are introduced, both PPS and the agent's constant compensation can serve as incentive devices.

In summary, the negative relation between PPS and risk and the positive relation between PPS and performance do not always hold in our model. The model may shed light on why the empirical evidence on these relations are mixed. This paper thus raises an important question: what do we really mean by more or fewer incentives? In the absence of a definitive measure for incentive, it would be difficult for a theoretical or empirical model to have predictive power or practical relevance.

In a related study, Prendergast [39] develops a model in which a positive relation between risk and incentives may arise. He accounts for an effect of uncertainty on incentives with the possibility of monitoring and delegation. The marginal returns to delegation are likely lower in more risky environments, as a principal may have little idea about the right actions to take. Therefore, higher incentives are needed to induce increased effort from an agent. In a more stable environment, a principal may be able to monitor an agent's input so that high incentives are unnecessary. Bitler et al. [4] consider a wealth effect in an entrepreneurial setting. Among other contributions, they find that entrepreneurial ownership shares increase with outside wealth and decrease with exogenously specified firm risk. Because the entrepreneur's expected utility function cannot be computed explicitly, it is intractable to verify, analytically or numerically, the sufficiency of the first-order conditions. See also [2,42]. These studies do not consider the relation between an agent's incentive and his performance as measured by the agent's expected net profit. [7]

Sung [43] allows the agent to control both mean and risk separately in a continuous-time setting. Proposition 2 of his paper obtains a static principal's maximization problem subject to two nonlinear constraints. Sung shows that the optimal contract is linear, due to the absence of wealth effect as well as a time-independent cost function for the agent's controls. Because the principal's constrained maximization problem is not solved, he does not discuss the relations between PPS and risk and between PPS and performance. Other related studies include [3,7,23,35]. In short, this stream of prior research neither discusses the two relations nor incorporates wealth effects.

The rest of this paper is organized as follows. Section 2 describes the model in which the ENE utility function is proposed for the study of wealth effects. Section 3 presents two benchmark cases under the exponential utility function. Section 4 examines the impact of both the wealth effect and the control of risk on PPS and performance. Section 5 discusses

[7] All agency models use the expected output as the agent's performance. We argue that the expected net profit (output less the agent's compensation) is a more relevant measure of performance from the principal's perspective.

some empirical implications of the model. Section 6 concludes the paper. Appendix A contains all the proofs.

2. The model

For tractability, consider a one-period principal–agent relationship. The principal hires the agent to manage an output, which is given by

$$y = A + \sigma\varepsilon, \tag{1}$$

where $\varepsilon \sim N(0, 1)$ is normally distributed with a mean of 0 and a variance of 1 and where A and σ are one-dimensional real, positive numbers. Both principal and agent observe the value of y at the end of the period. At time 0, the principal and agent sign a compensation contract, and the agent then takes actions that affect A and σ separately. As in [20], the agent makes a one-time choice of a vector of efforts (A, σ) at a personal cost $D(A, \sigma)$. Assume that the first-order derivatives of $D(A, \sigma)$ exist and that $D(A, \sigma)$ is convex with respect to A and σ^{-1}. The convexity of the cost function means that the agent's marginal costs of increasing the mean (A) and the precision (σ^{-1}) increase with the levels of A and σ^{-1}, respectively. We interpret $\sigma\varepsilon$ as the idiosyncratic risk of the output. In other words, the agent's two-dimensional efforts affect the mean and the idiosyncratic risk of the output. Here the systematic risk of the output is omitted. It would be reasonable to assume that systematic risk is beyond the control of the agent and that it is observable to both principal and agent.

Previous models typically interpret control of risk as an agent's selection of projects with certain risk levels. The cost of this process is assumed to be zero, that is, the agent does not incur costs in controlling risk. A key constraint imposed on the projects is that the higher the idiosyncratic risk, the higher the expected return or mean. Once a project is chosen, it is added to an existing output process. The agent then exerts costly effort to increase the mean of the combined output process. We argue that it is difficult to justify the selection of projects based on the assumption that higher idiosyncratic risks correspond to higher expected returns or means. It is well known that idiosyncratic risk does not affect expected asset returns in the absence of moral hazard. In an equilibrium model of asset pricing and moral hazard, Ou-Yang [36] further shows that when principals are risk neutral, which is a common assumption in the models on risk control, idiosyncratic risk only affects asset prices but does not affect expected asset returns.

In short, the literature does not offer a self-consistent mechanism on how agents select projects or control risk. We believe that if risk is to be determined endogenously, then it should be determined simultaneously with the mean of the output.

To endogenize risk in a consistent manner, we make a simplifying assumption that ex ante, Eq. (1) governs the outputs of all available projects, where y denotes the level rather than the return of the project. Ex ante, the mean (A) and the (idiosyncratic) risk (σ) of the project are not intrinsically related, but they are determined by the agent's actions.[8] In

[8] For simplicity, risk in this paper represents idiosyncratic risk.

particular, it does not have to be the case that a high risk corresponds to a high mean in equilibrium.

Another key difference between the current model and the previous ones is that we assume that it is costly to manage risk. This assumption is practically relevant. Most financial firms on Wall Street and many other firms engaged in risky businesses have their own risk management divisions or hire outside firms to manage firm-specific risks. Examples of firm-specific risks include extreme-position risk for a trading firm, currency risk for a firm engaged in international business, and business and bankruptcy risk in general. Mathematically, risk control may be understood as preventing certain extreme states from occurring and smoothing the distribution function of the output. Because the output follows a normal distribution in our model, the agent affects the shape of the distribution through his control on its mean and its variance. The agent has a two-dimensional action space: both reducing risk and increasing mean require separate costly actions, and the agent faces a tradeoff between the two actions.

Assume that the principal is risk neutral and that the agent is risk averse. Here the risk-neutrality for the principal is for simplicity, which implicitly assumes that the principal can diversify away idiosyncratic risks. To introduce wealth effects in a tractable way, we assume that the agent's utility function is of the following form:

$$U(C) = -\frac{t}{R_1}\exp\left(-R_1 C\right) - \frac{(1-t)}{R_2}\exp\left(-R_2 C\right), \tag{2}$$

where $0 \leqslant t \leqslant 1$, R_1 and R_2 are two positive constants, and C denotes the agent's consumption at the end of the period. Note that if $t = 0$ or 1, then this utility function reduces to the (negative) exponential utility function with a risk-aversion coefficient R_1 or R_2. This utility function is a weighted average of two exponential functions, but its absolute risk aversion decreases as the agent's consumption increases, as presented in the following lemma:

Lemma 1. *The agent's absolute risk aversion R_A is given by*

$$\begin{aligned} R_A \equiv -\frac{U_{cc}}{U_c} &= R_2 + \frac{t(R_1 - R_2)}{t + (1-t)\exp[(R_1 - R_2)C]} \\ &= R_1 + \frac{(1-t)(R_2 - R_1)}{(1-t) + t\exp[(R_2 - R_1)C]}, \end{aligned} \tag{3}$$

which decreases with his consumption C. Here U_c and U_{cc} denote the first- and second-order derivatives of U, respectively.

Fig. 1 depicts different patterns of this utility function with respect to consumption.

Remark 1. Because this utility function is a linear combination of two well-defined utility functions, it satisfies all the properties that define a utility function. For example, it is both strictly increasing and strictly concave with respect to consumption. Suppose that $R_1 \geqslant R_2$. It can be shown that $R_2 \leqslant R_A \leqslant R_1$. In other words, the absolute risk aversion of this utility falls in between the absolute risk aversions of the two exponential utility functions. Because the agent's initial wealth contributes to his terminal consumption, it affects his risk aversion. As a result, the agent's initial wealth affects his efforts as well as the optimal contract, unlike

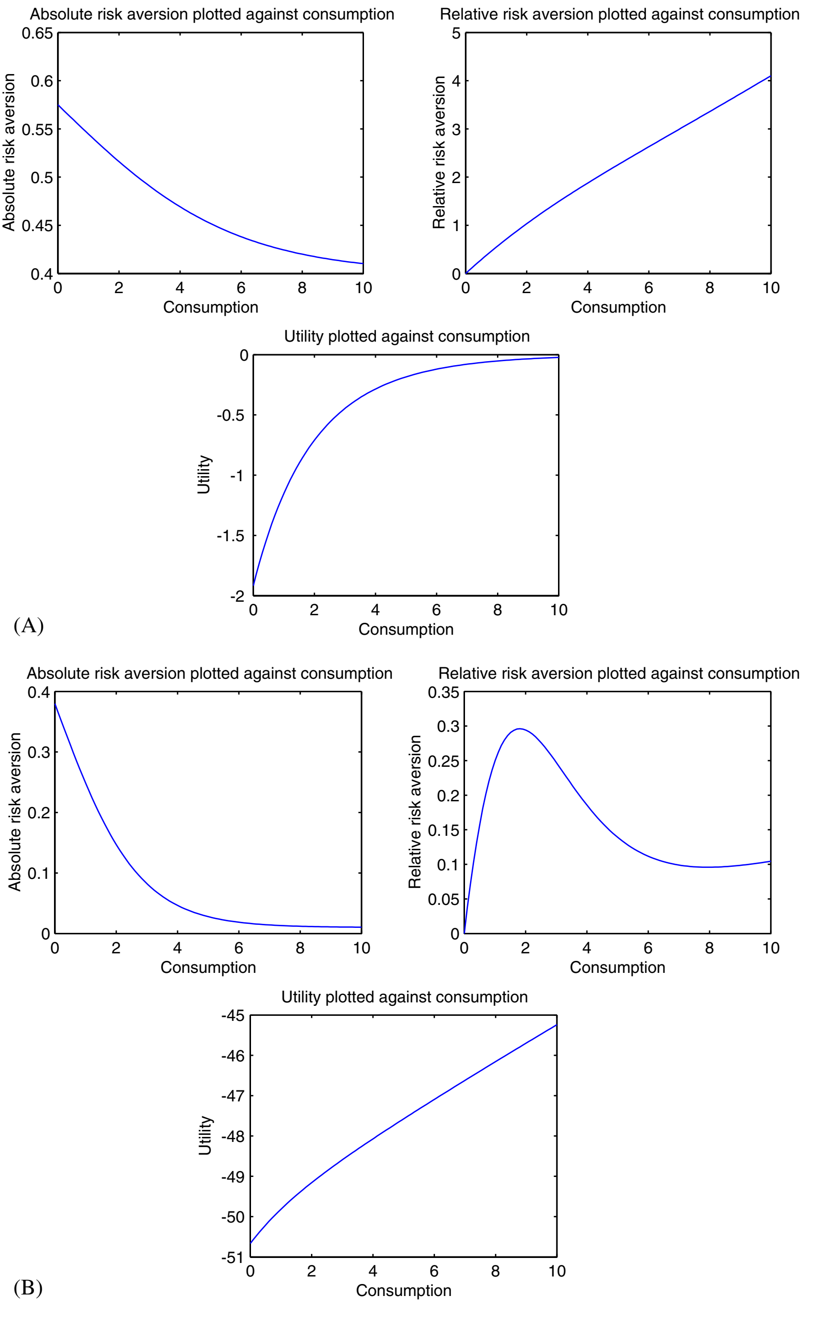


Fig. 1. The characteristics of the new utility function. Panel A: $t = 0.5$, $R_1 = 0.75$, and $R_2 = 0.4$; panel B: $t = 0.5$, $R_1 = 0.75$, and $R_2 = 0.01$.

in the exponential case. Although the absolute risk aversion of this utility function always decreases with consumption, Panel B of Fig. 1 shows that its relative risk aversion may be unimodal, increasing initially and then decreasing with consumption. We name this utility function as an *ENE*.

Remark 2. The ENE utility function can incorporate all linear combinations of exponential functions as follows:

$$U(C) = \sum_{i=1}^{N} -\frac{t_i}{R_i} \exp(-R_i C), \tag{4}$$

where N is a finite number and t_i represents the weight for each negative exponential utility function. Appendix A shows that when $t_i > 0$, the absolute risk aversion of this more general utility function also decreases when consumption C increases. Linear combinations of the negative exponential functions may be used to approximate general concave utility functions in a Stone–Weierstrass sense.[9] For $C \in (0, \infty)$, consider the set $\exp(-nC)$ $(n = 0, 1, 2, \ldots)$ and define a set z^n where $z = \exp(-C)$ on $z \in (0, 1)$.[10] The Stone–Weierstrass theorem states that all continuous functions $f(z)$ (including continuous concave utility functions) defined on the interval $[z_1, z_2]$ can be uniformly approximated by $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where $z_1 > 0$ and $z_2 < 1$ and where a_n is a real-valued coefficient. For example, one can approximate a power utility function $\frac{1}{1-l} C^{1-l}$ as follows. Define $f(z) = \frac{1}{1-l} \left[-\log(z) \right]^{1-l}$. The definition of $z = \exp(-C)$ yields $f(z) = \frac{1}{1-l} C^{1-l}$. One then obtains that $\frac{1}{1-l} C^{1-l} = f(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n \exp(-nC)$.

The principal–agent problem becomes intractable to us when $N > 2$. For tractability, we adopt a simplified version of the ENE utility function in which $N = 2$ as given in Eq. (2).

In the most general agency models, such as [14,15,18,32,40], optimal contracts are extremely difficult to characterize. Even if they can be explicitly characterized, their forms are typically complicated. As Prendergast [39] notes, it is difficult to generate an agency contract where one can talk about more or fewer incentives in a general model. Following him and many others, we confine the contract space to be linear:

$$S(y) = a + by, \tag{5}$$

where a and b represent the agent's constant compensation and his PPS, respectively. Linear contracts are used in situations such as those involving sharecroppers and franchisees. In [29] and many others, an entrepreneur sells a fraction of the firm to maximize his expected utility or the firm value. The sharing rule between the entrepreneur and outside investors is of the linear form. Compensations for corporate executives typically consist of three parts: cash, stock ownerships, and option ownerships. Following Jensen and Murphy [22], empirical tests often convert option ownerships into "equivalent" stock ownerships using the Black–Scholes [5] hedge ratio. An executive's total stock ownership plus his converted

[9] We thank the associate editor for this insight.

[10] One may also use the basis set, $-\exp(-nC)$, by adjusting the coefficients in the polynomial.

"equivalent stock ownership" from options divided by the firm's total shares outstanding constitutes the executive's proportional ownership of the firm. This proportion serves as a measure of incentive.

Given $S(y)$ in Eq. (5), the agent chooses A and σ to maximize the expected utility:

$$\begin{aligned} &E\left\{U\left[W_0+S(y)-D(A,\sigma)\right]\right\} \\ &\quad=-\frac{t}{R_1}\exp\left[-R_1\left(W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_1b^2\sigma^2\right)\right] \\ &\qquad-\frac{(1-t)}{R_2}\exp\left[-R_2\left(W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_2b^2\sigma^2\right)\right], \end{aligned} \tag{6}$$

where W_0 is the agent's initial wealth, and $[W_0+S(y)-D(A,\sigma)]$ then represents his terminal wealth or consumption. For tractability, we assume that the principal observes W_0. Note that the expectation is taken at time 0.

The ENE utility function allows us to take the agent's expected utility explicitly, which greatly simplifies the solution. Alternatively, one may introduce a wealth effect into a contracting problem through a simple power utility function, $\frac{1}{l}C^l$, where l denotes the agent's relative risk aversion coefficient. Under the power utility, one may adopt a lognormal process for the output y. The difficulty is that even if y is lognormal, the agent's end-of-period consumption, $W_0+S(y)-D(A,\sigma)$, is no longer lognormal. As a result, one cannot calculate the agent's expected utility in closed form and the problem becomes intractable even numerically. Because the agent's maximization problem cannot be explicitly characterized, the sufficiency of the agent's first-order conditions (FOCs) cannot be established. Consequently, it is not possible to verify the optimality of the solutions under the power utility functions. This ENE utility function, however, introduces wealth effects in a tractable way. As it shall be shown, the agent's FOCs are both necessary and sufficient, and the optimal contracting problem can be further simplified, due to the fact that the ENE utility function is a linear combination of two exponential functions.

The principal is assumed to be risk neutral, so her problem is to determine a and b to maximize the expected profit, less the agent's compensation:

$$E\left[y-(a+by)\right]=(1-b)A^*-a \tag{7}$$

subject to two constraints:

$$E\left\{U\left[W_0+S(y)-D(A,\sigma)\right]\right\}\geqslant U[W_0+\mathcal{E}_0], \tag{8}$$

$$\{A^*,\sigma^*\}\in\ \operatorname{argmax}_{\{A,\sigma\}\in\Re+}E\left\{U\left[W_0+S(y)-D(A,\sigma)\right]\right\}, \tag{9}$$

where $\{A^*,\sigma^*\}$ denote the agent's optimal actions given the principal's contract, where the notation "argmax" denotes the set of arguments that maximizes the objective function that follows, and where $\mathcal{E}_0$ denotes the agent's reservation wage. Eq. (8) is the agent's participation constraint, with $U[W_0+\mathcal{E}_0]$ representing the utility level that the agent would achieve elsewhere. We follow the literature by assuming that $\mathcal{E}_0$ is a constant known to both agent and principal. [11] We further assume that the principal has all the bargaining power or

[11] This is obviously a partial equilibrium approach. It would be of interest to endogenize $\mathcal{E}_0$ as a function of the agent's skill, his bargaining power, his initial wealth, and his experience.

that the agent receives his reservation wage in equilibrium. Eq. (9) is the agent's incentive compatibility constraint, which means that the principal's equilibrium contract must satisfy the agent's maximization problem or induce A^* and σ^*.

The principal is concerned only about the expected net profit at time 0, which we interpret as the agent's performance as judged by the principal. The expected net profit can also serve as a proxy for the market value of the output for which a risk-neutral investor is willing to pay. It is important to consider the market price as a measure of performance because many empirical studies employ Tobin's Q-ratio, which is defined as the market value of a firm divided by the replacement costs of its assets, as managerial performance (see, e.g., [17,31,33,37]). Although σ does not enter directly into Eq. (7), it does affect the agent's performance because a, b, and A all depend on it. For example, it may be the case that an agent has to spend a lot of time managing risk so that he has to sacrifice on the mean. Also, if the risk level is too high in equilibrium, the risk-averse agent will command a large amount of risk premium, which may offset the increase in the mean. The agent's performance may be reduced in both cases.

3. Special cases under the exponential utility function

Before solving the general model, this section studies two benchmark cases in which the agent has a negative exponential utility function. In the first case the agent controls only the mean of an output and in the second case the agent controls separately the mean and the volatility of an output.

3.1. Case I: control of mean

In this case, σ is an exogenously given constant in Eq. (1), and t equals 1 in Eq. (2). Assume that the agent's cost function is given by $D(A) = \frac{1}{2}kA^2$, where k is a positive constant. The results are standard and summarized in the next lemma.

Lemma 2. *The optimal contract $\{a, b\}$ and the optimal mean A are given by*

$$a = \mathcal{E}_0 + \frac{1}{2}R_1 b^2\sigma^2 - \frac{1}{2}kA^2, \quad b = \frac{1}{1 + kR_1\sigma^2}, \quad A = \frac{b}{k}, \tag{10}$$

respectively. The agent's performance is given by

$$P \equiv E\left[y - S(y)\right] = \frac{1}{2}A - \mathcal{E}_0 = \frac{1}{2}\frac{b}{k} - \mathcal{E}_0. \tag{11}$$

Holmström and Milgrom [19] have derived Eq. (10) in a continuous-time model. [12] The calculation for the agent's performance is new but straightforward. The proof of this lemma is thus omitted. The expressions for b and P contain the tradeoff between risk and incentives and the relation between performance and incentives, respectively. From the expression for

[12] See also [16,28,41].

b, it is important to control for the agent's risk aversion in the empirical test of the tradeoff, but this variable is difficult to control. Note that PPS is independent of both the agent's initial wealth W_0 and his reservation wage $\mathcal{E}_0$. It is clear from the expressions for b and P that PPS is a sufficient predictor for the agent's performance, that is, a high value of b leads to a high value of P. Notice that the presence of k in Eq. (11) does not affect the positive relation between P and b because k and b move in the opposite direction and because b is a decreasing function of k. Eq. (11) implies that to obtain the positive relation between P and b, it is important to control for the agent's reservation wage. This control appears to be missing in previous empirical studies.

3.2. Separate control of mean and risk

In this case, the agent's objective function given in Eq. (6) reduces to

$$\begin{aligned} &\max_{\{A,\sigma\}} EU\left[W_0 + S(y) - D(A,\sigma)\right] \\ &= \max_{\{A,\sigma\}} \left\{ -\frac{1}{R_1} \exp\left[-R_1 \left(W_0 + a + bA - D(A,\sigma) - \frac{1}{2} R_1 b^2 \sigma^2 \right) \right] \right\}. \end{aligned} \tag{12}$$

The FOCs are given by

$$b = D_A(A,\sigma), \tag{13}$$

$$0 = D_\sigma(A,\sigma) + R_1 b^2 \sigma, \tag{14}$$

where D_A and D_σ denote the partial derivatives of $D(A,\sigma)$ with respect to A and σ, respectively. We adopt the first-order approach in which the agent's maximization problem is replaced by these two FOCs. The proof of the sufficiency of these FOCs is a special case of the proof for the ENE utility function (2) in which $t = 1$. See the proof of Theorem 3 presented in Appendix A for detail.

The principal's constrained maximization problem becomes

$$\begin{aligned} &\max_{\{a,b\}} \quad E\left[y - S(y)\right] = \max_{\{a,b\}} \left[(1-b)A^* - a\right] \\ &(\text{s.t.}) \quad E\left[-\frac{1}{R_1} \exp\left(-R_1\left[W_0 + S(y) - D(A,\sigma)\right]\right) \right] \\ &\qquad \geqslant -\frac{1}{R_1} \exp\left[-R_1\left(W_0 + \mathcal{E}_0\right)\right], \\ &\qquad \{A^*, \sigma^*\} \in \operatorname{argmax}_{\{A,\sigma\}\in\Re^+} E\left[-\frac{1}{R_1} \exp\left(-R_1\left[W_0 + S(y) - D(A,\sigma)\right]\right) \right]. \end{aligned}$$

Consider a family of Cobb–Douglas-type cost functions:

$$D(A,\sigma) = kA^\alpha \sigma^{-\beta},$$

where k, α, and β are positive constants. This cost function implies that there is a tradeoff between increasing mean and reducing risk and that the marginal cost of changing one control variable depends on the level of the other. In general, we assume that the agent takes two actions, (e_1, e_2): e_1 increases mean and e_2 reduces risk. Mathematically, we may set $e_1 = A$ and $e_2 = \sigma^{-1}$. The cost function can then be written in terms of the actions as

$ke_1^{\alpha}e_2^{\beta}$. For tractability, this cost function makes a simplifying assumption that if the agent does not take any action to reduce risk, that is, $e_2 = 0$, the risk of the output approaches infinity. A more realistic assumption might be that $\sigma = 1/(\sigma_0^{-1} + e_2)$, meaning that if the agent takes no action to reduce risk, the risk of the output becomes the highest at σ_0 rather than infinity. This would, however, make the agent's FOCs with respect to e_1 and e_2 (or A and σ) nonlinear. As a result, the problem becomes intractable to solve.

We can rewrite the agent's cost function as $D(A, \sigma) = (A^m/\sigma)^{\beta}$, where $m = \alpha - \beta$. We assume that the agent's cost of effort depends on two components: his individual characteristics such as skill and experience as well as external factors such as the nature and competitiveness of the project.

The next theorem presents the main results of this case, which have not been obtained previously in the literature.

Theorem 1. *For fixed parameters* (R_1, k, α, β), *where* $\alpha \geqslant \beta + 1$ *and* $1 < \beta < 2$, *there exists a unique linear contract. The optimal A and σ are given by*

$$A = k_2 b^{(1+2n)}, \qquad \sigma = k_1 b^n, \tag{15}$$

where $n = \frac{2-\alpha}{2\alpha-2-\beta}$, $k_1 = \alpha^{\left(\frac{-\alpha}{2\alpha-2-\beta}\right)}(\beta/R_1)^{\left(\frac{\alpha-1}{2\alpha-2-\beta}\right)}k^{\left(\frac{1}{\beta+2-2\alpha}\right)}$, *and* $k_2 = \frac{\alpha R_1 k_1^2}{\beta}$. *The constant compensation* (a), *the PPS* (b), *and the performance* (P) *are given by*

$$a = \mathcal{E}_0 + \frac{1}{2}R_1 b^2\sigma^2 + D(A, \sigma) - bA, \quad b = \frac{(2-\beta)\alpha}{(\alpha-\beta)(2+\beta)},$$
$$P = k_2 b^{(1+2n)} - \frac{1}{\alpha}(1 + 0.5\beta)k_2 b^{2(1+n)} - \mathcal{E}_0. \tag{16}$$

Remark 1. Notice that PPS (b) happens to be independent of the agent's absolute risk aversion R_1. The principal's benefit A is given by $A = \alpha R_1 k_1^2 b^{(1+2n)}/\beta$, and the risk premium in the agent's compensation is given by $\frac{1}{2}R_1 b^2\sigma^2 = \frac{1}{2}R_1 k_1^2 b^{2(1+n)}$. Given a level of PPS, the benefit and cost are of the same proportion to R_1, the optimal PPS is thus independent of R_1 due to the agent's control of risk. Also, Eq. (15) shows that when R_1 increases, both A and σ decrease. In this way, a more risk-averse agent prefers a lower risk level, and as a result, the mean is lower because the marginal cost of increasing it increases with a lower risk level. Recall that in the case in which risk is exogenous, PPS strictly decreases with R_1. Given a risk level and a PPS, a risk-averse agent exerts the same level of effort (regardless of the value of R_1) while demanding a higher risk premium ($\frac{1}{2}R_1 b^2\sigma^2$), which is the cost to the principal for providing incentives. The marginal cost with respect to PPS, $R_1 b\sigma^2$, increases with R_1, whereas the marginal benefit is 1. Therefore, the optimal PPS decreases with R_1.

Remark 2. From the risk-PPS equation ($\sigma = k_1 b^n$), there may be a negative or positive relation between σ and b because k_1, n, and b depend on both α and β. Intuitively, when the cost of increasing mean is relatively high (a high α) for a given PPS, the risk-averse agent has an incentive to lower risk. Hence, giving the agent a high PPS will not increase the risk premium significantly because of a low level of risk. On the other hand, when the agent can raise mean more effectively (a low α), the principal may want to give the agent a high PPS, because the benefit to the principal reflected in the mean may outweigh the cost reflected in the risk premium in the agent's compensation. A positive relation between risk and PPS will thus arise.

In addition, from the relation between A and PPS, $A = \alpha R_1 k_1^2 b^{(1+2n)}/\beta$, where $(1+2n) > 0$, it can be seen that a higher PPS does not necessarily lead to a higher A because A also depends on other variables. [13] Similarly, the agent's performance, as measured by the principal's expected net profit, can increase or decrease with PPS. Appendix A points out that both the principal's problem and the agent's problem admit unique sets of optimal solutions.

3.2.1. Numerical illustrations

Figs. 2 and 3 present results illustrating the two distinct patterns for the relations between PPS and risk and between PPS and performance, respectively. Fig. 2 illustrates the impact of changing β on the two relations as well as on various endogenous variables. Panel A plots PPS, mean, risk, and the agent's performance against β. Note that β measures the agent's cost of managing risk. When β increases, the marginal cost of increasing (reducing) risk decreases (increases); hence the agent has an incentive to increase risk in equilibrium. A high level of risk has two advantages: a low absolute cost of managing risk and a low marginal cost of increasing mean. However, a high risk may lead to a high risk premium in the agent's compensation. To reduce risk premium, the principal reduces PPS. In equilibrium, PPS decreases with β, but both mean and risk increase with β. The net result is that the agent's performance increases with β. Consequently, there are negative relationships between PPS and risk and between PPS and performance.

Fig. 3 demonstrates the effect of α. As α increases, the cost of increasing mean increases, but the marginal cost of reducing risk decreases given a high mean. When α increases, PPS decreases because the principal does not want to force the agent to increase mean due to its high cost. [14] As a result, the mean decreases. Due to the lower marginal cost of reducing risk, the risk decreases so that the risk premium decreases. Overall, the agent's performance decreases with α. Consequently, positive relationships between PPS and risk and between PPS and performance arise.

In summary, this case demonstrates that the agent's PPS alone is not sufficient to predict performance and that PPS can be positively or negatively related to risk. Under the exponential utility function, the agent's initial wealth plays no role in the determination of incentives and performance, and the agent's reservation wage does not affect PPS. We argue that after controlling for the agent's skills, the agent's reservation wage partially reflects his bargaining power. Thus, the higher the agent's bargaining power, the higher his reservation wage. Therefore, the agent's bargaining power, just like his initial wealth, does not affect the provision of incentives. Similarly, the constant compensation (a) does not affect the

[13] Recall that when risk is exogenously given, a higher PPS leads to a higher level of effort, regardless of the values of R_1 and σ. In the current case, one cannot simply conclude that after controlling for R_1, α, β, and k_1, A increases with b, because if α and β are fixed, then b is fixed as well.

[14] In equilibrium, the principal must reimburse the agent's cost for the agent to meet his reservation utility.

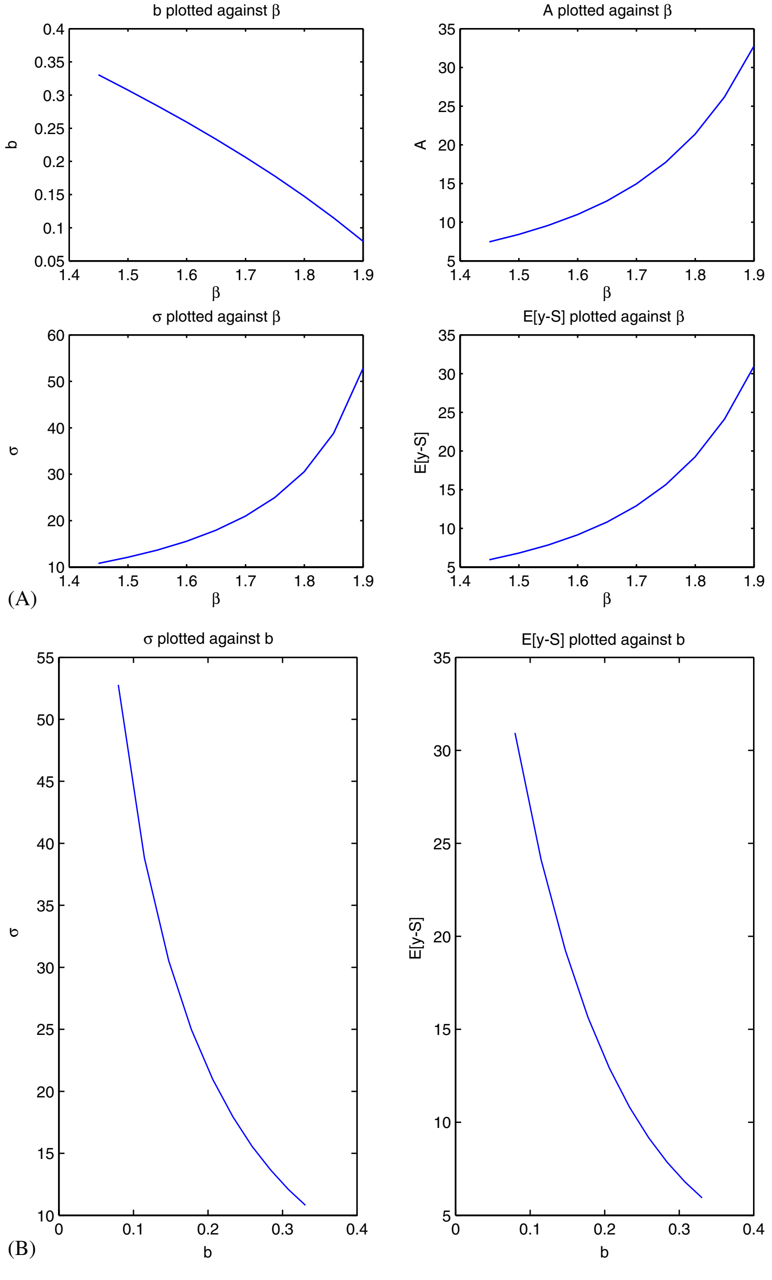


Fig. 2. The effects of b under the Cobb–Douglas function, $D(A,\sigma) = kA^{\alpha}\sigma^{-\beta}$, for the exponential utility function ($k = 0.1$, $\alpha = 2.8$, $R_1 = 0.1$, $W_0 = 0$, and $\varepsilon_0 = 0$).

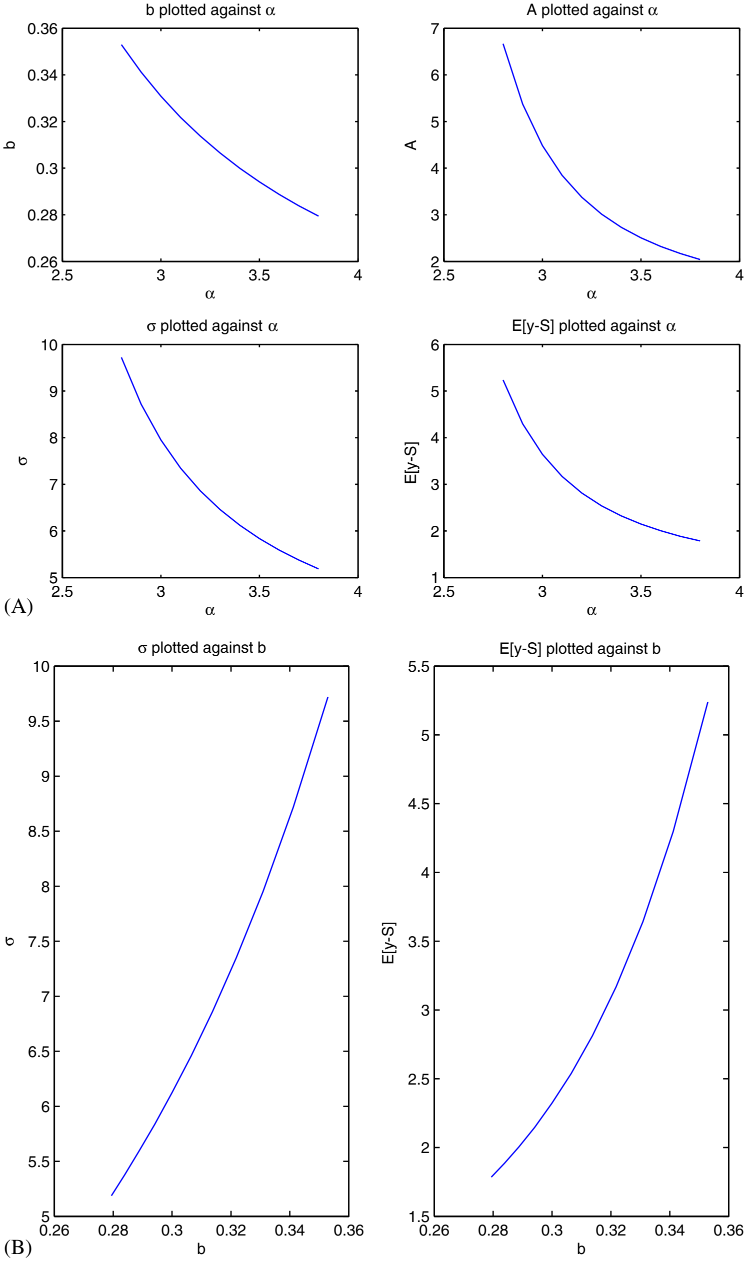


Fig. 3. The effects of α under the Cobb–Douglas function, $D(A, \sigma) = kA^{\alpha}\sigma^{-\beta}$, for the exponential utility function ($k = 0.1$, $\beta = 1.4$, $R_1 = 0.1$, $W_0 = 0$, and $\varepsilon_0 = 0$).

agent's decision to manage mean and risk, and PPS is the only factor that affects agent's effort choices.

4. Results under the ENE utility function

This section shows that under the ENE utility function, both the agent's initial wealth and his reservation wage affect incentives and performance. To examine the wealth effect exclusively, we first consider the case in which the agent's effort affects only the mean of an output.

4.1. Control of mean

As in the exponential case, consider a cost function $D(A) = \frac{1}{2}kA^2$, where k is a positive constant. The principal's objective is to maximize the expected net profit subject to the agent's incentive compatibility and participation constraints. The derivation given in Appendix A shows that the agent's FOC can be obtained explicitly, which yields a relation between PPS (b) and the agent's choice of mean (A). The agent's participation constraint relates his constant compensation a to his initial wealth W_0, reservation wage $\mathcal{E}_0$, and either b or A. After imposing the two constraints, the principal's maximization problem becomes unconstrained with only one control variable A.[15] Unlike the exponential case, the expression for a in terms of A can no longer be obtained in closed form because the agent's participation constraint is nonlinear. We use the relation between a and A as an implicit function in solving the principal's problem.

For notational convenience, we define

$$\bar{Y}_1(A) = -\frac{t}{R_1}\exp\left\{-R_1\left[W_0 + bA - D(A) - \frac{1}{2}R_1 b^2\sigma^2\right]\right\}, \tag{17}$$

$$\bar{Y}_2(A) = -\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0 + bA - D(A) - \frac{1}{2}R_2 b^2\sigma^2\right]\right\}, \tag{18}$$

where $\bar{Y}_1$ and $\bar{Y}_2$ represent the levels of the two exponential functions that form the agent's wealth-dependent ENE utility function defined in Eq. (2). Their derivatives with respect to A, while imposing the agent's FOC $b = kA$, satisfy:

$$\begin{aligned} -\frac{\bar{Y}_1'(A)}{R_1} &\equiv \frac{\bar{g}_1}{R_1} = (1 - kA - R_1k^2\sigma^2A)\bar{Y}_1, \\ -\frac{\bar{Y}_2'(A)}{R_2} &\equiv \frac{\bar{g}_2}{R_2} = (1 - kA - R_2k^2\sigma^2A)\bar{Y}_2. \end{aligned} \tag{19}$$

Recall that $U(W_0 + \mathcal{E}_0)$ denotes the agent's utility at time 0. The next theorem summarizes the results.

[15] Any one of the three variables, A, b, and a can be used as the control variable. There is a unique set of solutions to the principal's maximization problem.

Theorem 2. *The optimal effort A is determined by*

$$\frac{R_1}{R_2}=\frac{\log\left[\frac{-U(W_0+\mathcal{E}_0)g_2}{\bar{g}_1\bar{Y}_2-\bar{Y}_1\bar{g}_2}\right]}{\log\left[\frac{-U(W_0+\mathcal{E}_0)g_1}{\bar{g}_2\bar{Y}_1-\bar{Y}_2\bar{g}_1}\right]}=\frac{\log\left[\frac{-U(\cdot)R_2(1-kA-R_2k^2\sigma^2A)}{\bar{Y}_1(R_1-R_2)[1-kA-k^2\sigma^2A(R_1+R_2)]}\right]}{\log\left[\frac{U(\cdot)R_1(1-kA-R_1k^2\sigma^2A)}{\bar{Y}_2(R_1-R_2)[1-kA-k^2\sigma^2A(R_1+R_2)]}\right]}. \tag{20}$$

The optimal solutions for PPS and the constant compensation are then determined in terms of A by

$$b=kA \tag{21}$$

and

$$\bar{g}_1\exp(-R_1a)+\bar{g}_2\exp(-R_2a)=0, \tag{22}$$

respectively.

Remark. Note that $\bar{g}_1$ and $\bar{g}_2$ as defined in Eq. (19) must have opposite signs for Eq. (22) to have a solution. Because $\bar{Y}_1$ and $\bar{Y}_2$ are both negative, $(1-kA-R_1k^2\sigma^2A)$ in $\bar{g}_1$ and $(1-kA-R_2k^2\sigma^2A)$ in $\bar{g}_2$ must have opposite signs, which implies that the optimal solution for A is between $A_1=\frac{1}{1+R_1k\sigma^2}$ and $A_2=\frac{1}{1+R_2k\sigma^2}$. Suppose $R_1\geqslant R_2$, we have $A_1\geqslant A\geqslant A_2$. Let $T_1=\left[\frac{-U(W_0+\mathcal{E}_0)\bar{g}_2}{\bar{g}_1\bar{Y}_2-\bar{Y}_1\bar{g}_2}\right]$, $T_2=\left[\frac{-U(W_0+\mathcal{E}_0)\bar{g}_1}{\bar{g}_2\bar{Y}_1-\bar{Y}_2\bar{g}_1}\right]$, and $f(A)=T_1^{R_2}-T_2^{R_1}$, then Eq. (20) is equivalent to $f(A)=0$. Because $f(A_1)>0$, $f(A_2)<0$, and $f(A)$ is continuous on the support $[A_1,A_2]$, there exists a solution for $A\in[A_1,A_2]$. If we let $h(A)=|f(A)|$, then we can use numerical methods to search for a solution for A.

Appendix A shows that the agent's problem admits a unique set of solutions and that the principal's objective function is strictly concave. Therefore, all optimal (numerical) solutions are unique. We use Matlab to search for the optimal solution A, setting the tolerance level to be e^{-8}. All numerical solutions satisfy the agent's FOC, his participation constraint, and the principal's FOC to a precision of e^{-5}.

4.1.1. Numerical results

We first examine the significance of the wealth effect for optimal contracting and on the agent's performance. Panel A of Fig. 4 demonstrates that when the agent's initial wealth W_0 increases, both the agent's PPS and his performance increase. As in the exponential case, there is a negative tradeoff between PPS and the exogenously specified risk. Although we cannot prove these results analytically, similar patterns arise under numerous different parameter values. The reason is that the agent's risk tolerance increases with W_0, according to Lemma 1. Hence, his PPS increases. In addition, higher risk tolerance leads to lower risk premium in the agent's compensation (the cost to the principal), and therefore the agent's performance increases. Consequently, higher PPS leads to better performance, as is shown in Panel B.

We then study the impact of the agent's reservation wage $\mathcal{E}_0$ on optimal contracting and performance. We argue that after controlling for an agent's skill, $\mathcal{E}_0$ reflects partially the agent's bargaining power. Fig. 5 illustrates that when $\mathcal{E}_0$ increases, PPS increases but

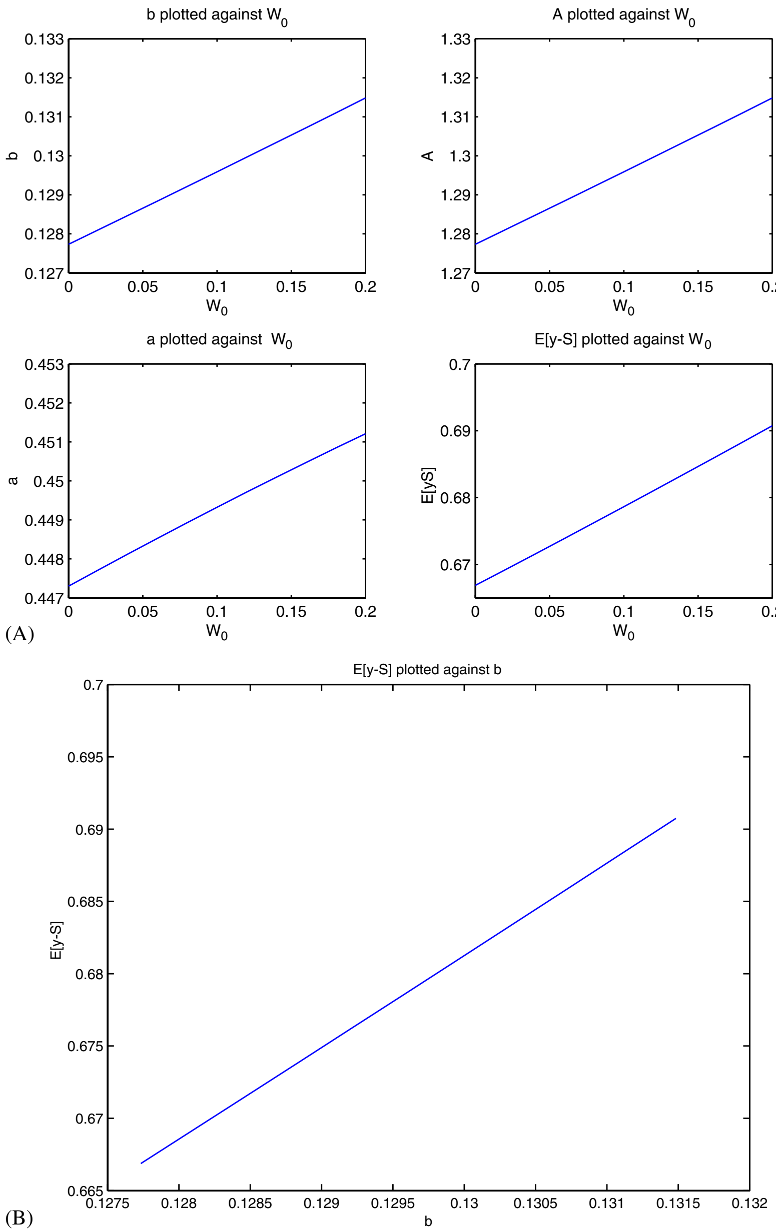


Fig. 4. The effects of initial wealth W_0 under cost function, $D(A) = 0.5kA^2$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $k = 0.1$, and $\sigma = 12$).

performance declines. Again, there is a negative relation between PPS and risk. The effect of $\mathcal{E}_0$ on PPS is similar to that of W_0, reducing the agent's aversion to risk. Even though the agent works harder due to a higher PPS, he commands a higher constant compensation

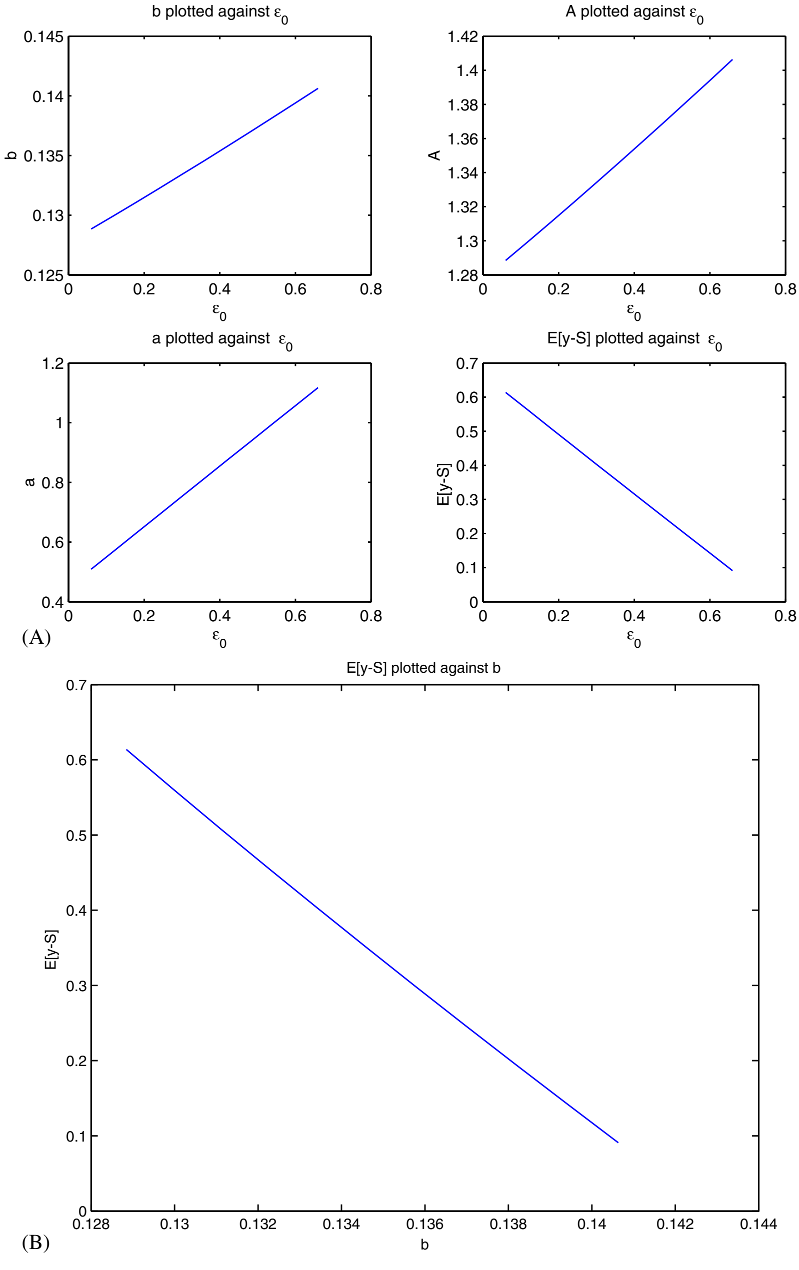


Fig. 5. The effects of reservation wage ε_0 under cost function, $D(A) = 0.5kA^2$, for the ENE utility function ($W_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $k = 0.1$, and $\sigma = 12$).

due to a higher $\mathcal{E}_0$, which outweighs the expected increase in the output. Consequently, the agent's performance decreases with PPS. Note that the agent's performance decreases with $\mathcal{E}_0$, whereas PPS increases with $\mathcal{E}_0$. These results remain robust under many different sets of parameter values.

In summary, this subsection demonstrates that PPS is not a unique predictor of performance. A higher PPS may be due to an agent's higher outside cash wealth, his enhanced bargaining power, or other factors. Different factors lead to varied performances of the agent. The negative tradeoff between risk and PPS, however, appears to be robust in this case. Although the agent's initial wealth and reservation wage are constants known to the principal, they affect optimal contracting and performance because they affect the agent's absolute risk aversion.

4.2. Separate control of mean and risk

For the principal's maximization problem, there are now four variables (a, b, A, σ) to be determined with three constraints from the agent's two FOCs and his participation constraint. The agent's participation constraint and his FOC with respect to σ are nonlinear. It is known that a maximization problem involving both nonlinear constraints and multiple control variables can be intractable even numerically and that even if numerical results could be obtained, the sufficiency of the results would be difficult to verify. Fortunately, we are able to linearize the nonlinear constraints and obtain closed-form solutions for all four variables in terms of a new variable, which greatly simplifies the principal's problem. The principal then maximizes the net profit with respect to this new variable alone without any constraint. Appendix A shows that the agent's FOCs are sufficient and also points out that the principal's problem admits a unique set of numerical solutions.

Define that

$$-\frac{D_\sigma(A, \sigma)}{\sigma b^2} \equiv \gamma R_1 + (1-\gamma) R_2 \equiv g_1(\gamma) \quad \text{or} \quad D_\sigma(A, \sigma) = -g_1(\gamma)\sigma b^2, \tag{23}$$

where $0 \leqslant \gamma \leqslant 1$ and $D_\sigma(A, \sigma)$ denotes the partial derivative of $D(A, \sigma)$ with respect to σ. Eq. (23) resembles Eq. (14) for the exponential case, and this definition is a key step in simplifying the problem. We interpret $g_1(\gamma)$ as a weighted averaged risk aversion coefficient, with γ being the weight that may depend on the agent's initial wealth and other factors. D_σ is the agent's marginal cost with respect to the control of risk, and $g_1(\gamma)\sigma b^2$ represents the marginal value of the agent's risk premium with respect to risk in terms of the weighted averaged risk aversion. Note that D_σ is negative because the agent's cost decreases when σ increases. Further define that

$$g_2(\gamma) \equiv \frac{1}{R_1} \log\left(-\frac{Y_1 R_1}{t}\right) - \frac{1}{R_2} \log\left(-\frac{Y_2 R_2}{1-t}\right) = \frac{1}{2}(R_1 - R_2) b^2 \sigma^2, \tag{24}$$

where $g_2(\gamma)$ represents the difference between the risk premia for the two exponential utility functions associated with PPS (b) and where Y_1 and Y_2 are defined as

$$Y_1 = -\frac{t}{R_1} \exp\left\{-R_1\left[W_0 + a + bA - D(A, \sigma) - \frac{1}{2} R_1 b^2 \sigma^2\right]\right\}$$

and

$$Y_2 = -\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0 + a + bA - D(A,\sigma) - \frac{1}{2}R_2 b^2\sigma^2\right]\right\},$$

respectively.

For tractability, we consider two types of cost functions for the agent's actions:

$$D_1(A,\sigma) = kA^{\alpha}\sigma^{-\beta}, \quad D_2(A,\sigma) = \tau_1 A^{\theta} + \tau_2\sigma^{-\psi},$$

where all parameters are positive constants. The first cost function is of the Cobb–Douglas type, as adopted in the exponential case. Again, it means that the agent's marginal cost of managing mean decreases with the level of risk and his marginal cost of managing risk increases with the level of mean. It captures the tradeoff between managing mean and managing risk. The second cost function assumes that the agent's cost of managing mean (risk) is independent of the level of risk (mean).[16] For the agent's FOCs to be sufficient, we require that $\beta > 1$ and $\alpha \geqslant \beta + 1$ for the Cobb–Douglas cost function or that $\psi > 1$ and $\theta > 1$ for the separate cost function.

Appendix A shows that all four control variables, a, b, A, and σ can be expressed in terms of γ plus other exogenous parameters, using the agent's two FOCs and his participation constraint. Consequently, the principal's problem is to determine γ to maximize the expected net profit. The main results are summarized in the next theorem.

Theorem 3. *Under the Cobb–Douglas cost function D_1, the optimal σ, b, and A are given, respectively, by*

$$\sigma = \left[g_3^2(\gamma)g_4^{\alpha-2}(\gamma)\right]^{-1/2(\alpha-\beta)}, \quad b = \left[g_3^2(\gamma)g_4^{2\alpha-\beta-2}(\gamma)\right]^{1/2(\alpha-\beta)},$$
$$A = \left[\frac{b\sigma^{\beta}}{k\alpha}\right]^{1/(\alpha-1)}, \tag{25}$$

where $g_3(\gamma) = \left[\frac{g_1(\gamma)}{\beta k}\right]^{\alpha-1}\left[\frac{1}{k\alpha}\right]^{-\alpha}$ *and* $g_4(\gamma) = \frac{2g_2(\gamma)}{(R_1-R_2)}$.

Under the separate cost function D_2, the optimal σ, b, and A are given, respectively, by

$$\sigma = \left[\frac{2g_1(\gamma)g_2(\gamma)}{\tau_2\psi(R_1-R_2)}\right]^{-1/\psi}, \quad b = \left\{\left[\frac{g_1(\gamma)}{\tau_2\psi}\right]^2\left[\frac{2g_2(\gamma)}{(R_1-R_2)}\right]^{\psi+2}\right\}^{1/2\psi},$$
$$A = \left(\frac{b}{\tau_1\theta}\right)^{1/(\theta-1)}. \tag{26}$$

[16] Again, these cost functions implicitly assume that if the agent exerts no effort in managing risk σ, then σ approaches infinity. Perhaps a more realistic assumption is that σ approaches a maximum level σ_0 instead of infinity when the agent does not manage risk. This would complicate the problem a great deal with a nonlinear agent's FOC with respect to σ and a truncated solution. We believe, however, that the patterns would not change because they are obtained for relatively small σ values.

Under both cost functions, the principal's problem is to choose γ to maximize

$$P(\gamma) \equiv [A - (a + bA)] = A - \left[-\frac{1}{R_1} \log\left(\frac{Y_1 R_1}{t}\right) + D(A, \sigma) + \frac{1}{2} R_1 b^2 \sigma^2 \right], \tag{27}$$

where $a = -\log(-\frac{Y_1 R_1}{t})/R_1 + D(A, \sigma) + \frac{1}{2} R_1 b^2 \sigma^2 - bA - W_0$ *and* $\frac{1}{R_1} \log(-\frac{Y_1 R_1}{t}) = \frac{1}{R_1} \log\left[-\frac{U(W_0 + \mathcal{E}_0) R_1 R_2 \gamma / t}{R_2 \gamma + R_1 (1-\gamma)} \right]$.

Remark 1. Note that $\left[-\frac{1}{R_1} \log\left(\frac{Y_1 R_1}{t}\right) + \frac{R_1}{2} b^2 \sigma^2 \right]$ represents the risk premium in the agent's compensation. It can be seen that under the Cobb–Douglas cost function, the equilibrium effort (A) depends on both PPS (b) and risk (σ). Given σ, the higher the value of b, the higher the value of A, because the agent receives a fraction b of the output. Given b, the higher the level of σ, the higher the value of A, because the agent's marginal cost of increasing A decreases at a higher level of σ. Since σ is endogenously determined, it is clear that a higher PPS does not necessarily induce a higher A. For instance, given a high PPS, a risk-averse agent may choose a low σ to avoid bearing too much risk. As a result, the agent's effort A may be low as well, both because the agent's marginal cost of increasing A is high and because the risk-averse agent can afford to have a low output, given a low risk level. The agent's equilibrium mean, however, depends only on PPS under the separate cost function because his marginal cost of managing mean is independent of risk.

Remark 2. The numerical procedure to solve the principal's maximization problem for an optimal γ is as follows. Appendix A shows that $0 \leqslant \gamma \leqslant 1$. Because the right-hand side of Eq. (24) is nonnegative, we have that $\left[\log(-\frac{Y_1 R_1}{t})/R_1 - \log(-\frac{Y_2 R_2}{1-t})/R_2 \right] \geqslant 0$. The expressions for the two terms in the inequality are given in Eqs. (56) and (57), which show that these two terms are increasing and decreasing with γ, respectively. Consequently, there exists a γ_0 so that the equality holds. Thus, we need to search for a γ only in the range of $(\gamma_0, 1)$. We use Matlab to search for the minimum solution in that range for the $-P(\gamma)$ function defined in Eq. (27).

4.2.1. Numerical results

Figs. 6–9 present the numerical results for the Cobb–Douglas cost function. Fig. 6 presents the results when the agent's initial wealth W_0 changes. When W_0 increases, PPS decreases and the optimal risk level increases. These patterns appear to be robust under many different sets of parameter values. Recall that when risk is exogenously given, PPS increases with W_0 because the agent is less risk averse with a higher W_0; he can bear more risk. When risk is endogenously determined, why then does PPS decrease with W_0? The reason is that the agent can always choose a higher risk level so that he bears more risk even under a lower PPS. A higher risk level lowers the agent's marginal cost of increasing mean, leading to a higher mean.

The numerical results show that when W_0 increases, the less risk-averse agent tends to increase risk too much so that the risk premium reflected in the agent's constant compensation a increases a lot. As a result, the agent's performance declines when W_0 increases. In other words, an increase in the mean is dominated by an increase in the risk premium.

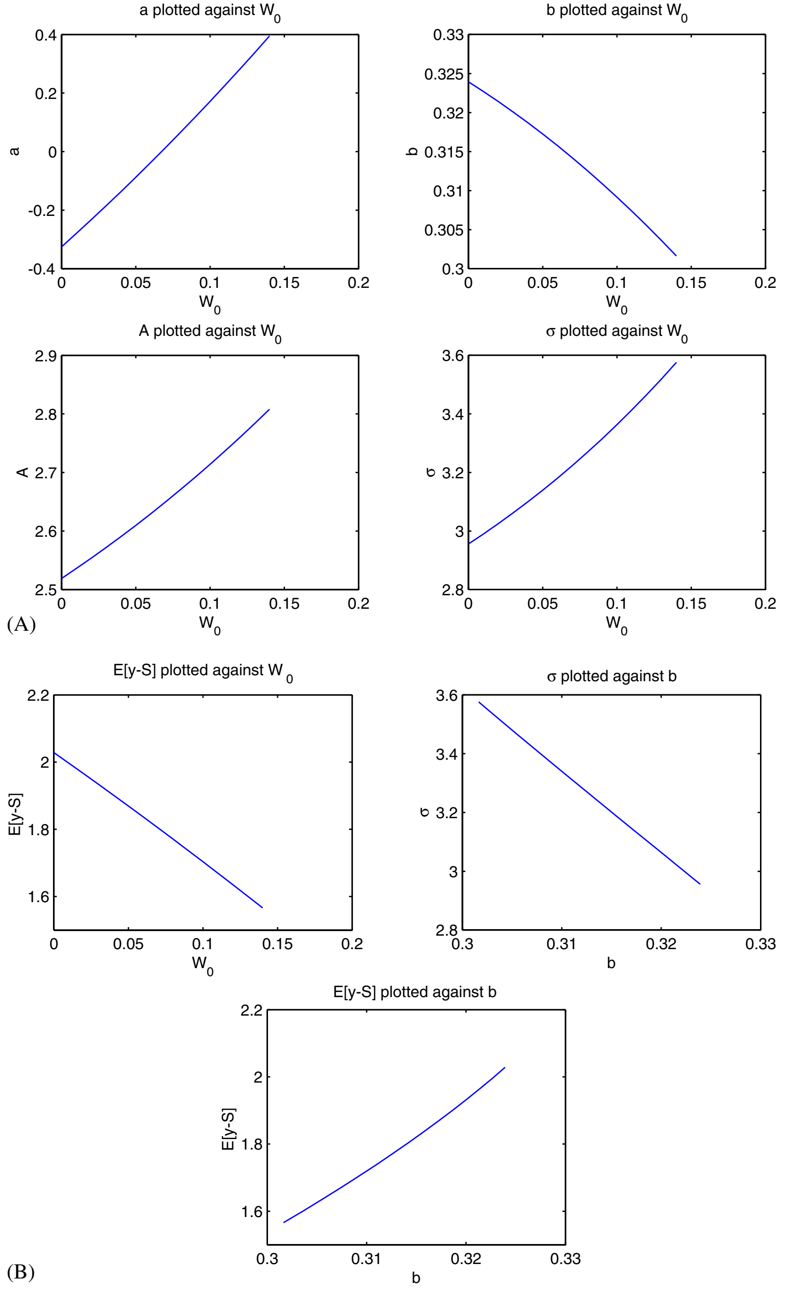


Fig. 6. The effects of initial wealth W_0 under the Cobb–Douglas cost function, $D(A) = kA^{\alpha}\sigma^{-\beta}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $k = 0.1$, $\alpha = 2.8$, and $\beta = 1.4$).

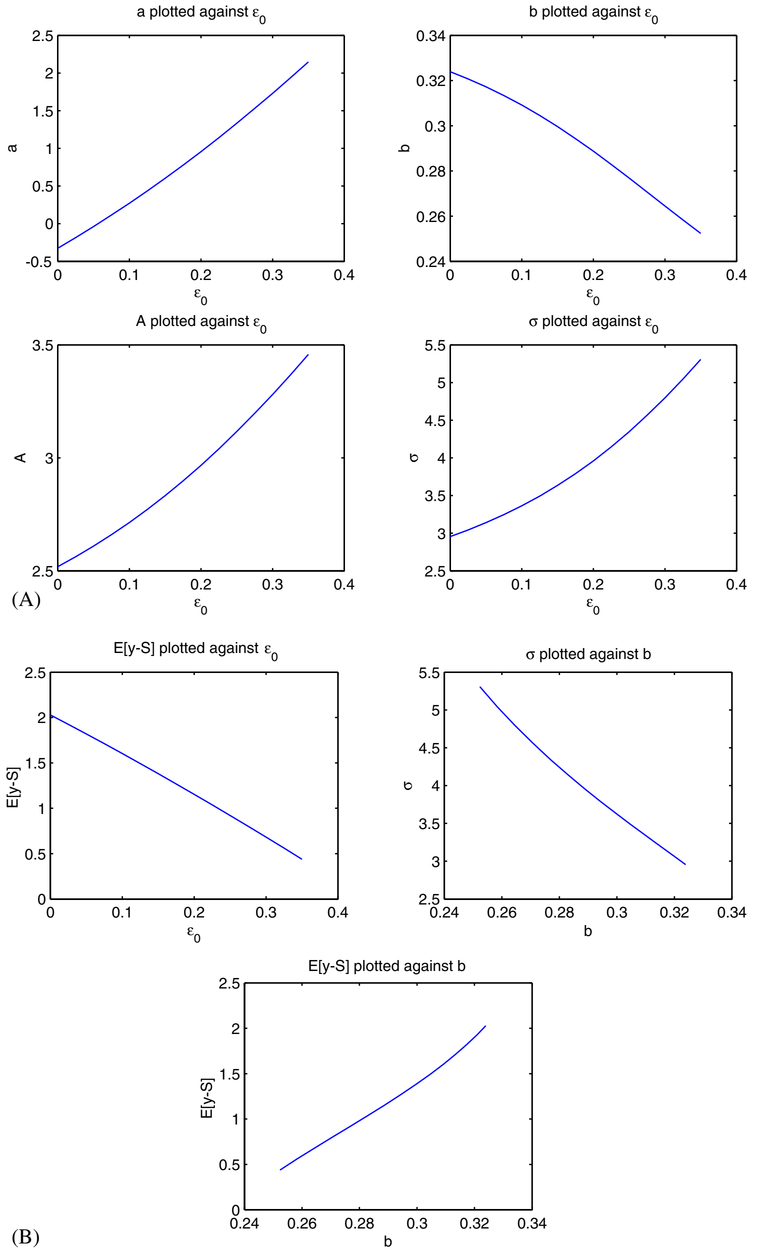


Fig. 7. The effects of reservation wage ε_0 under the Cobb–Douglas cost function, $D(A) = kA^{\alpha}\sigma^{-\beta}$, for the ENE utility function ($W_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $k = 0.1$, $\alpha = 2.8$, and $\beta = 1.4$).

In addition, there exists a negative relation between risk and PPS and a positive relation between performance and PPS. The effects of the agent's reservation wage $\mathcal{E}_0$ on both PPS and performance are similar to those of W_0, which are shown in Fig. 7.

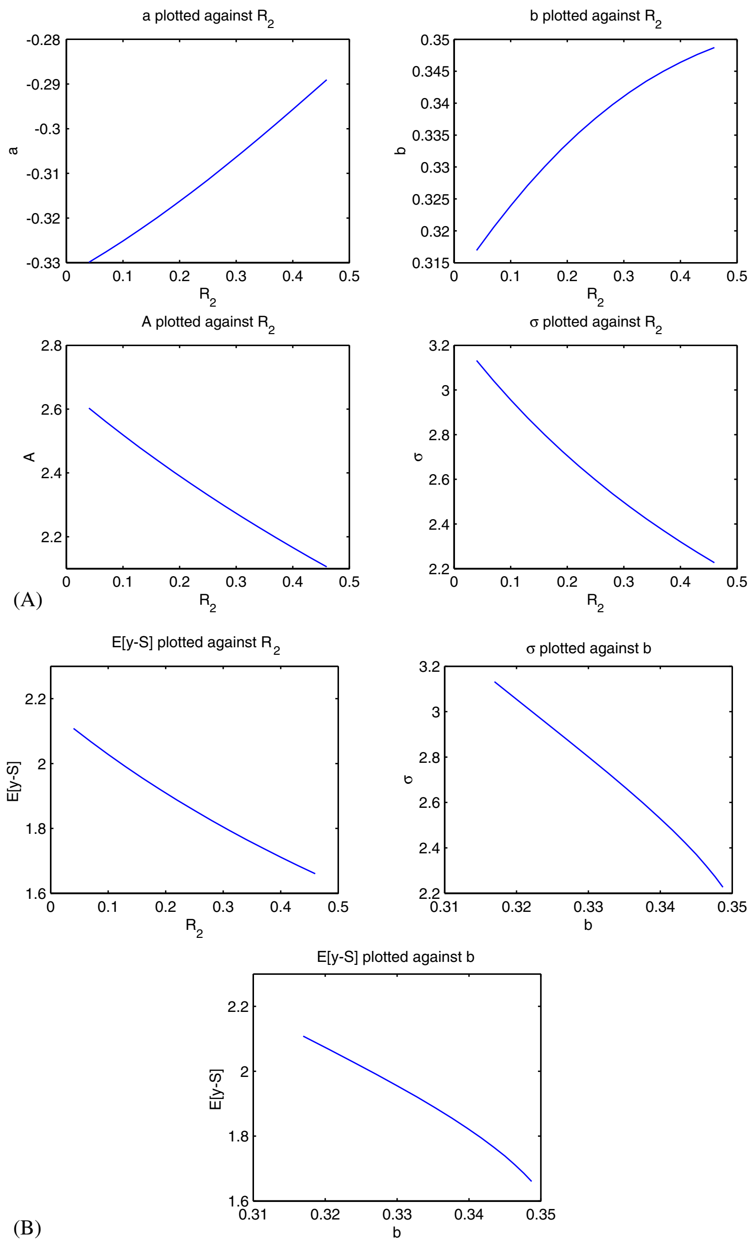


Fig. 8. The effects of R_2 under the Cobb–Douglas cost function, $D(A) = kA^{\alpha}\sigma^{-\beta}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $W_0 = 0$, $R_1 = 0.75$, $k = 0.1$, $\alpha = 2.8$, and $\beta = 1.4$).

When the coefficient R_2 increases, the agent effectively becomes more risk averse because the agent's risk aversion under the ENE utility function is in the range of (R_2, R_1), where $R_1 > R_2$. When the agent is more risk averse, he has more incentives to lower risk. At

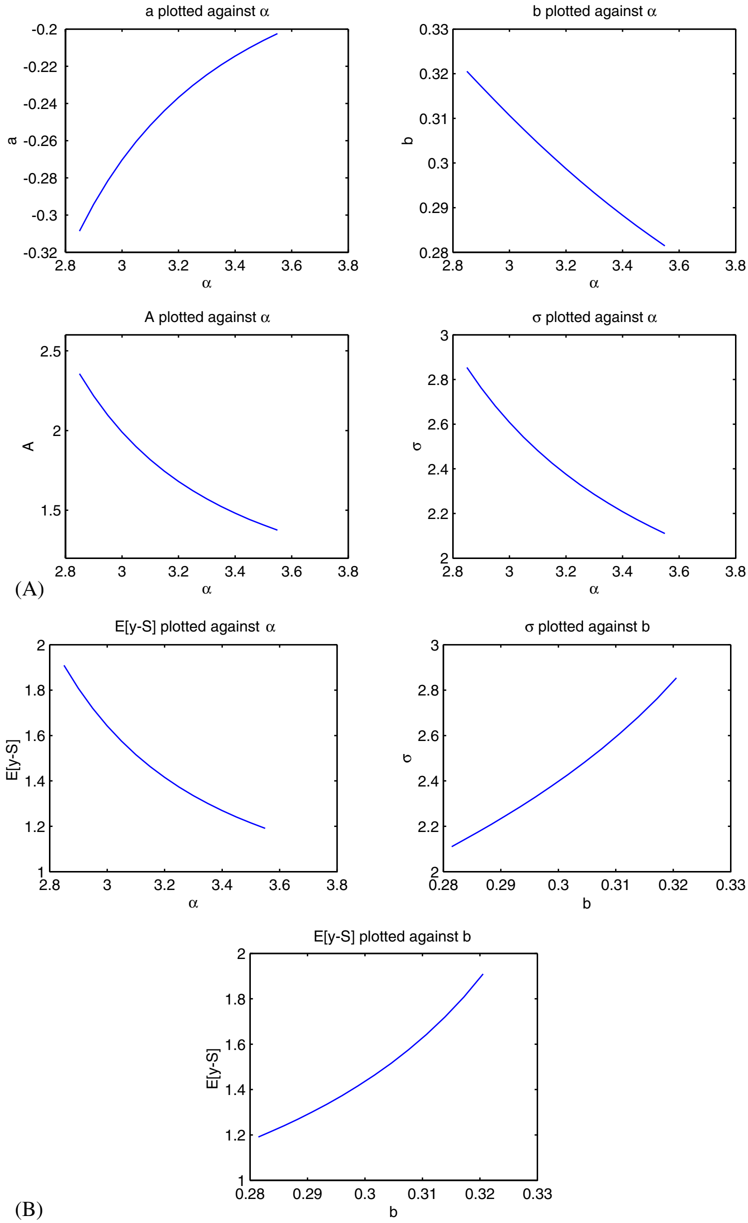


Fig. 9. The effects of α under the Cobb–Douglas cost function, $D(A) = kA^{\alpha}\sigma^{-\beta}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $W_0 = 0$, $R_1 = 0.75$, $k = 0.1$, $R_2 = 0.1$, and $\beta = 1.4$).

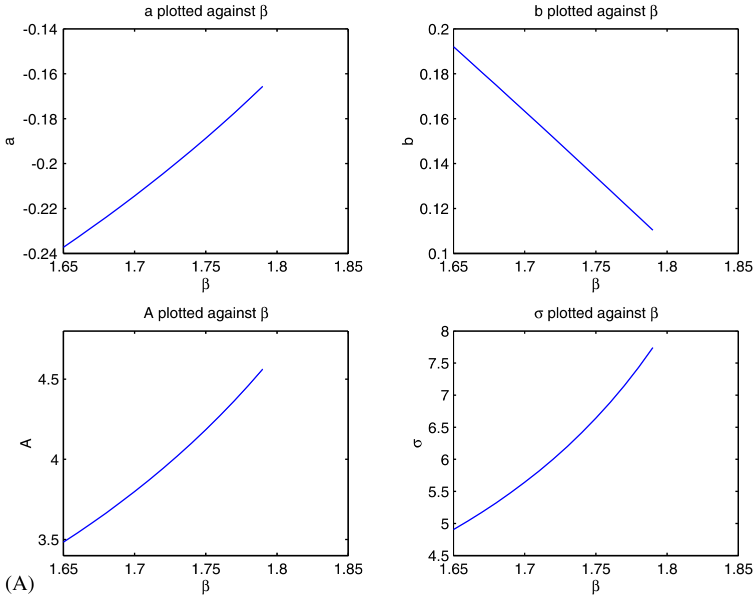
a lower risk level, however, the marginal cost of increasing mean is higher. To encourage the agent to increase mean, the principal gives him a higher PPS. The results show that in equilibrium, the agent chooses both a lower mean and a lower risk, as R_2 increases, and that a decrease in the output results in a lower performance. The relations both between risk and PPS and between performance and PPS are negative. These results are shown in Fig. 8.

Figs. 9 and 10 present the results with respect to the changes of α and β, respectively. When α increases, the cost of increasing mean goes up and the marginal cost of reducing risk goes down. Therefore, both mean and risk decrease. As a result, PPS, which equals the marginal cost of increasing mean $D_A(A, \sigma)$, as given in Eq. (44), decreases. Because the agent always achieves his reservation utility, his performance declines under both a higher cost and a lower mean; a decrease in PPS leads to a decrease in performance. When β increases, the cost of increasing risk decreases. As a result, the risk level increases, and the mean also increases because the marginal cost of increasing mean decreases. To reduce the risk premium, the principal assigns a lower PPS. In contrast to the case of increasing α, the agent's performance increases with β due to a drop in the agent's cost function. Consequently, performance goes down when PPS goes up, and a negative relation between risk and PPS arises.

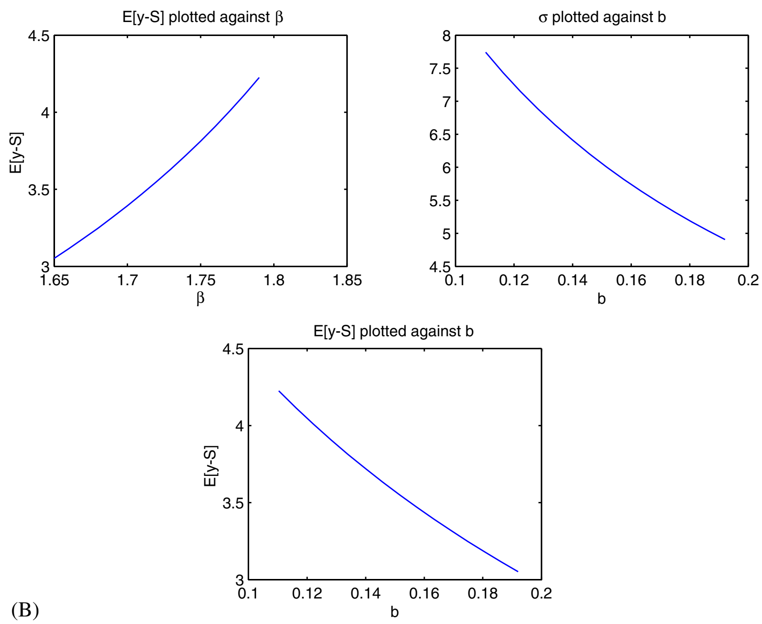
Figs. 11 and 12 summarize the results for the separate cost function. We find that when W_0 increases, PPS appears to increase in all of the numerical calculations performed, unlike in the Cobb–Douglas case in which PPS seems to always decrease. In the latter case the agent's marginal cost of increasing mean decreases with the level of risk. Given a lower PPS, a less risk-averse agent (with a higher W_0) chooses a higher level of risk so that he can increase the mean at a lower cost. Because the level of risk does not affect the agent's marginal cost of managing mean under the separate cost function, the agent with more wealth can bear more risk. As a result, PPS increases, which in this case determines entirely the agent's effort. When the agent's cost of managing risk is relatively low (e.g., $\tau_2 = 7$ in Fig. 11 compared to $\tau_2 = 20$ in Fig. 12), the absolute level of PPS is high because the agent can choose a low level of σ to help reduce the risk premium that he commands. Note that the risk premium in the agent's compensation depends on both PPS and σ. Because the absolute risk level is low and because the agent becomes less risk averse with a higher W_0, the agent can afford to take on more risk, so the risk level increases with W_0. As a result, a positive relation between PPS and risk arises, as shown in Panel B of Fig. 11. When the agent's cost of managing risk is relatively high, the absolute level of risk is relatively high. When PPS increases, the agent wants to lower the risk level to control his total share of risk ($b\sigma$). A negative relation between PPS and risk thus follows, as given in Fig. 12.

Under the separate cost function, the agent's performance decreases with his wealth. The reason is that when the agent becomes less risk averse as his wealth increases, he has fewer incentives to control risk and he appears to choose risk levels that are too high. The net result is that the increase in the risk premium dominates the increase in the mean, so that the principal's expected net profit decreases. In addition, we find that the agent's performance decreases with PPS because PPS increases with the agent's wealth.

In summary, under the Cobb–Douglas cost function, PPS decreases with the agent's initial wealth because a lower PPS encourages the agent to choose a higher risk level, which lowers the agent's marginal cost of increasing mean. Under the separate cost function, PPS increases with the agent's wealth. However, under both cost functions the agent's



(A)



(B)

Fig. 10. The effects of β under the Cobb–Douglas cost function, $D(A) = kA^{\alpha}\sigma^{-\beta}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $W_0 = 0$, $R_1 = 0.75$, $k = 0.1$, $R_2 = 0.1$, and $\alpha = 2.8$).

performance generally decreases with his wealth. Either positive or negative relations between PPS and risk and between PPS and performance can arise, and PPS is not a sufficient predictor for performance.

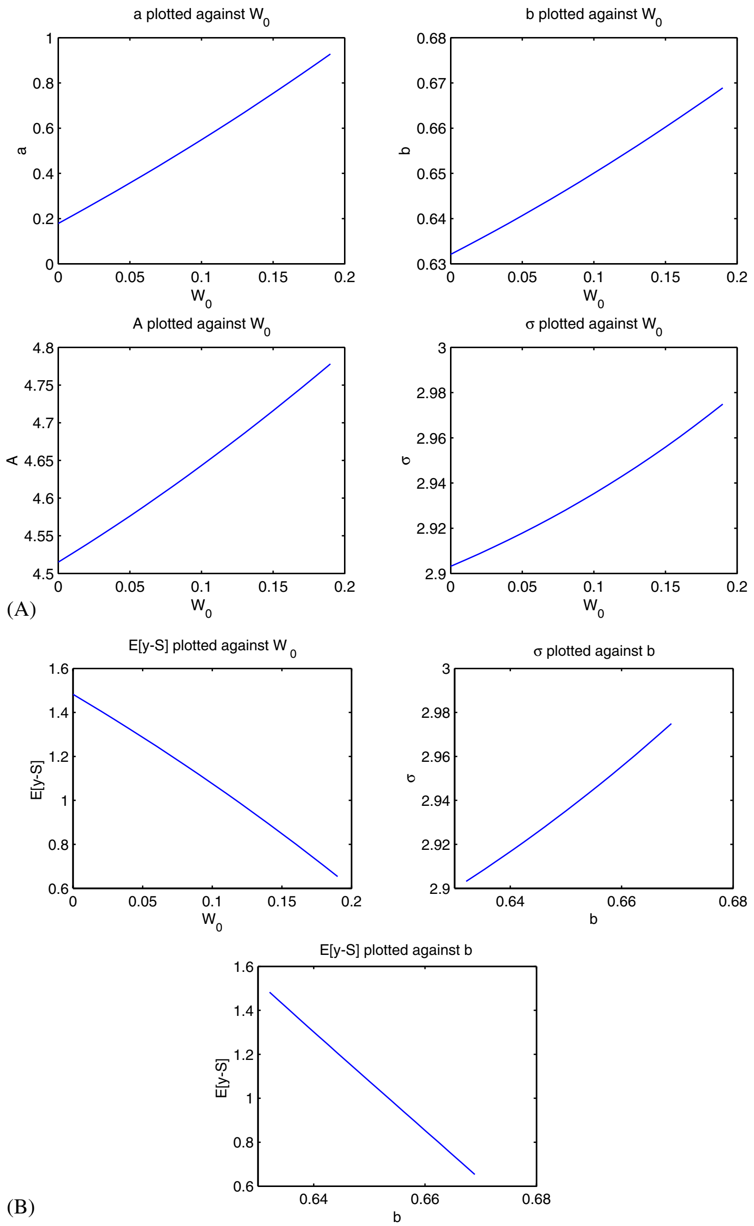


Fig. 11. The effects of initial wealth W_0 under the separate cost function, $D(A) = \tau_1 A^{\theta} + \tau_2 \sigma^{-\psi}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $\theta = 2$, $\psi = 2$, $\tau_1 = 0.07$, and $\tau_2 = 7$).

5. Some empirical implications

Our results in Section 3.1 demonstrates that when agents' actions do not affect risk, the two predictions of traditional agency theory still hold in the presence of wealth effects. For

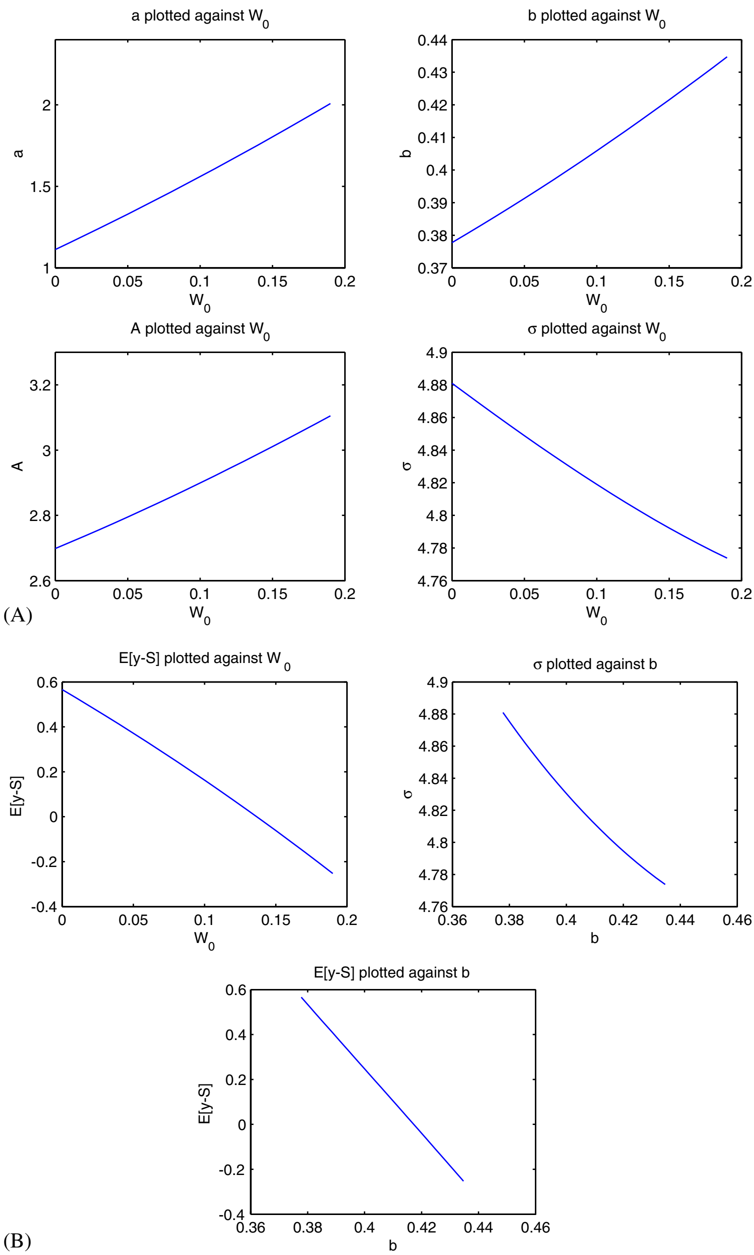


Fig. 12. The effects of initial wealth W_0 under the separate cost function, $D(A) = \tau_1 A^{\theta} + \tau_2 \sigma^{-\psi}$, for the ENE utility function ($\varepsilon_0 = 0$, $t = 0.5$, $R_1 = 0.75$, $R_2 = 0.1$, $\theta = 2$, $\psi = 2$, $\tau_1 = 0.07$, and $\tau_2 = 20$).

example, the negative relation between PPS and risk is more likely to hold in corporations such as diversified conglomerates and private equity firms where agents have less incentive or latitude to manipulate idiosyncratic risk. For a private equity firm, its manager does not have much latitude to manage idiosyncratic risk of the firm because the manager's major business plans must be approved by venture capitalists (VCs) of the firm. VCs typically hold a relatively diversified portfolio of private firms and would not have incentives to manage the idiosyncratic risk of an individual firm. This is perhaps the reason that Bitler et al. [4] find strong empirical support for the negative relation between PPS and risk in privately held firms. Similarly, the managers of public firms with strong corporate governance or prominent presence of institutional investors would have less latitude to manage idiosyncratic risk. Also, the manager of a large conglomerate with diversified holdings within the firm would have fewer incentives to manage idiosyncratic risk. Our results imply that the negative relation between incentives and risk are more likely to hold for those firms.

In addition, we show that when risk is exogenous and the manager's reservation wage or bargaining power is controlled for, the positive relation between PPS and performance holds for both the exponential utility and the ENE utility. It would be of interest to test this positive relation for private equity firms, diversified conglomerates, or firms that generally have strong corporate governance so that their managers do not have much latitude to manipulate firm-specific risks. It would also be of interest to revisit previous empirical studies on PPS and performance, with the additional control of managers' reservation wage or bargaining power.

When managers can easily manage (idiosyncratic) risk, we argue that a straightforward regression of PPS on risk would capture only an implied relation between the two, as presented in Figs. 2, 3, and 6–10. As these figures illustrate, a negative relation between PPS and risk arises under most of the situations. In one exception where when α changes, the implied relation is positive in both the exponential utility and the ENE utility. Since α is a parameter for the agent's cost of effort for increasing the mean of the output, it partially measures the agent's experience and skills. This result means that to obtain a negative relation between PPS and risk in empirical studies, it is important to control for managers' experience and skills. For example, Palia [37] obtains a negative relation between PPS and risk by controlling for managers' education, experience as chief executive officers, and age. Because Core and Guay [8] and Shi [42] find a positive relation without controlling for managerial characteristics such as education, experience, and age, it would be of interest to revisit their empirical studies by adding these control variables in the regressions.

If a manager can control idiosyncratic risk to some extent, then risk is endogenous. Consequently, it would be appealing to test the relation between risk and PPS in a structural model. To our knowledge, previous empirical studies on this relation do not explicitly take this point into account. In all three cases in which the agent can endogenize risk, [17] we find that there should be a negative coefficient for PPS when idiosyncratic risk is used as a dependent variable. Eqs. (14) and (15) present the optimal response of the agent given a level of PPS when the agent is of exponential utility, we can consider them as the agent's behavioral equations in a structural model. Our model predicts that given a higher PPS, the manager chooses a lower level of idiosyncratic risk. For the ENE utility function, we also find that the negative relation still holds by maneuvering Eqs. (25) and (26) for the Cobb–Douglas and separate cost functions, respectively.

[17] See Theorems 1 and 3.

6. Concluding remarks

Incentives are typically measured by the pay-performance sensitivity (PPS) of an agent's compensation in a linearized manner. It is widely believed that a higher level of PPS means a higher power of incentive, leading to better performance for corporations. For example, it is typically believed that higher ownership of a firm by its managers induces higher effort and thus leads to a higher firm value. This belief is supported by the theoretical literature that employs negative exponential utility functions for principal and agent as well as a normally distributed output whose mean is affected by the agent's effort. Under these assumptions, this literature can justify linear contracts to be optimal. It further predicts that there exists a negative tradeoff between incentives and risk and a positive relation between incentives and performance. These relationships have been tested extensively, but the empirical results are mixed.

It is well known that under the exponential-normal framework, the roles of an agent's constant compensation and his initial wealth have been ignored. As a result, PPS is a unique predictor for both the agent's effort and his performance. By introducing wealth effects and also allowing the agent to control both mean and risk, this paper revisits the two relations. For tractability of solutions as well as for a clear measure of incentive, the paper adopts the linear contract space.[18] The original ENE utility function introduces a wealth effect into the contracting problem, and the agent's ability to manage risk introduces a potential tradeoff between increasing the mean and decreasing the risk of an output.

Under the ENE utility, when an agent can affect only the mean of an output, an increase in the agent's wealth allows him to absorb more risk through a higher PPS, which induces a higher level of effort. The net result is that the benefit derived from an agent's higher effort dominates the higher risk premium due to a higher PPS, resulting in better performance. The agent's reservation wage, which may reflect partially his bargaining power, can affect PPS as well. As in the case of increasing wealth, an increase in an agent's reservation wage makes him less risk averse, leading to a higher level of effort because PPS increases. The principal, however, must pay the agent more on average due to a higher reservation wage, which dominates the benefits from a higher mean. Consequently, the agent's performance declines, and a negative relation between PPS and performance follows.

When an agent can manage both the mean and the risk of an output, an increase in the agent's wealth generally leads to lower performance. It appears that a less risk-averse agent (due to more wealth) bears too much risk. As a result, the increase in the risk premium that the agent commands dominates the benefits from a higher mean. Therefore, the performance as measured by the principal's net expected profit declines. Because PPS may increase or decrease, depending on the agent's cost function, either a positive or a negative relation between PPS and performance may arise. Furthermore, either a positive or a negative relation between PPS and risk may arise, depending on the agent's costs of managing mean and risk.

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[18] It is intractable to derive optimal contracts from and would be difficult to define incentives in a general contract space.

In conclusion, this paper introduces a wealth-dependent utility function and demonstrates that the two predictions based on the exponential and normal assumptions are not robust. Consequently, an empirical rejection of these relations may not constitute a rejection of the principal–agent theory in general. The paper also demonstrates that even within the simple linear contract space, PPS is not a sufficient predictor for an agent's effort and performance, and that the agent's cash compensation, his outside wealth, and his reservation wage may play important roles in the determination of performance. For an agency model to have both predictive power and practical relevance, it is essential to develop a sufficient measure for incentive that has a positive relation with performance. Otherwise, one cannot really discuss the magnitude of incentives. In addition, it might be a potentially fruitful avenue to apply the ENE utility function to other research fields such as market microstructure and corporate finance where wealth effects are largely ignored. It would be of great interest to examine the robustness of the existing results in these fields.

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Appendix A. Proofs

Proof of Lemma 1. Consider the ENE utility function:

$$U(C) = \sum_{i=1}^{N} -\frac{t_i}{R_i} \exp(-R_i C), \quad t_i > 0.$$

The agent's absolute risk aversion is then given by

$$R_A \equiv -\frac{U_{cc}}{U_c} = \frac{\sum_{i=1}^{N} t_i R_i \exp(-R_i C)}{\sum_{i=1}^{N} t_i \exp(-R_i C)}.$$

Taking the derivative of R_A with respect to C, we obtain

$$\frac{dR_A}{dC} = \left[\sum_{i=1}^{N} t_i \exp(-R_i C)\right]^{-2} \left\{ -\sum_{i=1}^{N} t_i R_i^2 \exp(-R_i C) \sum_{i=1}^{N} t_i \exp(-R_i C) + \left[\sum_{i=1}^{N} t_i R_i \exp(-R_i C)\right]^2 \right\}$$

$$\Rightarrow 2\sum_{i<j}^{N} t_i t_j R_i R_j \exp[-(R_i+R_j)C] - \sum_{i<j}^{N} t_i t_j (R_i^2+R_j^2)\exp[-(R_i+R_j)C]$$

$$= -\left\{\sum_{i<j}^{N} t_i t_j (R_i - R_j)^2 \exp[-(R_i+R_j)C]\right\} < 0,$$

where " $\Rightarrow$ " means that the term $\left[\sum_{i=1}^{N} t_i \exp(-R_i C)\right]^{-2}$ is omitted. Hence, we have shown that the agent's absolute risk aversion decreases when consumption increases.

Consider a simple version of the ENE utility function:

$$U(C) = -\frac{t}{R_1}\exp(-R_1 C) - \frac{(1-t)}{R_2}\exp(-R_2 C). \tag{28}$$

The agent's absolute risk aversion is then given by

$$\begin{aligned} R_A \equiv -\frac{U_{cc}}{U_c} &= \frac{tR_1\exp(-R_1C) + (1-t)R_2\exp(-R_2C)}{t\exp(-R_1C) + (1-t)\exp(-R_2C)} \\ &= \frac{tR_1 + (1-t)R_2\exp[(R_1-R_2)C]}{t + (1-t)\exp[(R_1-R_2)C]} \\ &= R_2 + \frac{t(R_1-R_2)}{t+(1-t)\exp[(R_1-R_2)C]}. \end{aligned} \tag{29}$$

Similarly, we have

$$R_A = R_1 + \frac{(1-t)(R_2-R_1)}{(1-t)+t\exp[(R_2-R_1)C]}. \tag{30}$$

It is clear that R_A decreases when agent's consumption C increases. Assuming that $R_1 \geqslant R_2$, we see that $R_2 \leqslant R_A \leqslant R_1$. □

Proof of Theorem 1. The agent's objective function given in Eq. (6) reduces to

$$\begin{aligned} &E\{U[W_0 + S(y) - D(A,\sigma)]\} \\ &\quad = -\frac{1}{R_1}\exp\left[-R_1\left(W_0 + a + bA - D(A,\sigma) - \frac{1}{2}R_1 b^2\sigma^2\right)\right]. \end{aligned} \tag{31}$$

The FOCs with respect to A and σ yield

$$A = \frac{\alpha R_1 k_1^2}{\beta} b^{(1+2n)} = k_2 b^{(1+2n)}, \tag{32}$$

$$\sigma = k_1 b^n, \tag{33}$$

where $n = \frac{2-\alpha}{2\alpha-2-\beta}$, $k_1 = \alpha^{\left(\frac{-\alpha}{2\alpha-2-\beta}\right)}(\beta/R_1)^{\left(\frac{\alpha-1}{2\alpha-2-\beta}\right)}k^{\frac{1}{(\beta+2-2\alpha)}}$, and $k_2 = \frac{\alpha R_1 k_1^2}{\beta}$. When $\alpha \geqslant \beta + 1$ and $\beta > 1$, these FOCs are sufficient.[19] From the agent's participation constraint, the constant compensation, a, is given by

$$a = \frac{1}{2} R_1 b^2 \sigma^2 + D(A, \sigma) - bA + \mathcal{E}_0. \tag{34}$$

Notice that the use of the agent's FOCs and his participation constraint enables us to express a, A, and σ in terms of b alone. The principal's constrained maximization problem then becomes an unconstrained one, that is, choosing an optimal b to maximize the following objective function:

$$\begin{aligned} P(b) &= E\left[y - (a + by)\right] = (1-b)A - a \\ &= (1-b)A - \left[\frac{1}{2} R_1 b^2 \sigma^2 + D(A, \sigma) - bA + \mathcal{E}_0\right] \\ &= A - \frac{1}{\alpha}(1 + 0.5\beta)bA - \mathcal{E}_0 \\ &= k_2 b^{1+2n} - \frac{1}{\alpha}(1 + 0.5\beta)k_2 b^{2+2n} - \mathcal{E}_0. \end{aligned} \tag{35}$$

The FOC with respect to b gives

$$(1 + 2n)k_2 b^{2n} - \frac{1}{\alpha}(2 + \beta)(1 + t)k_2 b^{1+2n} = 0, \tag{36}$$

yielding

$$b = \frac{(1 + 2n)(2\alpha)}{(2 + \beta)(2 + 2n)} = \frac{(2 - \beta)\alpha}{(\alpha - \beta)(2 + \beta)}. \tag{37}$$

It is easy to show that the principal's FOC is sufficient. □

Proof of Theorem 2. Given the linear contract form, $S(y) = a + by$, the agent's FOC yields

$$b = kA. \tag{38}$$

The agent's participation constraint is given by

$$\begin{aligned} &-\frac{t}{R_1}\exp\left\{-R_1\left[W_0 + a + bA - D(A) - \frac{1}{2} R_1 b^2 \sigma^2\right]\right\} \\ &\quad -\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0 + a + bA - D(A) - \frac{1}{2} R_2 b^2 \sigma^2\right]\right\} \\ &\quad = U(W_0 + \mathcal{E}_0). \end{aligned} \tag{39}$$

[19] The proof for the sufficiency is a special case of a general proof to be presented in the proof of Theorem 3 and is thus omitted here.

Let

$$\bar{Y}_1(A) = -\frac{t}{R_1}\exp\left\{-R_1\left[W_0 + bA - D(A) - \frac{1}{2}R_1b^2\sigma^2\right]\right\},$$
$$\bar{Y}_2(A) = -\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0 + bA - D(A) - \frac{1}{2}R_2b^2\sigma^2\right]\right\}.$$

For notational simplicity, we use $Y_1(A)$ and $Y_2(A)$ in place of $\bar{Y}_1(A)$ and $\bar{Y}_2(A)$, respectively. The derivatives of $Y_1(A)$ and $Y_2(A)$ with respect to A are then given by

$$Y_1'(A) = Y_1(A)\left[-R_1\left(kA - R_1k^2\sigma^2A\right)\right],$$
$$Y_2'(A) = Y_2(A)\left[-R_2\left(kA - R_2k^2\sigma^2A\right)\right].$$

Taking the derivative of Eq. (39) with respect to A and using the definitions of $Y_1(A)$ and $Y_2(A)$, we obtain

$$\partial a/\partial A = \frac{Y_1'(A)\exp(-R_1a) + Y_2'(A)\exp(-R_2a)}{R_1Y_1(A)\exp(-R_1a) + R_2Y_2(A)\exp(-R_2a)}. \tag{40}$$

Notice that both a and b can be expressed in terms of A, as implied by Eqs. (38) and (39). The principal's problem is then to determine A to maximize the expected net profit:

$$\max_A\left[A - a(A) - bA\right].$$

The FOC gives

$$1 - 2kA = \partial a/\partial A. \tag{41}$$

Let $\bar{g}_1 = R_1(1 - kA - R_1k^2\sigma^2A)Y_1$ and $\bar{g}_2 = R_2(1 - kA - R_2k^2\sigma^2A)Y_2$. Also, we use g_1 and g_2 in place of $\bar{g}_1$ and $\bar{g}_2$, respectively. Eqs. (40) and (41) then yield

$$g_1\exp(-R_1a) + g_2\exp(-R_2a) = 0. \tag{42}$$

Using Eqs. (39) and (42), we arrive at

$$\frac{R_1}{R_2} = \frac{\log\left[\frac{-U(W_0+\mathcal{E}_0)g_2}{g_1Y_2 - Y_1g_2}\right]}{\log\left[\frac{-U(W_0+\mathcal{E}_0)g_1}{g_2Y_1 - Y_2g_1}\right]}. \tag{43}$$

The proof for the sufficiency of the agent's FOC is a special case of the proof for the general case to be presented in the next subsection. It is thus omitted here. We next show that the principal's objective function is strictly concave or that its second-order derivative with respect to A is strictly negative.

Start with the second-order derivative of $a(A)$. From Eq. (40), we have

$$Y_1'\exp(-R_1a) - R_1Y_1\exp(-R_1a)a'(A) + Y_2'\exp(-R_2a)$$
$$-R_2Y_2\exp(-R_2a)a'(A) = 0,$$

where the "primes" denote the relevant partial derivatives with respect to A. The second-order derivative then yields

$$\begin{aligned}&Y_1'' \exp(-R_1 a) - 2R_1 Y_1' \exp(-R_1 a)a'(A) + R_1^2 Y_1 \exp(-R_1 a)[a'(A)]^2\\ &\quad + Y_2'' \exp(-R_2 a) - 2R_2 Y_2' \exp(-R_2 a)a'(A) + R_2^2 Y_2 \exp(-R_2 a)[a'(A)]^2\\ &\quad - R_1 Y_1 \exp(-R_1 a)a''(A) - R_2 Y_2 \exp(-R_2 a)a''(A) = 0,\end{aligned}$$

where the "double primes" denote the second-order derivatives and where $Y_1''(A)$ and $Y_2''(A)$ are given by

$$Y_1'' = Y_1 \left[-R_1 \left(kA - R_1 k^2 \sigma^2 A\right)\right]^2 - R_1 Y_1 \left(k - R_1 k^2 \sigma^2\right),$$

$$Y_2'' = Y_2 \left[-R_2 \left(kA - R_2 k^2 \sigma^2 A\right)\right]^2 - R_2 Y_2 \left(k - R_2 k^2 \sigma^2\right).$$

Express $a''(A)$ as $a''(A) \equiv I_1/I_2$. I_1 [the nominator of $a''(A)$] is then given by

$$\begin{aligned}I_1 &= \left[Y_1'' \exp(-R_1 a) + Y_2'' \exp(-R_2 a)\right]\\ &\quad -2\left[R_1 Y_1' \exp(-R_1 a) + R_2 Y_2' \exp(-R_2 a)a'(A)\right]\\ &\quad + \left[R_1^2 Y_1 \exp(-R_1 a)[a'(A)]^2 + R_2^2 Y_2 \exp(-R_2 a)[a'(A)]^2\right]\\ &= \exp(-R_1 a)Y_1 \left[R_1(kA - R_1 k^2 \sigma^2 A) - R_1 a'(A)\right]^2\\ &\quad + \exp(-R_2 a)Y_2 \left[R_2(kA - R_2 k^2 \sigma^2 A) - R_2 a'(A)\right]^2\\ &\quad - R_1 Y_1 \exp(-R_1 a)(k - R_1 k^2 \sigma^2) - R_2 Y_2 \exp(-R_2 a)(k - R_2 k^2 \sigma^2).\end{aligned}$$

I_2 [the denominator of $a''(A)$] is then given by

$$I_2 = R_1 Y_1 \exp(-R_1 a) + R_2 Y_2 \exp(-R_2 a).$$

It is straightforward to show that $\left[-a''(A) - 2k\right] < 0$. $\square$

Proof of Theorem 3. The agent's FOCs with respect to A and σ yield

$$b = D_A(A, \sigma), \tag{44}$$

$$\begin{aligned}&-\frac{t}{R_1} \exp\left\{-R_1 \left[W_0 + a + bA - D(A, \sigma) - \frac{1}{2} R_1 b^2 \sigma^2\right]\right\}\\ &\quad \times \left(R_1 \sigma^{-1} D_\sigma(A, \sigma) - R_1^2 b^2\right) - \frac{(1-t)}{R_2} \exp\left\{-R_2 \left[W_0 + a + bA - D(A, \sigma)\right.\right.\\ &\quad \left.\left. -\frac{1}{2} R_2 b^2 \sigma^2\right]\right\} \left(R_2 \sigma^{-1} D_\sigma(A, \sigma) - R_2^2 b^2\right) = 0.\end{aligned} \tag{45}$$

From Eq. (45), it is clear that both A and σ depend on the agent's initial wealth W_0 (Figs. 11 and 12). The agent's participation constraint then reduces to

$$\begin{aligned} &-\frac{t}{R_1}\exp\left\{-R_1\left[W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_1b^2\sigma^2\right]\right\}\\ &\quad-\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_2b^2\sigma^2\right]\right\}\\ &\quad\quad=U(W_0+\mathcal{E}_0). \end{aligned} \tag{46}$$

Let

$$Y_1=-\frac{t}{R_1}\exp\left\{-R_1\left[W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_1b^2\sigma^2\right]\right\}, \tag{47}$$

$$Y_2=-\frac{(1-t)}{R_2}\exp\left\{-R_2\left[W_0+a+bA-D(A,\sigma)-\frac{1}{2}R_2b^2\sigma^2\right]\right\}, \tag{48}$$

$$A_1=\left(R_1\sigma^{-1}D_\sigma-R_1^2b^2\right),\quad A_2=\left(R_2\sigma^{-1}D_\sigma-R_2^2b^2\right). \tag{49}$$

The agent's FOC with respect to σ and his participation constraint then lead to

$$U(W_0+\mathcal{E}_0)=Y_1+Y_2, \tag{50}$$
$$0=Y_1A_1+Y_2A_2. \tag{51}$$

A simple calculation yields

$$Y_1=\frac{U(W_0+\mathcal{E}_0)A_2}{A_2-A_1},\quad Y_2=\frac{U(W_0+\mathcal{E}_0)A_1}{A_1-A_2}. \tag{52}$$

From the definitions of Y_1 and Y_2 in Eqs. (47) and (48), we have

$$\begin{aligned} a&=-\frac{1}{R_1}\log\left(-\frac{Y_1R_1}{t}\right)+D(A,\sigma)+\frac{1}{2}R_1b^2\sigma^2-bA-W_0\\ &=-\frac{1}{R_2}\log\left(-\frac{Y_2R_2}{1-t}\right)+D(A,\sigma)+\frac{1}{2}R_2b^2\sigma^2-bA-W_0. \end{aligned} \tag{53}$$

Y_1 and Y_2 are then related to each other as follows:

$$\frac{1}{R_1}\log\left(-\frac{Y_1R_1}{t}\right)-\frac{1}{R_2}\log\left(-\frac{Y_2R_2}{1-t}\right)=\frac{1}{2}(R_1-R_2)b^2\sigma^2. \tag{54}$$

Define

$$-\frac{D_\sigma(A,\sigma)}{\sigma b^2}\equiv\gamma R_1+(1-\gamma)R_2\equiv g_1(\gamma). \tag{55}$$

Given this definition, A_1 and A_2 in Eq. (49) are given by

$$A_1 = (1-\gamma)R_1 b^2(R_2 - R_1), \quad A_2 = \gamma R_2 b^2 (R_1 - R_2).$$

Suppose that $R_1 > R_2$. We know that A_1 and A_2 must have opposite signs for Eq. (51) to have a solution because Y_1 and Y_2 are both negative. As a result, we must have $0 \leqslant \gamma \leqslant 1$. Substituting A_1 and A_2 into Eq. (52), we can express Y_1 and Y_2 in terms of γ:

$$\log\left(-\frac{Y_1 R_1}{t}\right) = \log\left[-\frac{U(W_0+\mathcal{E}_0)R_1R_2\gamma/t}{R_2\gamma + R_1(1-\gamma)}\right], \tag{56}$$

$$\log\left(-\frac{Y_2 R_2}{1-t}\right) = \log\left[-\frac{U(W_0+\mathcal{E}_0)R_1R_2(1-\gamma)/(1-t)}{R_2\gamma + R_1(1-\gamma)}\right]. \tag{57}$$

Using Eqs. (44), (53), (54), (55), (56), and (57), we can express all the control variables, a, b, A, and σ in terms of γ. When the cost function is given by $D_1(A,\sigma) = kA^{\alpha}\sigma^{-\beta}$, we have

$$\frac{\beta k A^{\alpha}\sigma^{(-\beta-1)}}{\sigma b^2} = \gamma R_1 + (1-\gamma)R_2 \equiv g_1(\gamma), \tag{58}$$

$$b = k\alpha A^{(\alpha-1)}\sigma^{-\beta}, \tag{59}$$

$$\frac{1}{2}(R_1-R_2)b^2\sigma^2 = \frac{1}{R_1}\log\left(-\frac{Y_1R_1}{t}\right) - \frac{1}{R_2}\log\left(-\frac{Y_2R_2}{1-t}\right) \equiv g_2(\gamma). \tag{60}$$

Let $g_3(\gamma) = \left[\frac{g_1(\gamma)}{\beta k}\right]^{\alpha-1}\left[\frac{1}{k\alpha}\right]^{-\alpha}$ and $g_4(\gamma) = \frac{2g_2(\gamma)}{(R_1-R_2)}$. It can be verified that the expressions for A, σ and b given in Eq. (25) satisfy these equations.

When the cost function is given by $D_2(A,\sigma) = \tau_1 A^{\theta} + \tau_2\sigma^{-\psi}$, we have

$$\frac{\tau_2\psi\sigma^{-\psi-1}}{\sigma b^2} = \gamma R_1 + (1-\gamma)R_2 \equiv g_1(\gamma), \tag{61}$$

$$b = \tau_1\theta A^{\theta-1}, \tag{62}$$

$$\frac{1}{2}(R_1-R_2)b^2\sigma^2 = \frac{1}{R_1}\log\left(-\frac{Y_1R_1}{t}\right) - \frac{1}{R_2}\log\left(-\frac{Y_2R_2}{1-t}\right) \equiv g_2(\gamma). \tag{63}$$

It can be verified that the solutions for A, σ, and b given in Eq. (26) satisfy these equations.

Consequently, the principal's problem is to choose an optimal γ to maximize the expected net profit:

$$\begin{aligned}\max_{\gamma}\ P(\gamma) &= \max_{\gamma}\,[A - (a+bA)] \\ &= \max_{\gamma}\left[A + \frac{1}{R_1}\log\left(\frac{Y_1R_1}{t}\right) - D(A,\sigma) - \frac{1}{2}R_1b^2\sigma^2\right].\end{aligned} \tag{64}$$

We cannot prove analytically that the principal's objective function is concave with respect to γ. But for all of the numerical calculations reported in the paper as well as many more calculations unreported, we plot $P(\gamma)$ against γ and find that $P(\gamma)$ is always strictly concave in γ. Therefore, the optimal γ obtained indeed maximizes the principal's expected net profits. In addition, the numerical results for γ, a, b, A, and σ satisfy the agent's FOCs, his participation constraint, and the principal's FOC to a precision of e^{-5}. □

We next show that the agent's FOCs are sufficient for his maximization problem as well.

A.1. Sufficiency of the agent's FOCs

Suppose that the agent's expected utility function or the agent's objective function is given by $F(m) = H_1(f_1(m)) + H_2(f_2(m))$, that $H_i(\cdot)$ and $f_i(\cdot)$ are concave functions, and that $H_i(\cdot)$ is increasing in $f_i(m)$, where m is a vector of variables, e.g., $m = (A, \sigma)$, and where $i = 1, 2$. Then for any $0 \leqslant \lambda \leqslant 1$, we have

$$f_1(\lambda m_1 + (1-\lambda)m_2) \geqslant \lambda f_1(m_1) + (1-\lambda) f_1(m_2),$$
$$f_2(\lambda m_1 + (1-\lambda)m_2) \geqslant \lambda f_2(m_1) + (1-\lambda) f_2(m_2).$$

Because $H_i(\cdot)$ is both concave and increasing, we obtain

$$H_1(f_1(\lambda m_1 + (1-\lambda)m_2)) \geqslant \lambda H_1(f_1(m_1)) + (1-\lambda) H_1(f_1(m_2)),$$
$$H_2(f_2(\lambda m_1 + (1-\lambda)m_2)) \geqslant \lambda H_2(f_2(m_1)) + (1-\lambda) H_2(f_2(m_2)).$$

We thus arrive at

$$F(\lambda m_1 + (1-\lambda)m_2) \geqslant \lambda F(m_1) + (1-\lambda) F(m_2),$$

which shows that $F(m)$ is concave in m or that the FOCs are both necessary and sufficient. Notice that the proof applies to the more general case in which $i = 1, 2, \ldots, N$.

In our model, $H_i(\cdot)$ corresponds to a negative exponential utility function, which is both concave and increasing in $f_i(A, \sigma)$, and $f_i(A, \sigma)$ is given by $f_i(A, \sigma) = W_0 + a + bA - D(A, \sigma) - \frac{1}{2} R_i b^2 \sigma^2$. Under the conditions that $\alpha \geqslant \beta + 1$ and $\beta > 1$ for the Cobb–Douglas cost function or that $\theta > 1$ and $\psi > 1$ for the separate cost function, it can be shown that $f_i(A, \sigma)$ is strictly concave with respect to A and σ.

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