Monetary-Fiscal Policy Mix and Risks of Nominal Bonds *

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Abstract

We propose a New Keynesian model with monetary-fiscal policy regime switch to explain the time-varying correlation between returns on the market portfolio and nominal Treasury bonds found in the data. In the active monetary and passive fiscal policy (AMPF) regime, neutral technology (NT) and marginal efficiency of investment (MEI) shocks are the most important drivers of economic fluctuations and the stock-bond correlation. In the passive monetary and active fiscal policy (PMAF) regime, the effect of the NT shock is depressed due to the weak reaction of short-term nominal interest rate to inflation, while the effect of the MEI shock remains strong. Because the NT shock leads to positive stock-bond correlation in the AMPF regime, while the MEI shock leads to negative correlation in the PMAF regime, our model provides a coherent explanation for the negative correlation between the market portfolio and long-term nominal Treasury bond returns during 1950s and 2000s when the fiscal policies are active, and for the positive correlation during 1980-2000 when monetary policies are active.

Keywords: bond-stock return correlation, monetary-fiscal policy regime, inflation risk premium

JEL classification codes: E52, E62, G12.

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1 Introduction

Stocks and long-term nominal Treasury bonds are the two largest classes of assets in the financial markets. Understanding the correlation between those two securities is tremendously important for the purpose of portfolio management and for the design of monetary policies. Empirically, an increasing amount of studies (Campbell et al., 2015, 2016; Christiansen and Ranaldo, 2007; Guidolin and Timmermann, 2007; Baele et al., 2010; David and Veronesi, 2013; Gourio and Ngo, 2016; Mumtaz and Theodoridis, 2017) have documented the time-varying feature of the correlation between returns on the market portfolio of stocks and long-term (5- and 10-year) nominal Treasury bonds. The overall stock-bond return correlation is slightly positive in the period 1953-2014. However, this correlation was negative in the 1950s, it becomes positive between the late 1960s to 2000 and the magnitude is especially large between 1980 and 2000. However, the correlation turned negative again after 2000. The literature has yet provide a satisfactory explanation about the dynamics of this covariance during the post-War period. In this paper, we build a new Keynesian model with monetary-fiscal policy regime switch to explain the time-variation of the correlation between returns on stocks and long-term nominal Treasury bonds.

Since the seminal work of Sargent and Wallace (1981) and Leeper (1991), a growing literature, e.g., Davig and Leeper (2011) and Bianchi and Ilut (2016), has shown that it is essential to consider joint behavior of monetary and fiscal authorities when examining the efficacy of government polices. Our model is built on the framework of Christiano, Motto and Rostagno (2014), which brings financial intermediaries and Risk shocks into an otherwise standard New Keynesian model. In addition, we incorporate the recursive preference with habit formation to generate realistic asset prices and the mix of different monetary-fiscal regimes to match the structural change in the policies found in the literature. The market portfolio, i.e., the stock in the economy, is defined in our model as a levered claim on all future consumption. For simplicity, long-term nominal Treasury bond is modeled as a nominal consol bond, the coupon payment of which decays exponentially over time. We include five structural shocks in the model to match the data: the neutral technology (NT) shock, marginal efficiency of investment (MEI) shock, the investment-specific technology shock (IST shock, i.e., the shock to the relative price of consumption to investment goods), Risk shock, and monetary policy (MP) shock.

Monetary policy is modeled as a simple Taylor rule, in which short-term nominal interest rate reacts to inflation and output positively. Under active monetary policy, interest rate reacts to inflation more than one-for-one, while less than one-for-one under passive monetary policy. Following Leeper (1991), fiscal policy is modeled as a lump-sum tax rule that reacts to government debt outstanding, government spending, and output. Higher government spending is coupled with an equivalent increase in lump-sum taxes (in the present value sense) to pay for the spending under passive fiscal policies. However, taxes are not expected to fully finance the increase in spending under active fiscal policies. As shown in Leeper (1991), among the four possible combinations of monetary-fiscal policy mix, the active monetary and passive fiscal (AMPF) and the passive monetary and active fiscal (PMAF) policy regimes lead to stabilized equilibrium. Those two regimes are our focuses in this paper.

Among the five shocks in the model, the NT and MEI shocks are shown to be the key drivers of both the macroeconomic dynamics and the time-varying riskiness of nominal Treasury bonds. The NT shock is a supply shock. After a positive NT shock, output goes up, but marginal cost goes down in the presence of nominal wage rigidity, resulting in lower price level. Under the AMPF regime, because nominal interest rate goes down more than one-for-one with a drop in inflation, real interest rate tends to go down. This further stimulates the output and encourages consumption. The rise in consumption and persistent drop in nominal interest rate leads to higher stock and long-term nominal Treasury bond prices. Therefore, the NT shock leads to positive correlation of returns on stock and long-term nominal bonds. However, under the PMAF regime, because nominal interest rate goes down less than one-for-one in reaction to a drop in inflation, real interest rate tends to go up. Consequently, the stimulus effect of the NT shock is largely muted and its impact on the economy and financial assets becomes minimal, even though the sign of resulting correlation between returns on the stock and long-term nominal bond stays positive.

On the contrary, the MEI shock is a demand shock. After a positive MEI shock, the transformation of investment into capital becomes more efficient. Demand for investment goes up so does the output. Consequently, the demand for labor and wage goes up and the price level rises. Under the AMPF regime, increases in inflation lead to larger increases in nominal interest rate and increases in real interest rate. Consumption drops temporarily due to higher returns on investments and savings but quickly bounces up due to larger capital stock in the economy. Overall, the price of the stock goes up while the price of the long-term nominal bond goes down due to the persistently higher nominal interest rate. Therefore, the MEI shock leads to a negative correlation of returns on the stock and the long-term nominal bond under the AMPF regime.

Since a positive MEI shock leads to higher price level, under the PMAF regime, the real interest rate tends to go down and the stimulating effects of the MEI shock stays strong. However, consumption goes down sharply immediately since taxes go up in response to higher output, irrespective of the lower government debt outstanding. The lower government deficits release the pressure on price level, resulting in a smaller inflation compared to that under the AMPF regime after a positive MEI shock. The nominal interest rate thus goes up less and goes down after a few quarters. As a result, the price of long-term nominal bond goes up and the price of the stock goes down due to the immediate large drop in consumption after the MEI shock, even though consumption bounces back eventually. Therefore, the MEI shock leads to a negative correlation of returns on the stock and the long-term nominal bond under both the AMPF and PMAF regimes.

The time-variation of the stock-bond correlation is driven by the changing relative importance of the NT and MEI shocks under different monetary-fiscal policy regimes. Under the AMPF regime, the effect of the NT shock dominates that of the MEI shock, while the opposite happens under the PMAF regime because the effects of the NT shock is largely weakened. Our model thus predicts that the stock-bond correlation is positive under the AMPF regime and negative under the PMAF regime.

Narrative accounts of the US monetary-fiscal policy history and the structural estimations in various studies, such as Davig and Leeper (2011), show consistently that the mid 1950s to the mid 1960s (with the exception of the late 1950s to early 1960s) and the post 2000 are the two longest period of PMAF regimes, while the 1980s to 2000 is a prolonged period of AMPF regime. Our model thus provides a coherent explanation for the negative correlation between stock and bond returns during the mid 1950s to mid 1960s and post 2000 and the positive correlation during 1980-2000 under the PMAF regime.

We show that our results are fairly robust. The results hold for alternative definitions of the market portfolio, alternative preferences including recursive preferences without habit formation, and the CRRA preference. Our results also hold at the effective lower bound (ELB), which is an extreme scenario of the PMAF regime and resembles the Great Recession post-2007. Empirically, the beta of the nominal Treasury bonds drop to an extremely low level during that period.

There are three papers that are closely related to our work. Campbell et al. (2015) propose that the change in the bond-stock return correlation is driven by the changes in the sensitivity of monetary policies to inflation and output gap, and the persistence of the monetary policy shocks. However, their model cannot explain the negative beta of nominal Treasury bonds in the 1950s.

Similar to our model, Mumtaz and Theodoridis (2017) also uses the switch of monetary-fiscal policy interaction to explain the sign change in the bond-stock return correlation. However, our mechanisms are different. Their model relies on the fiscal policy shock, which leads to different sign of correlation during different regimes. Our model, on the other hand, relies on the NT and MEI shocks, which have the largest contribution to business cycle fluctuations.

The most related study is Gourio and Ngo (2016), who focus on the zero lower bound

(ZLB) period post 2008. Their New Keynesian model generates positive term premia and inflation risk premia during normal times, but these premia fall when the economy are at or close to the ZLB. The similarity of this paper and ours lies in the fact that the ZLB regime, where monetary policy is completely inactive, is an extreme case of the passive monetary policy in our setup. However, our results do not rely on the extreme inactiveness of the monetary policy. In fact, negative correlation between bond and stock returns happen not only during the ZLB period, but also in 1950's and the period between 2000 and 2008. Therefore, our framework is capable of explaining the dynamics of the correlation between stock and nominal bonds in the post-War period.

Finally, our paper belongs to the growing literature that study the asset pricing implications of government policies in a general equilibrium framework. Recent papers in this literature includes Andreasen (2012), Van Binsbergen et al. (2012), DewBeck (2014), Kung (2015), Li and Palomino (2014), Palomino (2012), Rudebusch and Swanson (2012) in addition to the ones mentioned above.

The rest of the paper proceeds as follows. Section 2 provides empirical evidence on the shifts of bond-stock return correlation and monetary-fiscal policy regime. Section 3 proposes a New Keynesian model with bond, equity, and different monetary-fiscal policy regimes, and discusses the asset pricing implications of the model. Section 4 discusses the calibration of the model, the quantitative results, and the determinants of the correlation of stock and bonds. Section 5 concludes.

2 Empirical Evidence

In this section, we discuss the regime switches of monetary-fiscal policies and the betas of the nominal and real Treasury bonds during the post-war period.

2.1 Policy regimes

The post-war U.S. monetary and fiscal policies exhibit frequent changes of monetary and fiscal regimes. Both monetary and fiscal polices can be either active or passive. Structural estimation in Davig and Leeper (2006, 2011), corroborating with narrative accounts of policy behaviors, shows that the duration of each regime ranges from 11 to 22 quarters and the changing of fiscal policy regime is more frequent that that of monetary policy regime.

The regime of monetary polices is classified based on the response of short-term nominal interest to inflation. In the majority of macroeconomic studies, monetary policy is modeled as a Taylor rule of the following form:

$$i_t - i = \phi_i(i_{t-1} - i) + (1 - \phi_i)[\phi_\pi(\pi_t - \pi^*) + \phi_y(\Delta y_t - \Delta y^*)] + \sigma_i e_{i,t}\,. \tag{2.1}$$

where i_t is the short-term nominal interest rate. The policy rule has an interest-rate smoothing component captured by the sensitivity ϕ_i to the deviation of lagged interest rate, i_{t-1}, from the steady state value, i, and responds to the difference between inflation π_t and inflation target π^*, the deviation of output growth Δy_t from its value under flexible prices Δy^*, and a money policy (MP) shock $e_{i,t} \sim \text{IID}\mathcal{N}(0,1)$. The coefficients ϕ_π and ϕ_y capture the responses of the monetary authority to the deviations of inflation and the output growth from their targets, respectively. Under active monetary policy, the short rate increases more than one-for-one with increase in inflation, i.e., $\phi_\pi > 1$, while under passive monetary policy, $\phi_\pi < 1$.

The regime of fiscal policy is classified based on its role in balancing the government budget constraint. Under passive fiscal policy, the lump-sum taxes adjust to absorb the changes in government spending by reacting strongly to government debt outstanding and keep the budget constraint balanced. On the contrary, under active fiscal policy, increases in government spending is not expected to be fully financed by taxes when taxes do not react strongly enough to debt outstanding and the price level has to adjust so that the present value of future government surpluses equals the outstanding government liabilities in real terms. Therefore, passive fiscal policies do not influence the macroeconomic quantities except for the level of government debt, while active fiscal policies do.

In standard New Keynesian models (Davig and Leeper, 2011; Bianchi and Ilut, 2016), fiscal policy is modeled as a lump-sum tax rule that responds to economic fundamentals, similar to the form of the Taylor rule:

$$\tau_t - \tau = \varsigma_\tau(\tau_{t-1} - \tau) + \varsigma_b(b^\infty_{t-1} - b^\infty) + \varsigma_g(g_{yt} - g_y) + \varsigma_y(y_t - y), \tag{2.2}$$

where b^∞_{t-1} is the lagged government-debt-to-output ratio, g_t is the government-expenditure-to-output ratio, y_t is the detrended output, and y is the steady state of the detrended output. The coefficients ς_τ, ς_b, ς_g, and ς_y represent the persistency of the tax policy and the sensitivity of tax policy to government debt, government spending, and output, respectively. Under passive fiscal policy, taxes responds strongly to the movements of government debt with $\varsigma_b > \beta^{-1} - 1$, while under active fiscal policy, taxes do not respond or negatively respond to government debt.

Leeper (1991) is the first to show that, among four possible combinations of active/passive monetary and fiscal mixes, only two of them yield determinacy and unique solution: the active monetary policy and passive fiscal policy (AMPF) regime, and the passive monetary policy and active fiscal policy (PMAF) regime. The estimation of Davig and Leeper (2011) among many others, shows that except for a brief active period in 1959-60, monetary policy was passive from 1948 until the Fed changed operating procedures in October 1979 and policy became active. Monetary policy was consistently active except immediately after the two recessions in 1991 and 2000 and turns passive after the Great Recession in 2007.

After the World War II, Federal Reserve policy supported high bond prices to the exclusion of targeting inflation, an extreme form of passive monetary policy (Woodford 2001), until the Treasury Accord of March 1951. Through the Korean War (June 1950 - July 1953), monetary policy largely accommodated the financing needs of fiscal policy (Ohanian, 1997). From the mid-50s, through the Kennedy tax cut of 1964, and into the second half of the 1960s, fiscal policy was active, paying little attention to debt. Another prolonged period of active fiscal policy started with President Bush's tax reductions in 2002 and 2003 and continued with the drastically increased government spending and tax cuts included in the Economic Stimulus Act of 2008 and the American Recovery and Reinvestment Act of early 2009.

In summary, both the structural estimation and the narrative account of U.S. policy history single out two longest periods of passive monetary and active fiscal (PMAF) policy regime, mid-50s to mid-60s and post-2000, and one prolonged period of active monetary and passive fiscal (AMPF) policy regime, the beginning of 1980s to 2000.

2.2 Risks of nominal Treasury bonds and TIPS

To explore the correlation between stock and bond returns, we follow Campbell et al. (2016) to estimate the realized betas of 5-year zero-coupon nominal U.S. Treasury bonds and Treasury inflation-indexed securities (TIPS) using rolling window regressions of daily data. We use the Capital Market Asset Pricing Model (CAPM) and the return on CRSP value-weighted stock index as the market return.

Yields of 5-year nominal Treasury bonds and TIPS in daily frequency are available from April 5th, 1962 to September 29th, 2017 and from January 2nd, 2003 to September 29th, 2017, respectively, both of which are obtained from the Wharton Research Data Services (WRDS). Yields of 5-year nominal Treasury bond yield between January, 1947 and April, 1962 are in monthly frequency and obtained from McCulloch and Kwon (1993). Daily return on the market portfolio is from Kenneth French's website.[1] The classification for policy regimes between the first quarter of 1949 to the third quarter of 2008 is based on the estimation in Davig and Leeper (2011).[2]

[1]We thank Kenneth French for providing the data.
[2]We thank Eric Leeper for providing us the data.

Panel A of Figure 1 plots the beta of 5-year Treasury bond for the period of 1947-2017, which is obtained by regressing daily return of 5-year Treasury bond on the return on market portfolio using 3-month rolling window. The same figure also shows that the two longest PMAF regimes during 1947 to 2017 are the periods of 1956-1965 and 2002-2017. Even though the estimation in Davig and Leeper (2011) ends in 2008, the PMAF regime is likely to go well beyond 2008. After the financial crisis, the U.S. government implemented the $787 billion American Recovery and Reinvestment Act, approved in February 2009, and provided large fiscal stimulus to the economy. Meanwhile, the interest rate has stayed at zero between 2008 to 2015. All of these facts are strong signals of the PMAF regime for the post-2008 period. For these aforementioned reasons, we will treat the period of 2002-2017 as a PMAF regime. During those two periods, 1956-1965 and 2002-2017, the beta of 5-year nominal Treasury bonds is largely negative. On the contrast, the bond beta is consistently positive during the AMPF regimes, i.e., the periods of 1984-1990 and 1995-2001.

Panel B of Figure 1 plots the beta of 5-year TIPS for the period of 2003-2017 while date is available. Since TIPS are not as liquid as nominal Treasury bonds, we regress weekly return of 5-year TIPS on the return on market portfolio using 6 month rolling window. Opposite of the beta of nominal bonds, the beta of TIPS is largely positive during this PMAF period.

In sum, the data suggests that the correlation between stock and nominal bonds is negative in the PMAF regime while positive in the other regimes, especially in the AMPF regimes. On the contrary, the correlation between stock and real bond stays largely positive during the PMAF regime. In the rest of the paper, we explain these observed dynamics of the correlation between stock and nominal and real bonds in a DSGE model with different policy regimes.

3 Model

In this section, we generate stock and bond returns in a DSGE model with microfoundations. The main structure of our model follows Christiano et al. (2014) in modeling the households, financial intermediaries, final good sector, and intermediate good sector, and the setup for monetary and fiscal policies is consistent with the convention of Leeper (1991) and Bianchi and Ilut (2016).

3.1 Household

Household maximizes life-time utility

$$V_t \equiv \max_{\{C_t, L_t, B_t/P_t, B_t^\infty/P_t, I_t\}} \quad (1-\beta_t)U(C_{h,t}, L_t) + \beta_t \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1-\psi}{1-\gamma}} \tag{3.1}$$

and

$$U_t \equiv U(C_{h,t}, L_t) = \frac{C_{h,t}^{1-\psi}}{1-\psi} - A_t^L \int_0^1 \frac{L_{j,t}^{1+\phi}}{1+\phi} dj \,,$$

where $C_{h,t}$ is the habit adjusted consumption, defined as $C_{h,t} = C_t - b\bar{C}_t$ with $\bar{C}_t$ representing the aggregate consumption.[3] A_t^L is the disutility parameter of labor, growing at rate $(z_t^+)^{1-\psi}$, where (z_t^+) is the growth rate of the economy and is defined later in equation (3.17). $L_{j,t}$ is the number of household members with labor type j who are employed. The parameters are defined as follows: b is the habit parameter, ψ is the reciprocal of the degree of intertemporal elasticity of substitution, γ is the risk aversion parameter, and ϕ is the Frisch elasticity of labor supply parameter.

Households' utility maximization is subject to the budget constraint

$$\begin{aligned} & P_t C_t + Q_t^\infty B_t^\infty + B_t + Q_t^k (1-\delta)\bar{K}_{t-1} + \frac{P_t}{\Psi_t} I_t \\ \leq \quad & B_{t-1}^\infty (Q_t^\infty \rho + 1) + R_{t-1} B_{t-1} + Q_t^k \bar{K}_t + P_t L I_t + P_t D_t - P_t T_t + P_t T_t^e \,, \end{aligned}$$

[3] In equilibrium, $C_t = \bar{C}_t$. However, when making decisions, households at time t take $\bar{C}_{t-1}$ as given.

and the law of capital accumulation

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \left[1 - S\left(\frac{I_t}{\zeta_t^I I_{t-1}}\right)\right] I_t\,. \tag{3.2}$$

where P_t is the price of consumption goods, Q_t^k is the price of raw capital at t, I_t is investment made at t, and Ψ_t is the relative price of consumption to investment goods, which will be defined later. $S(\cdot)$ is the investment adjustment cost, defined as

$$S(x_t) = \frac{1}{2}\left\{\exp\left[\sigma_s\left(x_t - \exp(\mu_{z^+} + \mu_\psi)\right)\right] + \exp\left[-\sigma_s\left(x_t - \exp(\mu_{z^+} + \mu_\psi)\right)\right] - 2\right\},$$

where $x_t = \frac{I_t}{\zeta_t^I I_{t-1}}$, and $\exp(\mu_{z^+} + \mu_\psi)$ is the steady state growth rate of investment. The parameter σ_s is chosen such that $S(\exp(\mu_{z^+} + \mu_\psi)) = 0$ and $S'(\exp(\mu_{z^+} + \mu_\psi)) = 0$. ζ_t^I measures the marginal efficiency of investment, and evolves as follows:

$$\log\left(\frac{\zeta_t^I}{\zeta^I}\right) = \rho_{\zeta^I}\log\left(\frac{\zeta_{t-1}^I}{\zeta^I}\right) + \sigma_{\zeta^I} e_t^{\zeta^I}\,, \quad \text{and } e_t^{\zeta^I} \sim \text{IID}\mathcal{N}(0,1), \tag{3.3}$$

where $e_t^{\zeta^I}$ denotes the marginal efficiency of investment (MEI) shock. Note that investment I_t is measured in terms of investment goods instead of consumption goods. LI_t is the real wage income defined as

$$LI_t = \int \frac{W_{j,t}}{P_t} L_{j,t}\, dj\,,$$

D_t is the real dividend paid by firms, T_t is tax, T_t^e is the net transfer from entrepreneurs, and B_t is the face value of one-period debt lent to entrepreneurs at $t-1$ with gross nominal return R_t. To avoid numerical complication, we follow Woodford (2001) and define B_t^∞ as the amount of long-term government bonds issued at t, each of which has a stream of infinite coupon payments that starts in period $t+1$ with \$1 and decay every period at the rate of

ρ. The price of one such long-term bond, Q_t^∞, is given by

$$Q_t^\infty = \mathbb{E}_t\left[\sum_{s=1}^{\infty} M_{t,t+s}^{\$}\rho^{s-1}\right] = \mathbb{E}_t\left[M_{t,t+1}^{\$}\left(1+\rho Q_{t+1}^\infty\right)\right] ,$$

where $M_{t,t+s}^{\$}$ is the nominal stochastic discount rates (or pricing kernels) from period $t+s$ to t. The gross nominal return on long bond, R_t^B, is thus given by

$$R_t^B = \frac{1+\rho Q_t^\infty}{Q_{t-1}^\infty} . \tag{3.4}$$

It can be easily shown that the yield y_d on this bond is given by $1/Q_t^\infty - (1-\rho)$ and the effective duration is $1/(1-(1+y_d)\rho)$.

3.2 Financial Intermediation

The entrepreneurs have the ability to turn raw capital into productive capital, which is used in production. How much productive capital can be produced by entrepreneur e depends on his net worth $N_{e,t}$, leverage ratio $\chi_{e,t}$, the optimal capital utilization rate $u_{e,t+1}$, and a random productive realized at the end of t after raw capital $\bar{K}_{e,t}$ is purchased:

$$K_{t+1} = \int_0^\infty dF(\omega)\int_0^1 de\ \left[u_{e,t+1}\omega_{e,t}\bar{K}_{e,t}\right] = \int_0^\infty f(\omega)d\omega\int_0^1 de\ \left[u_{e,t+1}\omega_{e,t}\frac{N_{e,t}\chi_{e,t}}{Q_t^k}\right].$$

Entrepreneurs' productivity $\omega_{e,t}$ follows a lognormal distribution with time-varying standard deviation of $\sigma_{\omega,t}$, where

$$\log\left(\frac{\sigma_{\omega,t}}{\sigma_\omega}\right) = \rho_\omega \log\left(\frac{\sigma_{\omega,t-1}}{\sigma_\omega}\right) + \sigma_\omega e^{\omega,t}, \quad \text{and} \quad e_t^\omega \sim \text{IID}\mathcal{N}(0,1), \tag{3.5}$$

under the assumption that $\mathbb{E}_{t-1}[\omega_{e,t}] = 1$, $\log(\omega_{e,t}) \sim N(-\sigma_{\omega,t}^2/2, \sigma_{\omega,t})$. The shock $e^{\omega,t}$ is defined as the Risk shock.[4]

[4] If X follows a log-normal distribution with mean μ_X and standard deviation σ_X, that is, $\log(X) \sim \mathcal{N}(\mu_X, \sigma_X)$, then $\mathbb{E}[X] = \exp(\mu_X + \frac{1}{2}\sigma_X^2)$. According to the assumption, $\exp(\mu_X + \frac{1}{2}\sigma_X^2) = 1$, so $\mu_X = -\frac{1}{2}\sigma^2$.

The leverage ratio that an entrepreneur can take is

$$\chi_{e,t} = \frac{N_{e,t} + B_{e,t}}{N_{e,t}}\,,$$

where $B_{e,t}$ is the one-period loan from the banking industry to entrepreneur e that matures at $t+1$. At the aggregate level, we have

$$N_t = \int_0^1 N_{e,t} d\, e \quad \text{and} \quad B_t = \int_0^1 B_{e,t} d\, e\,.$$

It can be shown that the leverage ratio χ is the same to all entrepreneurs

$$\chi_{e,t} = \chi_t = \frac{N_t + B_t}{N_t}, \tag{3.6}$$

$$Q_t^k \bar{K}_t = N_t + B_t. \tag{3.7}$$

Assume that the banking industry is competitive and banks earn risk-free interest rate on loans in every state of $t+1$, i.e.,

$$[1 - F(\bar{\omega}_{t+1})]\mathcal{Z}_{t+1} B_{e,t} + (1 - \mu_b) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\ R_{t+1}^k Q_{k,t} \bar{K}_{e,t} = R_t B_{e,t}\,,$$

where $\mathcal{Z}_{t+1}$ is the $t+1$ state-contingent nominal return on bank loan, and μ_b is the bankruptcy cost, $\bar{\omega}_{t+1}$ is the threshold above which entrepreneur is productive enough to pay back the loan, and R_t^k is the nominal return on raw capital at t from the perspective of entrepreneur. Entrepreneurs with the threshold productivity $\bar{\omega}_{t+1}$ can just pay back the interest and principal from what they produce:

$$R_{t+1}^k \bar{\omega}_{t+1} Q_t^k \bar{K}_{e,t} = B_{e,t} \mathcal{Z}_{t+1}, \Rightarrow$$

$$\bar{\omega}_{t+1} = \frac{\mathcal{Z}_{t+1}(\chi_t - 1)}{\chi_t} \frac{1}{R_{t+1}^k}.$$

It can be shown that

$$R^k_{t+1}\left[\Gamma(\bar{\omega}_{t+1}) - \mu_b G(\bar{\omega}_{t+1})\right] = \frac{\chi_t - 1}{\chi_t} R_t \tag{3.8}$$

needs to hold in every state at period $t+1$. The definitions of $G(\bar{\omega}_{t+1})$ and $\Gamma(\bar{\omega}_{t+1})$ are given as follows:

$$\begin{aligned} G(\bar{\omega}_{t+1}) &= \int_0^{\bar{\omega}_{t+1}} \omega_t dF(\omega_t), & (3.9)\\ \Gamma(\bar{\omega}_{t+1}) &= [1 - F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G(\bar{\omega}_{t+1}). & (3.10) \end{aligned}$$

The return on raw capital, R^k_t, is defined as

$$R^k_t = \frac{(1-\tau^k)\left[u_t r^k_t - a(u_t)/\Psi_t\right] P_t + (1-\delta)Q^k_t + \tau^k \delta Q^k_{t-1}}{Q^k_{t-1}}, \tag{3.11}$$

where r^k_t is the real rental rate of productive capital paid by producers, and τ^k is the tax rate on capital income. The nominal cost of utilization per unit of raw capital is $\frac{P_t}{\Psi_t} a(u_t)$, where

$$a(u) = r^k[\exp(\sigma_a(u_t - 1)) - 1]/\sigma_a \, ,$$

with $\sigma_a > 0$. Note that the maintenance cost $a(u)$ is measured in terms of capital goods, whose relative price to consumption goods is $1/\Psi_t$. [5] The optimal value of u_t that maximizes the nominal return on raw capital, R^k_t, is

$$r^k_t = a'(u_t)/\Psi_t. \tag{3.12}$$

For the entrepreneur, his total worth at the end of t is given by

$$\begin{aligned} N_t &= \gamma_e R^k_t Q_{k,t-1} \bar{K}_{t-1} \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t) dF(\omega_t)\right] + W^e_t \\ &= \gamma_e [1 - \Gamma(\bar{\omega}_t)] R^k_t Q_{k,t-1} \bar{K}_{t-1} + W^e_t \, , & (3.13) \end{aligned}$$

[5] Note that our definition of utilization cost is different from Christiano et al, which is $\frac{P_t}{\exp((t)\mu_\Psi)} a(u_t)\omega_t \bar{K}_t$.

where $1-\gamma_e$ is fraction of raw capital return transferred from entrepreneur to households, and W_t^e is the transfer from household to entrepreneur. The latter serves as an insurance to entrepreneurs so that they can consume even if they bankrupt. Therefore, the net transfer from entrepreneurs to household is

$$T_t^e = (1-\gamma_e)[1-\Gamma(\bar{\omega}_t)]R_t^k Q_{k,t-1}\bar{K}_{t-1} - W_t^e \,.$$

Entrepreneurs choose the default level $\bar{\omega}_{t+1}$ to maximize their profits, and the optimal default value satisfies

$$\mathbb{E}_t \left\{ [1-\Gamma(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})} \left[\frac{R_{t+1}^k}{R_t} \left(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right) - 1 \right] \right\} = 0. \tag{3.14}$$

Since all entrepreneurs choose the same utilization rate and leverage ratio, we have the following aggregation:

$$K_t = u_t \bar{K}_{t-1}. \tag{3.15}$$

3.3 Final-Good Production Sector

There are two industries in the production sector: the final goods industry and the intermediate goods industry. The production of the final consumption goods uses a continuum of intermediate goods, indexed by $i \in [0,1]$, via the Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_p}} \, di \right]^{\lambda_p} , \quad \lambda_p > 1 \,,$$

where Y_t is the output of the final goods, $Y_{i,t}$ is the amount of intermediate goods i used in the final goods production, which in equilibrium equals the output of intermediate goods i, and λ_p measures the substitutability among different intermediate goods. When λ_p is larger, the intermediate goods are more substitutable and the demand to intermediate goods is more price elastic. The final goods industry is perfectly competitive. Profit maximization of the final goods producers leads to the demand function for intermediate goods i:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_p}{\lambda_p - 1}} ,$$

where P_t is the nominal price of the final consumption goods and $P_{i,t}$ is the nominal price of intermediate goods i. It can be shown that goods prices satisfy the following relation:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\lambda_p - 1}} di \right)^{-(\lambda_p - 1)} .$$

3.4 Intermediate-Good Production Sector

The production of intermediate goods i uses both capital and labor via the following homogenous production technology:

$$Y_{i,t} = (z_t L_{i,t})^{1-\alpha} K_{i,t-1}^{\alpha} - z_t^{+} \varphi, \tag{3.16}$$

where z_t is the level of the neutral technology, $L_{i,t}$ and $K_{i,t}$ are the labor and capital services, respectively, employed by firm i. α is the capital share of the output, and φ is the fixed production cost. Finally, z_t^{+} is defined as:

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \tag{3.17}$$

where Ψ_t is the level of the investment-specific technology, measured as the relative price of consumption goods to investment goods. We assume that z_t and Ψ_t evolve as follows:

$$\mu_t^z = \mu_z(1-\rho_z) + \rho_z\,\mu_{t-1}^z + \sigma_z e_t^z\,, \quad \text{and } \ e_t^z \sim \text{IID}\mathcal{N}(0,1), \tag{3.18}$$

$$\mu_t^\psi = \mu_\psi(1-\rho_\Psi) + \rho_\psi\,\mu_{t-1}^\psi + \sigma_\psi e_t^\psi\,, \quad \text{and } \ e_t^\psi \sim \text{IID}\mathcal{N}(0,1), \tag{3.19}$$

where

$$\mu_t^z = \Delta \log z_t, \tag{3.20}$$

$$\mu_t^{z^+} = \Delta \log z_t^+, \tag{3.21}$$

$$\mu_t^\psi = \Delta \log \Psi_t. \tag{3.22}$$

e_t^z and e_t^ψ represent the neutral (NT) and investment-specific technology (IST) shocks, respectively. The intermediate goods industry is assumed to have no entry and exit, which is ensured by choosing a fixed cost ψ that brings zero profits to the intermediate goods producers in the steady state.

Cost minimization problem gives the relationship between capital rental rate and wage:

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{P_t r_t^k}. \tag{3.23}$$

Intermediate goods producer i rents capital service $K_{i,t}$ from entrepreneurs and its net profit at period t is given by $P_{i,t}Y_{i,t} - P_t r_t^k K_{i,t} - W_t L_{i,t}$, where $L_{i,t}$ is the labor service demanded by firms. $L_{i,t}$ is a combination of all labor types and will be defined later. The producer takes the nominal rent of capital service $P_t r_t^k$ and nominal wage rate W_t as given but has market power to set the price of its product in a Calvo (1983) staggered price setting to maximize profits. With probability ξ_p, producer i cannot reoptimize its price at period t, and has to set it according to the following rule,

$$P_{i,t} = \tilde{\pi}_{p,t}\, P_{i,t-1},$$

where

$$\tilde{\pi}_{p,t} = \left(\pi_t^*\right)^{\ell} \left(\pi_{t-1}\right)^{1-\ell} \tag{3.24}$$

is the inflation indexation, π_t^* is the target inflation rate or steady state inflation rate, and $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate. Producer i sets price $P_{i,t}$ with probability $1-\xi_p$ to maximize its profits, i.e.,

$$\max_{\{P_{i,t}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \left[\tilde{\theta}_{p,t\oplus\tau} P_{i,t} Y_{i,t+\tau\,|\,t} - s_{t+\tau} P_{t+\tau} Y_{i,t+\tau\,|\,t}\right]$$

subject to the demand function

$$Y_{i,t+\tau} = Y_{t+\tau} \left(\frac{\tilde{\theta}_{p,t\oplus\tau} P_{i,t}}{P_{t+\tau}}\right)^{-\frac{\lambda_p}{\lambda_p-1}}$$

where $\tilde{\theta}_{p,t\oplus\tau} = \left(\prod_{s=1}^{\tau} \tilde{\pi}_{p,t+s}\right)$ for $\tau \geq 1$ and equals 1 for $\tau = 0$. Here, $Y_{i,t+\tau\,|\,t}$ is the output by producer i at time $t+\tau$ if the last time P_i is reoptimized is period t, and $s_{t+\tau}$ is the real marginal cost, given by

$$s_{t+\tau} \equiv MC_{t+\tau} = \frac{1}{z_{t+\tau}^{1-\alpha} P_{t+\tau}} \left(\frac{W_{t+\tau}}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_{t+\tau}^k}{\alpha}\right)^{\alpha}. \tag{3.25}$$

The value of $s_{t+\tau}$ depends on the economic condition at $t+\tau$,and does not depend on firm i's actions.[6] The first order condition of this problem with respect to $P_{i,t}$ is

$$\sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \left[\tilde{\theta}_{p,t\oplus\tau}^{1+\epsilon_p} (1+\epsilon_p) P_{i,t}^{\epsilon_p} P_{t+\tau}^{-\epsilon_p} Y_{t+\tau} - \epsilon_p s_{t+\tau} \tilde{\theta}_{p,t\oplus\tau}^{\epsilon_p} P_{i,t}^{\epsilon_p-1} P_{t+\tau}^{1-\epsilon_p} Y_{t+\tau}\right] = 0$$

[6] Equation (3.25) can be derived by minimizing input costs $W_{t+\tau} L_{j,t+\tau\,|\,t} + r_{t+\tau}^k K_{j,t+\tau\,|\,t}$ given $Y_{i,t+\tau\,|\,t}$.

where $\epsilon_p = \lambda_p/(1-\lambda_p)$. Define the following auxiliary variables

$$\begin{aligned}
H_t &= \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \tilde{\theta}_{p,t\oplus\tau}^{1+\epsilon_p} \left(\frac{Y_{t+\tau}}{Y_t}\right) \left(\frac{P_{t+\tau}}{P_t}\right)^{-\epsilon_p}, \\
J_t &= \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \tilde{\theta}_{p,t\oplus\tau}^{\epsilon_p} \left(\frac{s_{t+\tau}}{s_t}\right) \left(\frac{Y_{t+\tau}}{Y_t}\right) \left(\frac{P_{t+\tau}}{P_t}\right)^{1-\epsilon_p}.
\end{aligned}$$

Then the law of motion for inflation can be expressed as:

$$1 = (1-\xi_p) \left[\frac{\epsilon_p}{1+\epsilon_p} \frac{J_t}{H_t} s_t\right]^{\frac{1}{1-\lambda_p}} + \xi_p \left[\frac{\tilde{\pi}_{p,t}}{\pi_t}\right]^{\frac{1}{1-\lambda_p}}. \tag{3.26}$$

3.5 Labor Unions

There are labor contractors who hire workers of different labor types through labor unions and produce homogenous labor service L_t, according to the following production function:

$$L_t = \left[\int_0^1 L_{j,t}^{\frac{1}{\lambda_w}} \, dj\right]^{\lambda_w}, \quad \lambda_w > 1,$$

where λ_w measures the elasticity of substitution among different labor types. The intermediate goods producers employ the homogenous labor service for production. Labor contractors are perfectly competitive, and their profit maximization leads to the demand function for labor type j:

$$L_{j,t} = L_t \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}}.$$

It is easy to show that wages satisfy the following relation:

$$W_t = \left(\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} \, dj\right)^{1-\lambda_w},$$

where $W_{j,t}$ is the wage of labor type j and W_t is the wage of the homogenous labor service.

Assume that labor unions face the same Calvo (1983) type of wage rigidities. In each period, with probability ξ_w, labor union j cannot reoptimize the wage rate of labor type j

and has to set the wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} e^{\tilde{\mu}_{w,t}} W_{jt-1} \,,$$

where

$$\tilde{\pi}_{w,t} = \left(\pi_t^*\right)^{\ell_w} \left(\pi_{t-1}\right)^{1-\ell_w} \tag{3.27}$$

is the inflation indexation and $\tilde{\mu}_{w,t} = \ell_\mu \mu_{z^+,t} + (1-\ell_\mu)\mu_{z^+}$ is the growth indexation. With probability $1-\xi_w$, labor union j chooses $W_{j,t}^*$ to maximize households' utility.

The optimal wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\left(W_t^*\right)^{1-\phi\epsilon_w} = \mu_w P_t C_{h,t}^{\varphi} A_t^L L_t^{\phi} W_t^{-\epsilon_w \phi} \left(\frac{J_{w,t}}{H_{w,t}}\right) \,, \quad \text{and} \quad \mu_w = \frac{\epsilon_w}{1+\epsilon_w} \,, \tag{3.28}$$

where

$$J_{w,t} = 1 + \xi_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \frac{L_{t+1}}{L_t} \left(\frac{W_{t+1}}{W_t}\right)^{-\epsilon_w} \left(\tilde{\pi}_{w,t+1} e^{\tilde{\mu}_{w,t+1}}\right)^{\epsilon_w} J_{w,t+1} \right] , \tag{3.29}$$

$$\begin{aligned} H_{w,t} = 1 + \xi_w \mathbb{E}_t \Bigg[& M_{t,t+1}^{\$} \frac{P_{t+1}}{P_t} \frac{A_{t+1}^L}{A_t^L} \left(\frac{C_{h,t+1}}{C_{h,t}}\right)^{\psi} \left(\frac{L_{t+1}}{L_t}\right)^{1+\phi} \left(\frac{W_{t+1}}{W_t}\right)^{-\epsilon_w(1+\phi)} \\ & \times \left(\tilde{\pi}_{w,t+i} e^{\tilde{\mu}_{w,t+i}}\right)^{\epsilon_w(1+\phi)} H_{w,t+1} \Bigg] . \end{aligned} \tag{3.30}$$

The dynamics for aggregate wage level is:

$$W_t^{1/(1-\lambda_w)} = (1-\xi_w)(W_t^*)^{1/(1-\lambda_w)} + \xi_w \left(\tilde{\pi}_{w,t} \exp(\tilde{\mu}_t^w) W_{t-1}\right)^{1/(1-\lambda_w)} . \tag{3.31}$$

3.6 Policies

The central bank implements a Taylor (1993)-type monetary policy rule, specified in Equation 2.1:

$$i_t - i = \phi_i(i_{t-1} - i) + (1 - \phi_i)[\phi_\pi(\pi_t - \pi^*) + \phi_y(\Delta y_t - \Delta y^*)] + \sigma_i e_{i,t}\,;$$

and the fiscal authority adjusts taxes according to the tax policy Equation 2.2:[7]

$$\tau_t - \tau = \varsigma_\tau(\tau_{t-1} - \tau) + \varsigma_b(b^\infty_{t-1} - b^\infty) + \varsigma_g(g_{yt} - g_y) + \varsigma_y(y_t - y),$$

Government's flow budget identity follows:

$$\frac{Q^\infty_t B^\infty_t}{P_t} = R^B_t \frac{Q^\infty_{t-1} B^\infty_{t-1}}{P_t} + G_t - T_t$$

holds at any time t. Equivalently, government budget constraint can be written in the following form:

$$b^\infty_t = \frac{R^B_t b^\infty_{t-1} Y_{t-1}}{\Pi_t Y_t} + g_{yt} - \tau_t \tag{3.32}$$

The government spending G_t is exogenously given to be a fixed proportion of output.

There are two relevant monetary/fiscal policy regimes that yield determinacy and unique solution of the model according to the policy regime literature, as discussed in Subsection 2.1: the AMPF regime and PMAF regime.

3.7 Equilibrium

In equilibrium, all intermediate good producers take the same actions and all markets clear:

$$P_{i,t} = P_t, \quad Y_{i,t} = Y_t, \quad L_{i,t} = L_t.$$

[7] For simplicity, we do not include a fiscal shock to the tax policy. Our unreported result shows that fiscal shock is not important for the stock-bond return correlation quantitatively.

— Resource constraint:

$$Y_t = C_t + I_t/\Psi_t + G_t + a(u_t)\bar{K}_{t-1} + \mathcal{D}_t, \tag{3.33}$$

where $\mathcal{D}_t$ is the bankruptcy cost, equal to $\mu_b G(\bar{\omega}_t) R_t^k \frac{Q_{k,t-1}\bar{K}_{t-1}}{P_t}$.

3.8 Asset Pricing Implications

In this section, we discuss the asset pricing implications of the model.

3.8.1 Returns on Stock

We define a stock in two ways. The first definition follows Abel (1999), in which a stock is the claim on consumption raised to a power of λ, C_t^λ, and $\lambda > 1$ reflects leverage. Since dividend in the data is four to five times more volatile than consumption, the leverage ratio λ is needed to create the wedge between dividend and consumption. The stock price and stock return are thus given by

$$\begin{aligned} S_t^c &= P_t C_t^\lambda + \mathbb{E}_t\left[M_{t,t+1}^{\$} S_{t+1}^c\right] && (3.34) \\ R_{s,t+1}^c &= \frac{S_{t+1}^c}{S_t^c - P_t C_t^\lambda}\,. && (3.35) \end{aligned}$$

The second definition follows Christiano et al. (2014), in which a stock is the claim on entrepreneur wealth. Assuming zero return at bankruptcy, the return on entrepreneur's wealth is given by

$$\begin{aligned} R_{s,t}^e(\omega) &= \max\{0, \omega R_t^k \chi_{t-1} - \mathcal{Z}_t(\chi_{t-1} - 1)\} \\ &= \max\{0, [\omega - \bar{\omega}_t]\} \times R_t^k \chi_{t-1}\,. \end{aligned}$$

It can be easily shown that the average return on entrepreneur's wealth is given by

$$R^e_{s,t} = [1 - G(\bar{\omega}_t) - \bar{\omega}_t]\, R^k_t \chi_{t-1}\,. \tag{3.36}$$

There are two main differences between the Christiano et al. (2014) and Abel (1999) definitions of a stock. First, the Christiano et al. (2014) definition allows for time-varying leverage ratio χ_t, financing cost $\mathcal{Z}_t$, and bankruptcy probability captured by $\bar{\omega}_t$, while the definition in Abel (1999) assumes a constant leverage ratio. Second, the value of a stock in the Christiano et al. (2014) definition crucially depends on the output of capital, captured by the rental rate of capital, and the resale value of capital Q^k_t, while the value of stock in the Abel (1999) definition only depends on consumption. Even though consumption is positively related to the output of capital, its correlation with the resale value of capital can be positive or negative, depending on both the shock and regime. As a result, the two stock returns $R^e_{s,t}$ and $R^c_{s,t}$ do not always react to shocks in the same manner.

3.8.2 Returns on Long-Term Real and Nominal Bonds

In our model, the long-term bond has a maturity of infinity and pays coupon every period. The duration of the bond is finite though because the coupon exponentially decays. To illustrate the intuition behind the return on the bond, we analyze the risk premium in a default-free zero-coupon bond with maturity of n periods. Real and nominal default-free zero-coupon bonds with maturity at $t + n$ pay a unit of real and nominal consumption, respectively, at maturity. Their prices are

$$B^{c,(n)}_t = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad B^{\$,(n)}_t = \mathbb{E}_t[M^{\$}_{t,t+n}], \tag{3.37}$$

for real and nominal bonds, respectively, where $M_{t,t+n}$ and $M^{\$}_{t,t+n}$ are the real and nominal discount factors for payoffs at $t+n$.[8] The associated real and nominal yields are defined, respectively, as

$$r_t^{(n)} = -\frac{1}{n}\log B_t^{c,(n)}, \quad \text{and} \quad i_t^{(n)} = -\frac{1}{n}\log B_t^{\$,(n)}.$$

The returns on real and nominal bonds are given by

$$R_{b,t+1}^{c,(n)} = \frac{B_{t+1}^{c,(n-1)}}{B_t^{c,(n)}}, \quad \text{and} \quad R_{b,t+1}^{\$,(n)} = \frac{B_{t+1}^{\$,(n-1)}}{B_t^{\$,(n)}}, \tag{3.38}$$

respectively.

It is useful to decompose expected excess returns on real and nominal bonds into real term and inflation risk premia, which are compensations for real and nominal risks, respectively. The one-period real term premium of a bond with maturity n-period is defined as

$$rTP_t^{(n)} \equiv \log \mathbb{E}_t\left[R_{b,t+1}^{c,(n)}\right] - r_t, \tag{3.39}$$

and the one-period inflation risk premium $\pi TP_t^{(n)}$ is the log difference between the real returns for investing in an n-period nominal bond and an n-period real bond for one-period:

$$\pi TP_t^{(n)} \equiv \log \mathbb{E}_t\left[R_{b,t+1}^{\$,(n)} P_t/P_{t+1}\right] - \log \mathbb{E}_t\left[R_{b,t+1}^{c,(n)}\right], \tag{3.40}$$

where r_t, $R_{b,t+1}^{\$,(n)}$, and $R_{b,t+1}^{c,(n)}$ are the net real interest rate, returns on nominal and real bonds, respectively.

Next, in order to illustrate the mechanism that drives the return on long-term bond, we derive the bond risk premium analytically under the simplifying assumption that all the variables follow log-normal distribution and are homoskedastic.

[8]Notice that $B_t^{c,(n)}$ is the real price of the real bond, while $B_t^{\$,(n)}$ is the nominal price of the nominal bond.

3.8.3 Real Term Premium and Inflation Risk Premium

The one-period real term premium of a bond with maturity n-period can be written as

$$rTP_t^{(n)} = -\mathrm{cov}_t\left[m_{t,t+1}, \sum_{s=2}^{n} m_{t+s-1,t+s}\right]. \tag{3.41}$$

The above equation indicates that the real term premium of a long-term bond is positive if the stochastic discount factor (SDF) of the first period is negatively correlated with the SDFs of the future periods until maturity on average, and vice versa.

Inflation risk premium can be written as

$$\pi TP_t^{(n)} = \mathrm{cov}_t\left(m_{t,t+1}, \sum_{s=1}^{n} \pi_{t+s}\right). \tag{3.42}$$

Therefore, inflation risk premium of a bond with maturity n depends on the covariance between the $t+1$-period pricing kernel and the inflation between $t+1$ to maturity. To compute the inflation risk premium of the nominal consol bond, we define the long-term inflation as $\pi_t^{\infty} = \sum_{s=1}^{\infty} \rho^{s-1}\pi_{t+s}$ and examine the correlation between π^{∞} and the pricing kernel M.

The details of deriving the real term premium and inflation risk premium can be found in Section Appendix B.

4 Results and Analysis

4.1 Calibration

We calibrate the model to match key macroeconomic moments. Table 1 lists the calibrated values of structural parameters. The steady state growth rate of the technology μ_z is set to be 0.0041, and the steady state growth rate of the investment-specific technological change μ_ψ is set to 0.0042, implying the average annual growth rate of the economy is 2.4%. Steady state or targeted inflation rate, π^*, is 0.6075%, indicating the targeted annual inflation rate is 2.43%. Government spending is assumed to be 20% of total output. Following the convention of the macro literature, the power on capital in production function α is 0.33; deprecation rate on capital δ is 0.025; price and wage markup, λ_p and λ_w, are 1.2 and 1.05, respectively. The long-term bond parameter ρ is 0.99 so that the duration of the bond is 10 years. The preference parameters are taken from the long-run risk literature: the elasticity of intertemporal substitution, ψ, is 1.2, and the risk aversion parameter γ is 40 so that the return on stock (consumption claim) has the same Sharpe ratio as the return on CRSP market portfolio. The Frisch elasticity of labor supply ϕ is 1, taken from Christiano et al. (2014). We set the habit parameter, b, at 0.85, which is within the wide range of values estimated from the literature. The objective discount factor β is chosen to yield a 3.97% annual risk free rate, closer to 4.14% in the data. Price markup λ_p and wage markup λ_w are 1.2 and 1.05, respectively, from Christiano et al. (2014). The tax rate on capital income, τ^k, is set to 0 for simplicity. The financial sector parameter values are set following the literature including Christiano et al. (2014) and Bernanke et al. (1999): the steady state transfer received by the entrepreneurs from the households, W^e, is 0.003, the fraction of entrepreneurial net worth transferred to households, $1-\gamma_e$, is 0.027, the bankruptcy cost, μ_b, is 0.3, and the steady state probability of default, $F(\bar{\omega})$, is 0.008. The leverage ratio of the claim on consumption, λ, is set to 3, following the convention of the finance literature. The persistence and volatility of shock processes are chosen to match the macroeconomic moments.

Policy parameters in different regimes are set according to the estimation in Bianchi and Ilut (2016). In the AMPF regime, monetary policy responds strongly to inflation to stabilize price and fiscal policy adjusts according to government debt position to satisfy government budget constraint: the sensitivity of interest rate to inflation ϕ_π is 2.7372, the sensitivity of interest rate to output gap ϕ_y is 0.7037, the interest rate persistent parameter ϕ_r is 0.91, the sensitivity of tax to bond ς_b is 0.0609, the sensitivity of tax to output ς_y is 0.3504, the sensitivity of tax to government spending ς_g is 0.3677, and the tax persistent parameter ς_τ is 0.9844. In the PMAF regime, fiscal policy is active and responsible for stabilizing price. Tax-to-GDP ratio no longer adjusts according to debt position, meaning $\varsigma_b = 0$,[9] and because less persistent, with $\varsigma_\tau = 0.8202$. At the same time, monetary policy is passive and has a lower sensitivity to macro fluctuation, $\phi_y = 0.1520$, and a lower persistency, where $\phi_r = 0.6565$. More important, the sensitivity of policy rate to inflation drops below one.

The magnitude of ϕ_π turns out to be critical to the dynamics of the economics in the PMAF, although not critical to the negative sign of the stock-bond return correlation. When ϕ_π is too low, the model generates a counterfactual implication that consumption and output respond negatively to a positive NT shock. In fact, this is the most criticized feature of the new Keynesian model with the zero lower bound (ZLB), which is an extreme case of the PMAF regime. Wieland (2015) and Garín et al. (2017) demonstrate empirically that the sign of output response is the same as the sign of the shock both during normal time and at the ZLB, and Wu and Zhang (2017) proves that the economy has similar behavior when central banks implement unconventional monetary policy at the ZLB in a New Keynesian model. To avoid the aforementioned counterfactual implication, we choose $\phi_\pi = 0.5305$ according to Davig and Leeper (2011), instead of $\phi_\pi = 0.4991$ in Bianchi and Ilut (2016). Note that the negative correlation between stock and nominal bond is robust to the value of ϕ_π. In fact, the lower the value of ϕ_π is, the more negative the correlation is. We discuss the details in Subsection 4.7. Persistence and standard deviation of the shock processes are taken from Christiano, Motto and Rostagno (2014) and Justiniano, Primiceri and Tambalotti (2011), both of which have the similar model structure and shocks as ours, and are presented in Panel D of Table 1.

The model is solved under this set of parameter values using second-order perturbation method. The moments of the key macroeconomic and finance variables generated from the model are presented together with the corresponding moments in the data in Panel A of

[9] Leeper (1991) shows that any value of ς_b less than $1/R^B - 1$ would lead to passive fiscal policies, where R^B is the return on government debt.

Table 2. Data moments are computed from the AMPF and PMAF periods during 1954q3 - 2016q3 and, following Davig and Leeper (2011), are the average of the moments during these two regimes weighted by their corresponding frequency.

Our model matches these macroeconomic moments reasonably well given the intentionally small set of structural shocks in our model in order to illustrate the economic intuition behind the results. Our calibration matches half of the return on nominal 10-year Treasury bonds and one third of the return on market portfolio observed in the data with stock defined as consumption claim, after matching the Sharpe ratio. On one hand, shocks in our model have constant volatilities for simplicity, while volatility shocks have been shown to help generating higher risk premium (Bansal, Kiku, Shaliastovich and Yaro, 2014). On the other hand, the return on entrepreneur wealth, used as a proxy for return on market portfolio in Christiano, Motto and Rostagno (2014), is large and volatile, with annualized mean and standard deviation being 6.96% and 64.4%, respectively. In reality, the return on market portfolio likely reflects both the return on (aggregate) consumption claim and the return on owning capital.

4.2 Variance Decomposition

Panel B of Table 2 reports the variance decomposition of the key variables under the AMPF and PMAF regimes at business cycle frequency (between 8 to 32 quarters). Results in both regimes show that the MEI shock contributes most to the dynamics of macroeconomic variables, followed by the NT shock. For example, the MEI shock contributes 12.6% and 97.68% of variations in consumption and investment growth rates, respectively, in the AMPF regime, and 57.7% and 98.55% in the PMAF regime. This is consistent with the finding in Justiniano et al. (2011) that the MEI shock contributes most to business cycle fluctuations and on the contrary, the IST shock is of little importance. The Risk shock does not play as important a role as it does in the model of Christiano et al. (2014) for the following reasons. First, our parameter values are calibrated mainly based on the estimation of a regime-switching model in Bianchi and Ilut (2016), while the parameter values in Christiano et al. (2014) are estimates from the model with a single AMPF regime. Second, Christiano et al. (2014) includes both anticipated and unanticipated Risk shocks while our model only includes the latter. These two types of shocks are correlated and reinforce the effects of each other. The variance decomposition in Christiano et al. (2014) shows that the anticipated Risk shock is much more important for business cycle fluctuations than the unanticipated one. However, in order to keep our model simple and straightforward, we do not include the anticipated Risk shock. We expect that adding the anticipated Risk shock, which amplifies the effect of the unanticipated Risk shock, will makes our results stronger. The impulse responses to the (unanticipated) Risk shock in Figure B.4 in the Appendix shows that the Risk shock leads to positive stock-bond return correlation in the AMPF regime, and negative correlation in the PMAF regime, which reinforces the effects of the NT and MEI shocks. [10]

For a shock to have substantial effect on the correlation between stock and bond returns, it has to contribute significantly to the variations of both returns. Table 2 shows that in the AMPF regime, the correlation between the return on consumption claim and return on nominal bonds mainly depends on the NT shock, which contributes 89.51% and 64.43% of the variations in these two returns, respectively, while the correlation between the return on consumption claim and return on real bond returns mainly depends on the MP shock, which contributes 7.82% and 45.51% of the variations in these two returns, respectively. In contrast, the correlation between the return on entrepreneur wealth and returns on nominal and real bond returns mainly depends on the MEI shock, which contributes 22.28%, 26.1%, and 42.94% of the variations in these three returns, respectively.

In the PMAF regime, the effect of the NT shock on the return on consumption claim become significantly weaker while the effect of the MEI shock stays strong. As a result, the correlation between stock and bond returns largely depends on the MEI shock, regardless of

[10] Another difference between our model and Christiano et al. (2014) is the preference function. We use recursive preference while they use log utility. However, our experiment shows that the preference function makes little difference in the contribution of the Risk shock to macroeconomic dynamics.

the definition of stock and the nominal or real feature of the bonds. Specifically, the MEI shock contributes 77.29%, 20.63%, 12.08%, and 13.57% of the variations in the returns on consumption claim, entrepreneur wealth, and nominal and real bonds, respectively.

Note that even though the Risk shock contributes over 70% of the variations in the return on entrepreneur wealth R_s^e in both the AMPF and PMAF regimes, it never contributes more than 3% of the variations in the return on real and nominal bonds. Therefore, Risk shock is not crucial for understanding the correlation between stock and bond returns.

Given that the NT, MEI, and MP shocks are the most important drivers of the correlation of stock and long-term bond returns, they are the focus of our analysis. The case of the MP shock is quite straight forward. A positive MP shock leads to higher nominal and real interest rates, and thus contracts the economy. Consequently, the values of stock and bond go down, resulting in a positive correlation between stock and bond returns, regardless of the definition of the stock, the nominal or real feature of the bond, and the regimes. However, the effects of the NT and MEI shocks on the correlation between stock and bond returns are much more complex and we provide detailed analysis below.

4.3 The NT Shock

The AMPF regime — Under the AMPF regime, the values of the consumption claim and capital, the long-term nominal bond, and the long-term real bond all go up after a positive NT shock, resulting in a positive correlation between stock and bond returns, regardless of the definition of the stock and the nominal or real feature of the bond. Impulse responses of relevant variables under a positive NT shock in the AMPF regime are presented by the solid blue lines in Figure 2.

The NT shock is a supply shock. Higher productivity leads to increase in output and consumption, but decrease in inflation. Inflation goes down because nominal rigidity prevents real wage from rising as quickly as the productivity, resulting in a reduction in real marginal cost. In the AMPF regime, the nominal interest rate reacts strongly to inflation. Even though real interest rate rises at the beginning due to the reaction of nominal interest rate to higher output, it quickly drops due to the strong reaction of nominal interest rate to inflation. The lower real interest rate boosts the economy further. Taken all together, a positive NT shock under the AMPF regime leads to a strong and long-lasting boom in the economy. Consequently, the claim on consumption goes up and return on stock gets higher.

The price of the real long-term bond crucially depends on the changes in average real interest rates from the current period till maturity. As illustrated before, real interest rate goes up at the beginning but goes down in the long-run. And the reduction in interest rate in the long-run dominates its increase in the short-run and the price of the real long-term bond goes up. Thus, return on the real bond goes up.

The price of the nominal long-term bond depends on both the inflation and the real interest rate. Since inflation drops, and the real interest rate from the current period till maturity also drops on average, the price of the nominal bond goes up and the return on nominal long-term bond also goes up.

Therefore, the NT shock leads to a positive correlation between stock and (real and nominal) bond returns under the AMPF regime. Moreover, the covariance of the short-term inflation and stochastic pricing kernel is positive.

The PMAF regime — The impulse responses after a positive NT shock in the PMAF regime are presented by the dotted red lines in Figure 2. After a positive NT shock, inflation again goes down. However, under the Taylor rule in the PMAF regime, nominal interest rate weakly reacts to inflation. Consequently, the real interest rate rises persistently in the short and long run, leading to a lower price of the real long-term bond. The price of nominal bond still rises because of lower inflation, however at a much smaller magnitude.

More importantly, the contractionary effect of a higher real interest rate largely cancels out the direct stimulus effect of the higher NT shock on the the economy. The demand for labor and capital decreases substantially so that the price of capital drops. Consequently, the return on entrepreneur wealth goes down. The increase in consumption is also significantly smaller than that in the AMPF regime. The return on consumption claim still rises, however, the magnitude is about 2% of that in the AMPF regime.

In sum, under the PMAF regime, the return on consumption claim is still positively correlated with the return on nominal bond, but becomes negatively correlated with the return on real bond. On the opposite, the return on entrepreneur wealth is negatively (positively) correlates with return on nominal (real) bond. However, the effect of the NT shock on consumption claim in the PMAF regime is significantly weaker than that in the AMPF regime due to the weak reaction of consumption. Note that the covariance of short-term inflation and stochastic pricing kernel stays positive.

4.4 The MEI Shock

The AMPF regime — As shown by the solid blue lines in Figure 3, after a positive MEI shock, the transformation of investment goods into raw capital becomes more efficient, leading to higher investments, lower price of capital, and larger amount of end-of-period capital. The substitution effect of a positive MEI shock leads to higher investment and lower consumption, while the wealth effect leads to higher consumption because the households anticipate a higher level of capital and consumption in the future. The substitution effect dominates the wealth effect and consumption drops initially, but quickly goes up afterwards. The price of capital drops due to the lower cost of capital production and higher supply of capital, leading to a lower return on entrepreneur wealth. The negative effect of the MEI shock on the value of existing capital is discussed in Greenwood and Jovanovic (1999), who document a deep drop of S&P 500 stocks in the 1990s when the new internet and computing technology came out.

Since the MEI shock is a demand shock, a positive MEI shock leads to higher demand for output, and thus higher capital utilization rate and higher demand for labor supply. Consequently, the marginal cost of output go up, resulting in higher inflation. Due to the strong reaction of nominal interest rate to inflation in the AMPF regime, the real interest rate also goes up, dampening the expansionary effect of the positive MEI shock.

The price of real long-term bond goes down due to the higher real interest rate.[11] Even though the nominal interest rate eventually goes down, the effect of the higher rate at the short run dominates and the price of the nominal long-term bond goes down. Therefore, the return on consumption claim negatively correlates with, while the return on entrepreneur wealth positively correlates with, the returns on nominal and real long-term bonds in the AMPF regime.

The PMAF regime — In this regime, the tax policy is active and responsible for price level adjustments. The dotted red lines in Figure 3 presents the impact of a positive MEI shock in the PMAF regime. After a positive MEI shock, output and inflation go up initially for the same reason as in the AMPF regime. However, the government increases taxes in reaction to higher output. Constrained by the government budget balance, increase in government surplus is accompanied by lower price level and thus higher government debt outstanding in real terms.[12] Therefore, the increase in inflation after a positive MEI shock is muted by the active tax policy. Inflation goes up weakly after the shock but goes down quickly afterwards. The nominal interest rate increases less than one-for-one with inflation. The real interest rate goes down first and goes up afterwards. Large increase in the lump-sum taxes lead to sharp drop in consumption. Consequently, the return on consumption claim goes down. The return on entrepreneur wealth also goes down due to the drop in the price of capital.

Since nominal interest rate quickly goes down after the initial rise, the price of nominal long-term bond goes up and so does the return on this bond. The real interest rate, on the opposite, goes down for the first two periods and goes up afterwards, resulting in a fall in the price of the real long-term bond and the return on this bond. Therefore, under the PMAF regime, the stock return negatively correlates with the return on nominal long-term bonds,

[11] The real interest rate goes down at the very beginning but stay higher afterwards, because the policy rate also reacts to higher output

[12] A more intuitive way to interpret the reduction in price is that after an increase in tax, government lowers money supply given that it has more fiscal income and needs not to rely on inflation to balance the budget.

but positively correlates with the return on real long-term bonds.

Note that the MEI shock becomes the main driver of movements in consumption claim in the PMAF regime, while the NT shock is the main driver in the AMPF regime. This change is critical to the change in the sign of the stock-bond return correlation because the NT shock leads to positive correlation but the MEI shock leads to negative correlation of returns on the consumption claim and the nominal long-term bond.

4.5 Other Shocks

We plot the impulse responses under other shocks, that is, the IST, Risk, and MP shocks, in Figures B.3, B.4, and B.5, respectively, in the Appendix. The blue solid lines represent impulse responses in the AMPF regime, and the red dashed lines represent impulse responses in the PMAF regime. Opposite to the MEI shock, the investment price shock leads to negative stock-bond return correlation in the AMPF regime and positive correlation in the PMAF regime. The generates consistent return correlation in different regimes to what we observed in the data. That is, in the AMPF regime, the causes the stock return and bond return to move in the same direction, and in the PMAF regime, the shock causes the two returns to move in the opposite directions. As we mentioned before, the monetary policy shock generates positive stock-bond return correlation in both regimes, since a positive monetary policy shock always increases the real interest rate, which contracts the economy and the values of both stocks and bonds. However, in both the AMPF and PMAF regimes, the effects of the NT and MEI shocks dominate those of the other shocks.

4.6 Correlation of Stock and Bond Returns

The variance decomposition and impulse response analyses above lead to the following proposition.

Proposition. *The correlation of return on stockand return on nominal long-term bond is* *positive in the AMPF regime, and negative in the PMAF regime. The correlation of return on stock and return on real long-term bond is positive in either regime.*

Table 3 reports the correlation matrix of the return on consumption claim (R_s^c), return on entrepreneur wealth (R_s^e), return on long-term nominal bonds ($R_b^{\$}$), and return on long-term real bonds (R_b^c), under both the AMPF and PMAF regimes, from the baseline model with all five shocks. Panels A and B of Table 4 report the correlation matrix of returns and key variables when either the NT shock or MEI shock is shut down, respectively. The following conclusion can be drawn from these two tables. First, the NT shock is key driver of the positive and strong correlation between return on the consumption claim and return on the nominal long-term bond in the AMPF regime. Without the NT shock, even though the correlation is still positive, its magnitude is almost halved, reduced from 0.85 to 0.48. Second, the MEI shock is the reason why the correlation between return on the consumption claim and return on the nominal long-term bond becomes negative in the PMAF regime. Without the MEI shock, the correlation is positive in either regime. Third, the conclusion in the Proposition is robust to the alternative definition of the return on stock, i.e., the average return to entrepreneur wealth. More important, the MEI shock is also the reason of the negative correlation between the return on entrepreneur wealth and return on long-term nominal bond in the PMAF regime. That is, regardless of the definition of stock, the MEI shock is the reason for the negative stock-bond return correlation. Fourth, correlation between return on stock, in either definition, and return on long-term real bond is positively correlated with return on consumption claim under either regime. Finally, the correlation between the nominal pricing kernel and the long-term inflation π^{∞} is positive under both regimes, indicating a positive inflation risk premium in nominal long-term bond.

4.7 The PMAF Regime at the Effective Lower Bound (ELB)

The zero lower bound is an extreme case of the PMAF regime where policy rate does not react to economic fluctuations at all, i.e., ϕ_π and ϕ_y are equal to zero. To keep the model simple and avoid the computational difficulty, we neither include additional preference or inflation shocks to create the ZLB environment nor discuss the case with completely inactive monetary policy, in which $\phi_\pi = 0$ and $\phi_y = 0$. Instead, we assume an ELB scenario, in which the policy rate is almost constant at its steady state level, and discuss the case with ϕ_π and ϕ_y close to zero. When ϕ_π is lower than certain threshold, the model implies that consumption and output respond negatively to a positive NT shock. Because the nominal interest rate is kept constant, the lower inflation caused by a positive NT shock induces higher real interest rate, which has a significant contractionary impact on the economy. In that case, stock prices decreases due to the pessimistic future economic outlook, hence bond and stock returns move in the opposite directions in response to NT shocks, which reinforces our result that in the PMAF regime — the bond-stock return correlation is even more negative since both NT shocks and MEI shocks generate negatively correlated bond and stock returns.

Therefore, our result does not rely on the extreme inactiveness of the monetary policy, as in Gourio and Ngo (2016). The negative correlation between stock and nominal Treasury bond holds as long as the monetary policy is passive (and fiscal policy is active to ensure determinacy).

4.8 Alternative Preference

In our benchmark model, we use recursive preference with habit formation in order to generate a risk premium with reasonable magnitude. We show in this section that the relation between stock-bond return correlation and policy regime is robust to the choice of preference.

4.8.1 CRRA preference

We first change the preference to constant relative risk aversion (CRRA) preference. Panels A and B of Figure B.1 plot the impulse responses to NT and MEI shocks in both regimes, respectively. These impulse responses are qualitatively similar to their counterparts under the recursive preference in the benchmark model. Specifically, a positive NT shock again leads to increases in the returns on consumption claims and long-term nominal bonds in both regimes, while a positive MEI shock leads to opposite movements in these two returns. Panel A of Table B.1 shows that the main result in the baseline model, i.e., positive stock-bond return correlation under the AMPF regime and negative under the PMAF regime, holds under the CRRA preference.

4.8.2 Recursive preference without habit

We also solve a model under a recursive preference without habit formation. Panels A and B of Figure B.2 plot the impulse responses to NT and MEI shocks in both regimes, respectively. Panel B of Table B.1 presents the correlation matrix under the recursive preference without habit. Both the impulse responses and the correlation matrix share similar qualitative characteristics with those under the baseline preference and the relation between the stock-bond correlation and policy regime holds.

5 Conclusion

We build a New Keynesian model with the recursive preference, financial intermediaries, and monetary-fiscal policy interaction, which coherently explains the positive bond-stock return correlation during 1980-2000 when the monetary policy is active, and the negative correlation during 1950s and 2000s when the fiscal policy is active. When the monetary policy is active and the fiscal policy is negative, the NT and MEI shocks together drive the economy, and both shocks induce a positive correlation between bond and stock return. However, when the fiscal policy is active and the monetary policy is negative, the MEI shock dominates in driving the economic dynamics, and induces a negative bond-stock return correlation.

In the next step, we plan to solve and estimate a model with regime switching among the four possible monetary-fiscal regimes following Davig and Leeper (2011). Such a model allows us to understand the stock-bond correlation in the other two regimes, namely active monetary and fiscal policies (AMAF) and passive monetary and fiscal policies (PMPF) regimes, both of which have no equilibrium solutions independently. Adding switching policy regimes could potentially change the economic dynamics for any stand-alone regimes as agents anticipate possible future changes of policy regimes. Moreover, we plan to study the effects of volatility shocks on the stock-bond correlation as more and more studies find the importance of time-varying volatility in explaining economic dynamics.

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Table 1: **Parameter Values in the Baseline Model**

This table presents the calibrated parameter values used in the baseline model. Policy parameters are different under the AMPF and PMAF regimes, while other parameters are kept the same. The superscripts "1" and "2" for the policy coefficients represent the AMPF and PMAF regimes, respectively.

Parameter	Description	Value
Panel A: Preference		
β	discount factor	0.9989
ψ	reciprocal of elasticity of intertemporal substitution	1/1.2
γ	risk aversion	40
ϕ	labor supply aversion	1
b	habit parameter	0.85
Panel B: Production		
α	capital share	0.33
δ	capital depreciation rate	0.025
σ_s	investment adjustment cost parameter	10.78
σ_a	utilization rate cost parameter	2.6263
ξ_p	probability that cannot re-optimize price	0.74
ℓ	price indexation parameter	0.91
λ_p	degree of elasticity of substitution for goods aggregation	1.2
ξ_w	probatility that cannot re-optimize wage	0.81
ℓ_w	wage indexation parameter	0.94
λ_w	degree of elasticity of substitution for labor aggregation	1.05
μ_{z^+}	growth rate of permanent TFP	0.0041
μ_ψ	growth rate of investment specific technology	0.0042
π^*	target inflation rate	1.006
W^e	transfer received by entrepreneurs	0.003
μ_b	bankruptcy cost	0.3

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Table 1 – continued from previous page

Parameter	Description	Value
$F(\bar{\omega})$	steady-state probability of default	0.008
$1-\gamma_e$	fraction of entrepreneurial net worth transferred to households	0.027
ρ	decay rate of long-term government bonds coupon payment	0.99
λ	leverage ratio	3
Panel C: Policies		
ϕ_π^1	sensitivity of interest rate to inflation (AMPF)	2.7372
ϕ_π^2	sensitivity of interest rate to inflation (PMAF)	0.5305
ϕ_y^1	sensitivity of interest rate to output (AMPF)	0.7037
ϕ_y^2	sensitivity of interest rate to output (PMAF)	0.1520
ϕ_i^1	interest rate persistence (AMPF)	0.91
ϕ_i^2	interest rate persistence (PMAF)	0.6565
ς_b^1	sensitivity of tax to debt (AMPF)	0.0609
ς_b^2	sensitivity of tax to debt (PMAF)	0
ς_y^1	sensitivity of tax to output (AMPF)	0.3504
ς_y^2	sensitivity of tax to output (PMAF)	0.3504
ς_g^1	sensitivity of tax to government spending (AMPF)	0.3677
ς_g^2	sensitivity of tax to government spending (PMAF)	0.3677
ς_τ^1	tax persistence (AMPF)	0.9844
ς_τ^2	tax persistence (PMAF)	0.8202
g_y	steady-state government-spending-to-output ratio	0.20
Panel D: Shocks		
$\rho_{\mu^{z+}}$	persistence of the NT shock	0.15
ρ_{μ^ψ}	persistence of the IST shock	0.16
ρ_{ζ^I}	persistence of the MEI shock	0.77
ρ_ω	persistence of the Risk shock	0.97
$\sigma_{\mu^{z+}}$	standard deviation of the NT shock	0.80

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Table 1 – continued from previous page

Parameter	Description	Value
σ_{μ^ψ}	standard deviation of the IST shock	0.40
σ_{ζ^I}	standard deviation of the MEI shock	2.75
σ_ω	standard deviation of the Risk shock	7.00
σ_i	standard deviation of the MP shock	0.06

Table 2: **Summary Statistics**

Panel A of this table reports the first and second moments of key macroeconomic and financial variables. Column 1 are variable names. Column 2 and 3 give the annualized mean and standard deviation in the data in percentage. Column 4 and 5 give the corresponding simulated mean and standard deviation from the model. Panel B of this table reports the variance decompositions at the business cycle frequency of the key variables in the model: return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), inflation (π), nominal interest rate (i), real interest rate (r), nominal pricing kernel ($M^\$$), growth rate of consumption (ΔC), growth rate of investment (ΔI), and price of capital (Q^k). The second to sixth columns are contributions of the NT shock, MEI shock, Risk shock, IST shock, and MP shock, respectively. The numbers before and after the slash (/) represent the contribution of the shocks to fluctuations in variables in the AMPF regime and PMAF regime in percentage, respectively. Business cycle frequency is measured as a periodic component with cycles between 8 to 32 quarters. All returns are excess returns, and all variables are annualized.

Panel A: Simulated Moments

Variables	Data		Model	
	Mean	Std.Dev	Mean	Std.Dev.
consumption growth (ΔC)	1.60	1.72	2.48	1.93
investment growth (ΔI)	2.23	12.70	4.37	13.04
nominal short-term interest rate (i)	4.14	3.06	3.97	1.44
inflation (π)	1.11	0.83	1.98	2.19
return on stock (consumption claim, R_s^c)	6.95	33.77	2.04	11.21
return on entrepreneur wealth (R_s^e)	-	-	6.96	64.40
return on long-term (10-year) nominal bond ($R_b^\$$)	2.48	15.34	0.97	3.38

Panel B: Variance Decomposition at Business Cycle Frequency

Variables	NT (e_z)	MEI (e_{ζ^I})	Risk (e_{σ_ω})	IST (e_ψ)	MP (e_r)
R_s^c	89.51 / 0.58	0.04 / 77.29	0.93 / 2.47	1.70 / 1.76	7.82 / 17.89
R_s^e	1.06 / 5.73	22.28 / 20.63	74.42 / 72.02	0.65 / 1.23	1.58 / 0.39
$R_b^\$$	64.43 / 79.50	26.10 / 12.08	2.56 / 1.60	0.14 / 1.27	6.78 / 5.57
R_b^c	7.92 / 79.93	42.94 / 13.57	1.43 / 1.25	2.20 / 1.96	45.51 / 3.28
π	59.84 / 76.74	35.56 / 19.66	3.55 / 3.17	0.22 / 0.40	0.83 / 0.03
i	18.57 / 47.32	74.16 / 41.60	1.11 / 1.25	0.16 / 0.32	6.00 / 9.50
r	30.84 / 73.09	58.79 / 16.85	1.49 / 2.64	0.07 / 0.25	8.81 / 7.17
$M^\$$	94.75 / 94.37	0.03 / 0.11	0.01 / 0.01	5.21 / 5.52	0.00 / 0.00
ΔC	81.42 / 38.75	12.60 / 57.70	1.52 / 0.58	0.84 / 0.41	3.62 / 2.56
ΔI	0.64 / 0.03	97.68 / 98.55	1.60 / 1.39	0.05 / 0.03	0.03 / 0.00
Q^k	3.64 / 2.84	27.36 / 23.72	67.69 / 73.03	0.04 / 0.12	1.26 / 0.29

Table 3: **Correlation Matrix**

This table reports the correlation matrix of financial and macroeconomic variables with all shocks in the baseline model. The variables include return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^{\$}$), inflation ($\pi$), consumption growth ($\Delta C$), real pricing kernel ($M$), and the sum of future inflation (π^{∞}). The numbers before and after the slash (/) represent the correlations in the AMPF regime and PMAF regime, respectively.

	Panel A: Correlation Matrix							
Variables	R_s^c	R_s^e	$R_b^{\$}$	R_b^c	π	ΔC	M	π^{∞}
R_s^c	1.00	0.21 / 0.25	0.85 / -0.14	0.46 / 0.33	-0.47 / -0.14	0.40 / 0.30	-0.95 / -0.08	-0.29 / 0.09
R_s^e		1.00	0.49 / -0.26	0.53 / 0.31	-0.25 / -0.04	0.10 / 0.11	-0.10 / 0.26	-0.09 / 0.06
$R_b^{\$}$			1.00	0.76 / -0.91	-0.54 / -0.35	0.36 / 0.00	-0.78 / -0.91	-0.24 / -0.32
R_b^c				1.00	-0.30 / 0.35	0.20 / 0.05	-0.25 / 0.91	-0.05 / 0.33
π					1.00	-0.76 / -0.81	0.49 / 0.43	0.44 / 0.61
ΔC						1.00	-0.39 / -0.08	-0.76 / -0.34
M							1.00	0.33 / 0.31

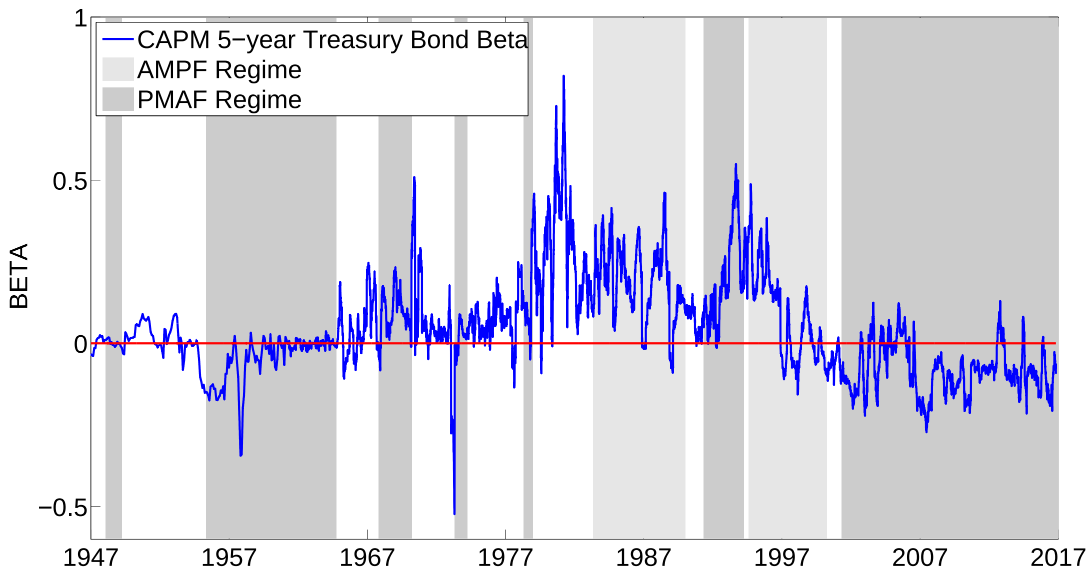
Table 4: **Correlation Matrices without the NT or MEI Shocks**

Panels A to B of this table report the correlation matrices of financial and macroeconomic variables in the baseline model with all shocks except the NT shock and all shocks except the MEI shock, respectively. The variables include return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), inflation (π), consumption growth (ΔC), real pricing kernel (M), and the sum of future inflation (π^∞). The numbers before and after the slash (/) represent the correlations in the AMPF regime and PMAF regime, respectively.

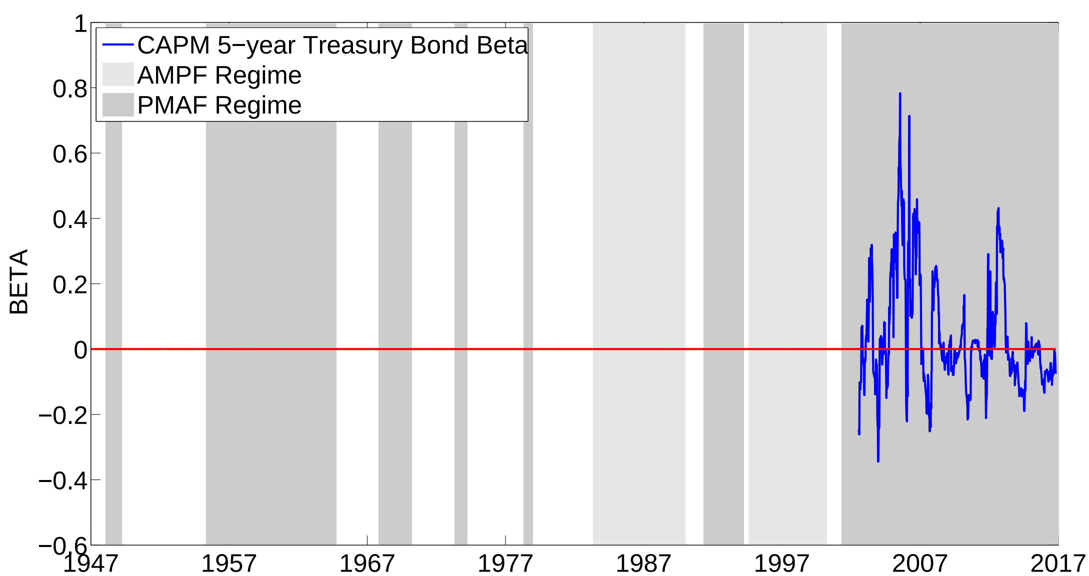
	Panel A: All Shocks except the NT Shock							
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	M	π^∞
R_s^c	1.00	0.36 / 0.28	0.48 / -0.47	0.62 / 0.89	-0.05 / -0.18	0.20 / 0.40	-0.44 / -0.03	0.03 / 0.14
R_s^e		1.00	0.69 / -0.11	0.52 / 0.22	-0.32 / -0.22	0.11 / 0.15	0.00 / 0.13	-0.07 / 0.01
$R_b^\$$			1.00	0.93 / -0.57	-0.40 / 0.12	0.14 / -0.21	-0.01 / -0.36	0.02 / -0.14
R_b^c				1.00	-0.27 / -0.13	0.15 / 0.37	0.10 / 0.41	0.05 / 0.16
π					1.00	-0.50 / -0.84	0.09 / 0.05	0.17 / 0.34
ΔC						1.00	-0.12 / 0.00	-0.78 / -0.02
M							1.00	0.09 / 0.10
	Panel B: All Shocks except the MEI Shock							
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	M	π^∞
R_s^c	1.00	0.23 / -0.32	0.98 / 0.36	0.59 / 0.02	-0.58 / -0.05	0.50 / 0.17	-0.95 / -0.22	-0.56 / -0.08
R_s^e		1.00	0.34 / -0.12	0.34 / 0.17	-0.17 / 0.04	0.10 / -0.02	-0.11 / 0.28	-0.13 / 0.07
$R_b^\$$			1.00	0.66 / -0.90	-0.58 / -0.51	0.49 / 0.15	-0.90 / -0.95	-0.55 / -0.47
R_b^c				1.00	-0.20 / 0.52	0.30 / -0.08	-0.32 / 0.97	-0.13 / 0.48
π					1.00	-0.88 / -0.80	0.61 / 0.52	0.88 / 0.91
ΔC						1.00	-0.48 / -0.12	-0.79 / -0.80
M							1.00	0.61 / 0.49

Figure 1: **CAPM Betas of Long-Term Treasury Bonds**

Panels (a) and (b) plot the time series of estimated CAPM betas of the nominal 5-year Treasury bond and the 5-year TIPS, respectively. CAPM betas in blue are estimated from a rolling window of 3 months of daily return from 1947 to 2017. The shaded areas with light and dark grey represent the AMPF and PMAF regimes, respectively. The x-axis is time, and y-axis is the size of beta.



(a) CAPM Beta of the 5-year Treasury Bond



(b) CAPM Beta of the 5-year TIPS

Figure 2: **Impulse Responses of a Positive NT Shock**

This figure plots the impulse responses of key macro and finance variables in the model after a positive NT shock. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^{\$}$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^{\$}$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

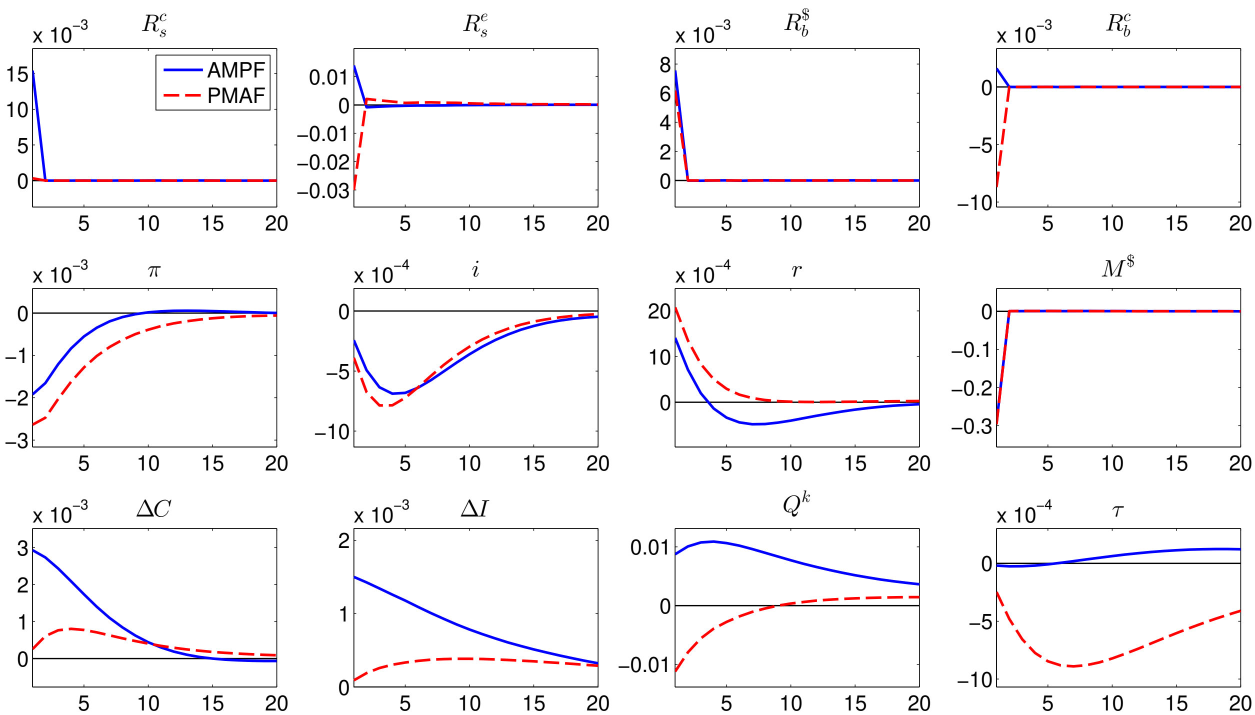
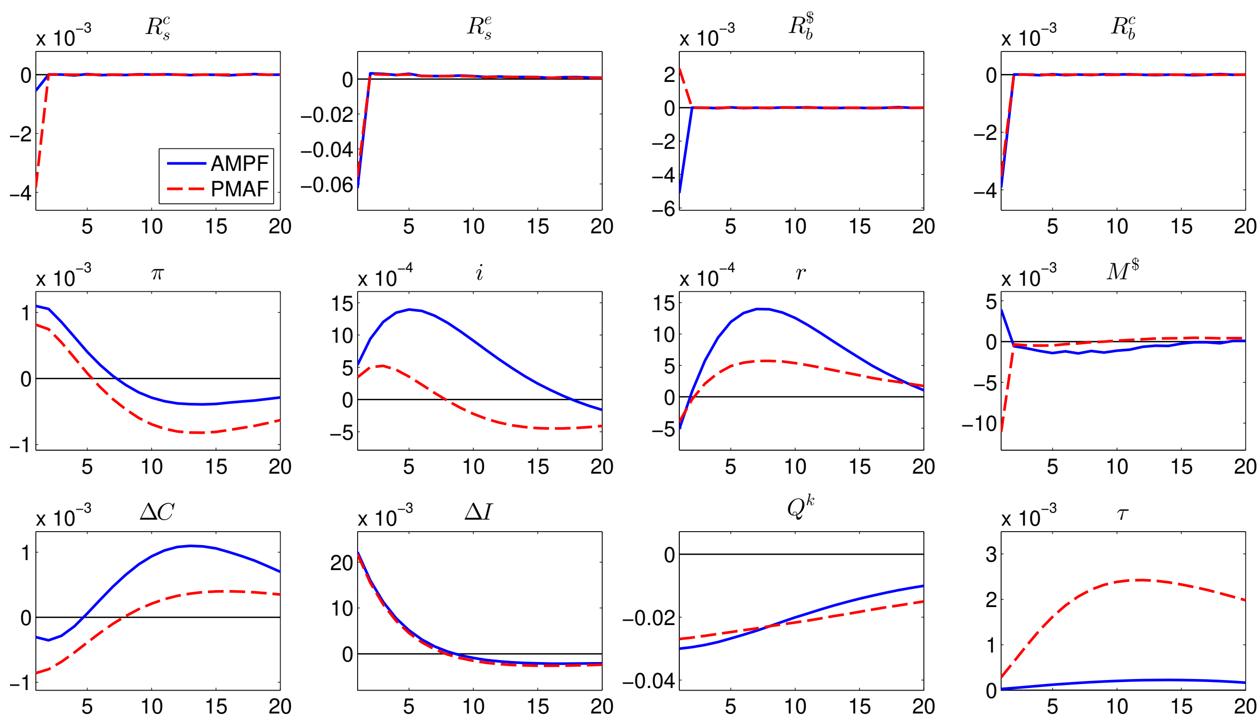


Figure 3: **Impulse Responses of a Positive MEI Shock**

This figure plots the impulse responses of key macro and finance variables in the model after a positive MEI shock. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^\$$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.



Appendix A Data

The raw data in quarterly frequency used in constructing the observed macroeconomic variables are:
GDP Deflator (P)**:** price index of nominal gross domestic product, index numbers, 2005=100, seasonally adjusted, NIPA.
Nominal nondurable consumption ($C^{nom}_{nondurables}$)**:** nominal personal consumption expenditures: nondurable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal durable consumption ($C^{nom}_{durables}$)**:** nominal personal consumption expenditures: durable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal consumption services ($C^{nom}_{services}$)**:** nominal personal consumption expenditures: services, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal investment (I^{nom})**:** nominal gross private domestic investment, billions of dollars, seasonally adjusted at annual rates, NIPA.
Price index (PC^{nom})**:** price index of nondurable goods, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.
Price index (PI^{nom})**:** nominal investment: price index of nominal gross private domestic investment, Nonresidential, Equipment & Software index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.
Federal Funds Rate: (FF) effective federal funds rate, H.15 selected interest rates, percent, averages of daily figures, FRED2.

Here NIPA, BLS and FRED2 stand for
FRED2: Database of the Federal Reserve Bank of St. Louis available at:
http://research.stlouisfed.org/fred2/.
BLS: Database of the Bureau of Labor Statistics available at: http://www.bls.gov/.
NIPA: Database of the National Income And Product Accounts available at:
http://www.bea.gov/national/nipaweb/index.asp.
BGOV: Database of the Board of Governors of the Federal Reserve System available at:
http://www.federalreserve.gov/econresdata/default.htm.

The variables used in the estimation is constructed as follows:

- inflation = growth rate of P
- consumption = $C^{nom}_{nondurables} + C^{nom}_{services}$
- nominal investment = $I^{nom} + C^{nom}_{durables}$

The financial market data used include:
Stock return: Market portfolio excess return, percent, Kenneth French's website.
5-yr TIPS: 5-year TIPS yield, percent, WRDS.
5-yr nominal bond (D): 5-year nominal Treasury bonds yield, percent, WRDS.
5-yr nominal bond (M): 5-year nominal Treasury bonds yield, percent, McCulloch and Kwon (1993).
10-yr nominal bond: 10-year nominal Treasury bonds yield, percent, WRDS.

Here Kenneth French's website, WRDS and McCulloch and Kwon (1993) stand for
Kenneth French's website: Kenneth French's data library available at:
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
WRDS: Wharton Research Data Services available at: https://wrds-web.wharton.upenn.edu/wrds/.
McCulloch and Kwon (1993): U.S. Term Structure Data, 1947-1991 available at:
http://www.econ.ohio-state.edu/jhm/ts/mcckwon/mccull.htm.
D: Daily frequency data.
M: Monthly frequency data.

Appendix B Bond Risk Premium

Appendix B.1 Real Term Premium

In equilibrium, $R_{b,t+1}^{c,(n)} = \exp\left(-(n-1)r_{t+1}^{(n-1)} + nr_t^{(n)}\right)$ satisfies

$$\mathbb{E}_t\left[M_{t,t+1}R_{b,t+1}^{c,(n)}\right] = 1\,.$$

Under the assumption of log-normality, the above equation leads to

$$-nr_t^{(n)} = -r_t - \mathbb{E}_t\left[(n-1)r_{t+1}^{(n-1)}\right] + \frac{1}{2}\mathrm{var}_t\left[(n-1)r_{t+1}^{(n-1)}\right] - \mathrm{cov}_t\left[m_{t,t+1}, (n-1)r_{t+1}^{(n-1)}\right]\,. \tag{B.1}$$

The one-period real term premium of a bond with maturity n-period is thus given by

$$\begin{aligned} rTP_t^{(n)} &\equiv \log\mathbb{E}_t\left[\exp\left(-(n-1)r_{t+1}^{(n-1)} + nr_t^{(n)}\right)\right] - r_t \\ &= n\,r_t^{(n)} - \mathbb{E}_t\left[(n-1)r_{t+1}^{(n-1)}\right] + \frac{1}{2}\mathrm{var}_t\left[(n-1)r_{t+1}^{(n-1)}\right] - r_t\,. \end{aligned}$$

Substituting equation (B.1) into the above equation leads to:

$$rTP_t^{(n)} = \mathrm{cov}_t\left(m_{t,t+1}, (n-1)r_{t+1}^{(n-1)}\right)\,. \tag{B.2}$$

Based on the definition of $r_{t+1}^{(n-1)}$, we have

$$\begin{aligned} -(n-1)r_{t+1}^{(n-1)} &= \log B_{t+1}^{c,(n-1)} = \log\mathbb{E}_{t+1}\left[e^{\sum_{s=2}^{n} m_{t+s-1,t+s}}\right] \\ &= \mathbb{E}_{t+1}\left[\sum_{s=2}^{n} m_{t+s-1,t+s}\right] + \mathrm{var}_{t+1}\left[\sum_{s=2}^{n} m_{t+s-1,t+s}\right]\,. \end{aligned} \tag{B.3}$$

Under the assumption of log-mornality and homoskedasticity, variance and covariance are constant. Therefore, the combination of equations (B.2) and (B.3) leads to

$$rTP_t^{(n)} = -\mathrm{cov}_t\left[m_{t,t+1}, \sum_{s=2}^{n} m_{t+s-1,t+s}\right]\,. \tag{B.4}$$

The above equation indicates that the real term premium of a long-term bond is positive if the stochastic discount factor (SDF) of the first period is negatively correlated with the SDFs of the future periods until maturity on average, and vice versa. For example, the real term premium of a 2-period bond is given by

$$rTP_t^{(2)} = -\mathrm{cov}_t\left(m_{t,t+1}, m_{t+1,t+2}\right)\,.$$

If the SDF is negatively (positively) autocorrelated, the real term premium of the 2-period bond is positive (negative). The real term premium of a 3-period bond is given by

$$rTP_t^{(3)} = -\mathrm{cov}_t\left(m_{t,t+1}, m_{t+1,t+2} + m_{t+2,t+3}\right)\,.$$

Therefore, if $m_{t,t+1}$ is negatively (positively) correlated with $m_{t+1,t+2}$ and $m_{t+2,t+3}$ on average, the real term premium of a 3-period bond is positive (negative).

Appendix B.2 Inflation Risk Premium

Given that $R_{b,t+1}^{\$,(n)} = \exp\left(-(n-1)i_{t+1}^{(n-1)} + ni_t^{(n)}\right)$, inflation risk premium can be written as

$$\begin{aligned}
\pi TP_t^{(n)} &= ni_t^{(n)} - nr_t^{(n)} + \log \mathbb{E}_t\left[\exp\left(-(n-1)i_{t+1}^{(n-1)} - \pi_{t+1}\right)\right] - \log \mathbb{E}_t\left[\exp\left(-(n-1)i_{t+1}^{(n-1)}\right)\right] \\
&= ni_t^{(n)} - nr_t^{(n)} - \mathbb{E}_t\left[(n-1)i_{t+1}^{(n-1)}\right] + \frac{1}{2}\text{var}_t\left[(n-1)i_{t+1}^{(n-1)}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] \\
&\quad + \frac{1}{2}\text{var}_t\left[\pi_{t+1}\right] + \mathbb{E}_t\left[(n-1)r_{t+1}^{(n-1)}\right] - \frac{1}{2}\text{var}_t\left[(n-1)r_{t+1}^{(n-1)}\right] \qquad \text{(B.5)}
\end{aligned}$$

From the equilibrium relation $\mathbb{E}_t\left[M_{t,t+1}^{\$} R_{b,t+1}^{\$,(n)}\right] = 1$, we get

$$-ni_t^{(n)} = -i_t - \mathbb{E}_t\left[(n-1)i_{t+1}^{(n-1)}\right] + \frac{1}{2}\text{var}_t\left[(n-1)i_{t+1}^{(n-1)}\right] - \text{cov}_t\left[m_{t,t+1}^{\$}, (n-1)i_{t+1}^{(n-1)}\right]. \qquad \text{(B.6)}$$

In addition,

$$\begin{aligned}
i_t - r_t &= -\log \mathbb{E}_t\left[e^{m_{t,t+1}^{\$}}\right] + \log \mathbb{E}_t\left[e^{m_{t,t+1}^{\$} + \pi_{t+1}}\right] \\
&= \mathbb{E}_t\left[\pi_{t+1}\right] + \frac{1}{2}\text{var}_t\left[\pi_{t+1}\right] + \text{cov}_t\left[m_{t,t+1}^{\$}, \pi_{t+1}\right]. \qquad \text{(B.7)}
\end{aligned}$$

Substituting equations (B.1), (B.6) and (B.7) into equation (B.5) leads to

$$\begin{aligned}
\pi TP_t^{(n)} &= \text{var}_t\left[\pi_{t+1}\right] + \text{cov}_t\left[m_{t,t+1}^{\$}, \pi_{t+1}\right] + \text{cov}_t\left[m_{t,t+1}^{\$}, (n-1)i_{t+1}^{(n-1)}\right] \\
&\quad - \text{cov}_t\left[m_{t,t+1}, (n-1)r_{t+1}^{(n-1)}\right] + \text{cov}_t\left[\pi_{t,t+1}, (n-1)i_{t+1}^{(n-1)}\right] \\
&= \text{cov}_t\left[m_{t,t+1}, \pi_{t+1}\right] + \text{cov}_t\left[m_{t,t+1}, (n-1)\left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)}\right)\right], \qquad \text{(B.8)}
\end{aligned}$$

where the second equality follows from $m_{t,t+1} = m_{t,t+1}^{\$} + \pi_{t+1}$. Realizing that under the log-normality and homoskedasticity assumptions the nominal-real bond spread is

$$\begin{aligned}
&(n-1)\left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)}\right) = \log\left[B_{t+1}^{c,(n-1)} / B_{t+1}^{\$,(n-1)}\right] = \log \mathbb{E}_t[M_{t+1,t+n}] - \log \mathbb{E}_t[M_{t+1,t+n}^{\$}] \\
&= \log \mathbb{E}_{t+1}\left[\exp\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1}\right)\right] - \log \mathbb{E}_{t+1}\left[\exp\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1} - \pi_{t+s,t+s+1}\right)\right] \\
&= \sum_{s=1}^{n-1} \mathbb{E}_{t+1}[\pi_{t+1+s}] - \frac{1}{2}\text{var}_t\left(\sum_{s=1}^{n-1} \pi_{t+1+s}\right) + \text{cov}_{t+1}\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1}, \sum_{s=1}^{n-1} \pi_{t+s+1}\right).
\end{aligned}$$

Since the variance and covariance terms are constant under the log-normality and homoskedasticity assumptions, it follows that

$$\pi TP_t^{(n)} = \text{cov}_t\left(m_{t,t+1}, \sum_{s=1}^{n} \pi_{t+s}\right). \qquad \text{(B.9)}$$

Therefore, inflation risk premium of a bond with maturity n depends on the coveriance between the $t+1$-period pricing kernel and the inflation between $t+1$ to maturity.

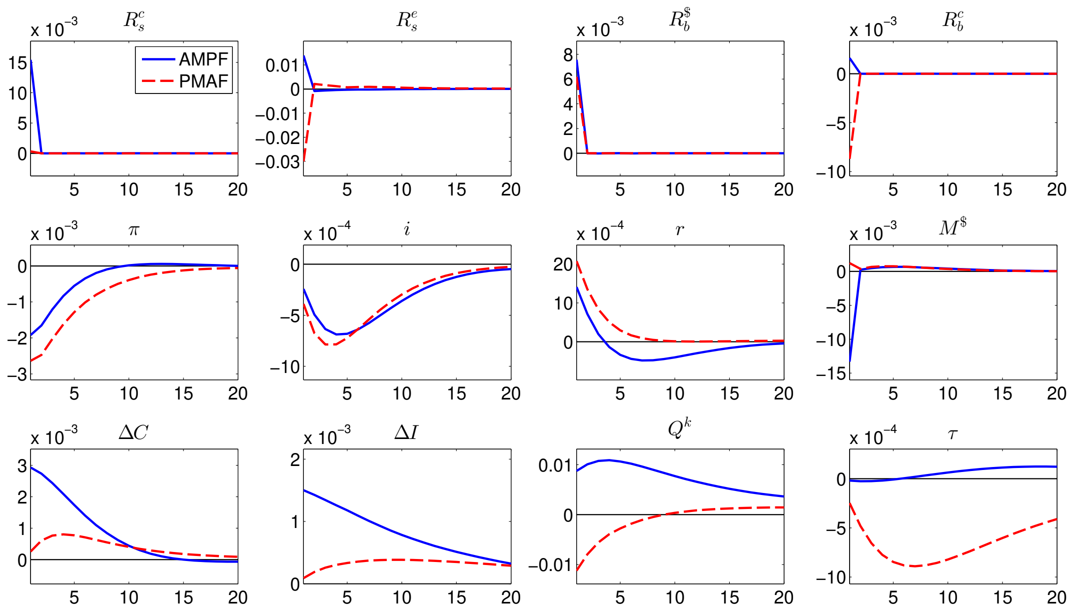
Table B.1: **Correlation Matrices — Alternative Preferences**

Panels A and B of this table report the correlation matrices of financial and macroeconomic variables in the models with CRRA preference and recursive preference without habit formation, respectively. The variables include return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), inflation (π), consumption growth (ΔC), real pricing kernel (M), and the sum of future inflation (π^∞). The numbers before and after the slash (/) represent the correlations in the AMPF regime and PMAF regime, respectively.

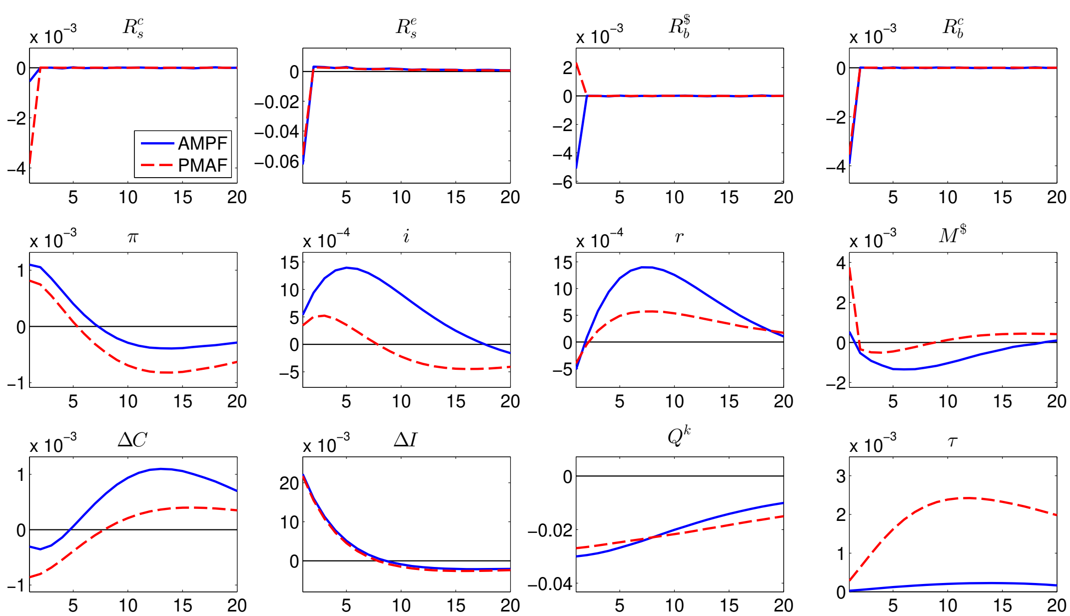
Panel A: CRRA Preference								
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	M	π^∞
R_s^c	1.00	0.21 / 0.25	0.85 / -0.14	0.46 / 0.33	-0.47 / -0.14	0.40 / 0.30	-0.95 / -0.80	-0.29 / 0.09
R_s^e		1.00	0.49 / -0.26	0.53 / 0.31	-0.25 / -0.04	0.10 / 0.11	-0.26 / -0.28	-0.09 / 0.06
$R_b^\$$			1.00	0.76 / -0.91	-0.54 / -0.35	0.36 / 0.00	-0.84 / -0.01	-0.24 / -0.32
R_b^c				1.00	-0.30 / 0.35	0.20 / 0.05	-0.47 / -0.13	-0.05 / 0.33
π					1.00	-0.76 / -0.81	0.57 / 0.63	0.44 / 0.61
ΔC						1.00	-0.55 / -0.57	-0.76 / -0.34
M							1.00	0.53 / 0.34
Panel B: Recursive Preference without Habit Formation								
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	M	π^∞
R_s^c	1.00	0.29 / 0.28	0.52 / -0.15	0.01 / 0.31	-0.40 / -0.09	0.93 / 0.79	-0.95 / -0.11	-0.27 / 0.09
R_s^e		1.00	0.59 / -0.24	0.45 / 0.29	-0.25 / -0.03	0.32 / 0.30	-0.14 / 0.22	-0.11 / 0.06
$R_b^\$$			1.00	0.76 / -0.92	-0.42 / -0.38	0.54 / 0.00	-0.37 / -0.90	-0.13 / -0.33
R_b^c				1.00	-0.10 / 0.39	0.06 / 0.12	0.24 / 0.91	0.07 / 0.34
π					1.00	-0.51 / -0.61	0.42 / 0.45	0.45 / 0.68
ΔC						1.00	-0.88 / -0.22	-0.56 / -0.37
M							1.00	0.30 / 0.32

Figure B.1: **Impulse Responses under CRRA Preference**

Panels (a) and (b) of this figure plots the impulse responses of key macro and finance variables after positive NT and MEI shocks, repectively, in the model with CRRA preferences. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^\$$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.



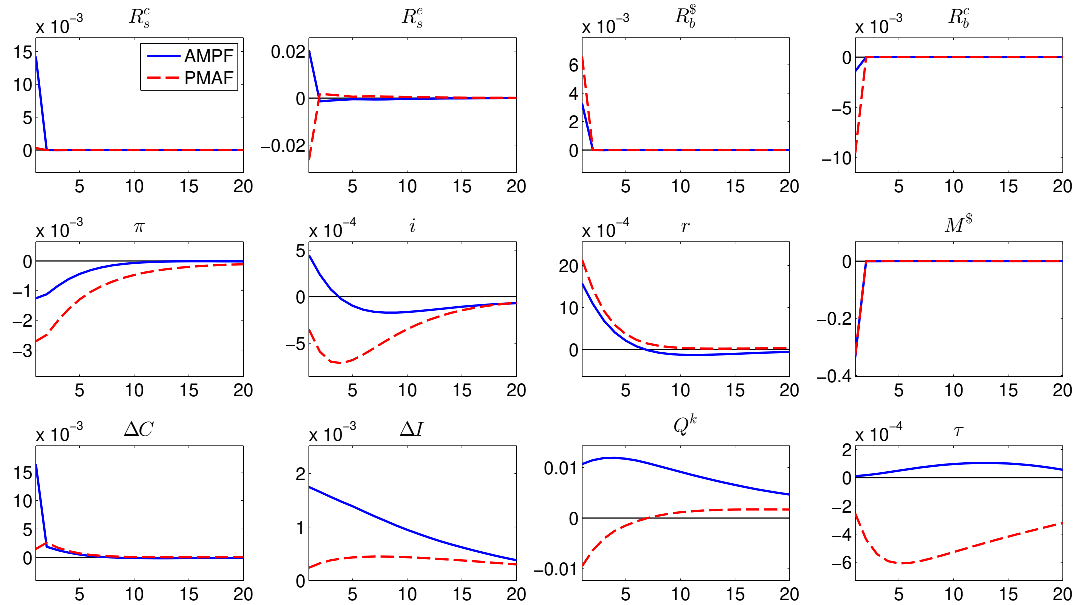
(a) Impulse Responses after a Positive NT Shock



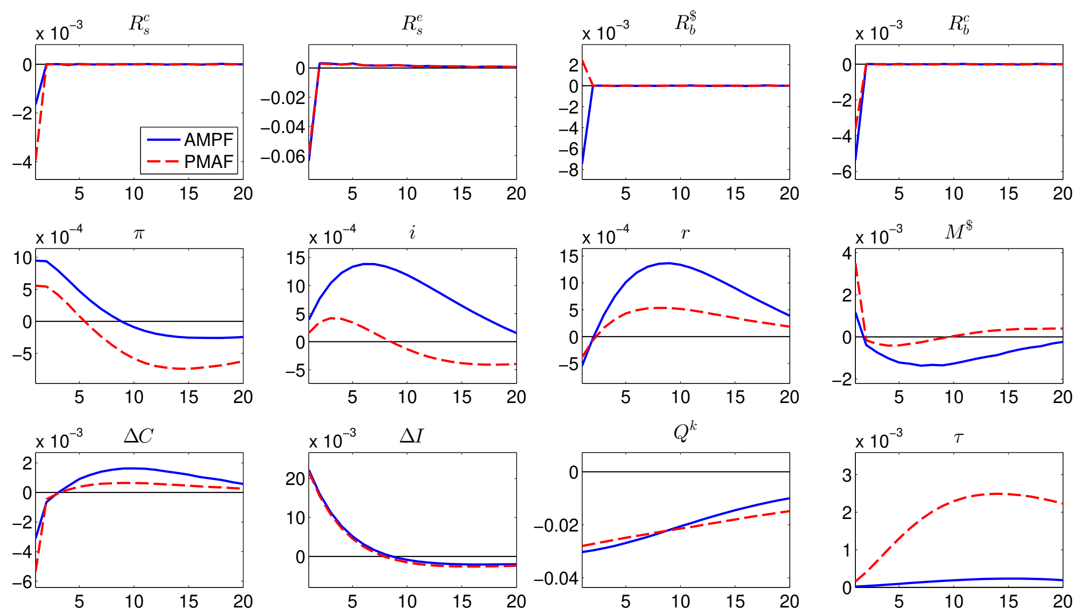
(b) Impulse Responses after a Positive MEI Shock

Figure B.2: **Impulse Responses under Recursive Preference without Habit Formation**

Panels (a) and (b) of this figure plots the impulse responses of key macro and finance variables after positive NT and MEI shocks, respectively, in the model with recursive preference without habit formation. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^\$$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.



(a) Impulse Responses after a Positive NT Shock



(b) Impulse Responses after a Positive MEI Shock

Figure B.3: **Impulse Responses of a Positive IST Shock**

This figure plots the impulse responses of key macro and finance variables in the model after a positive IST shock. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^{\$}$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^{\$}$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red solid lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

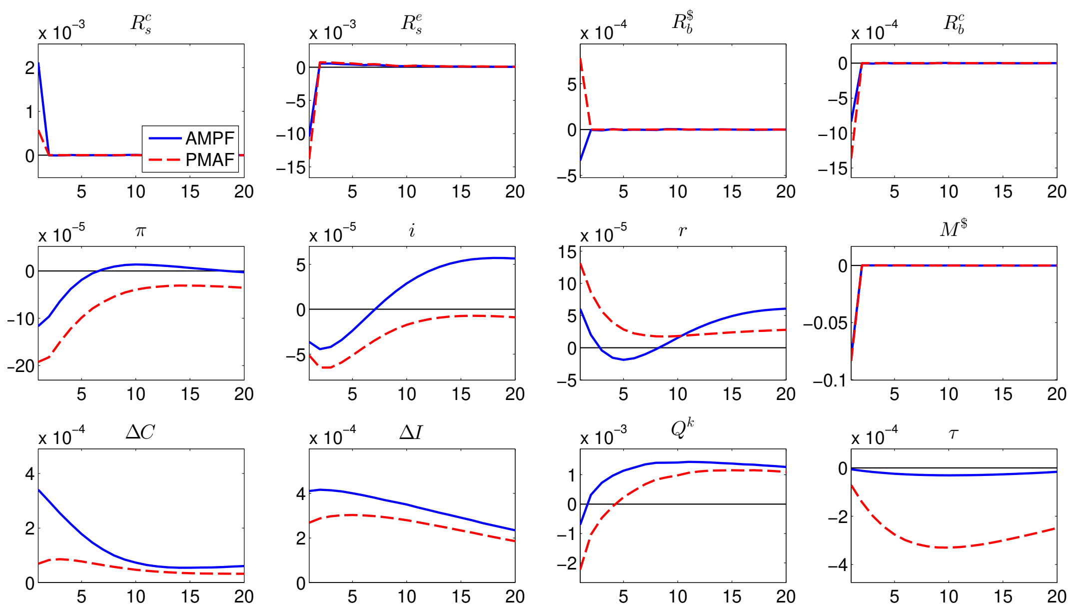


Figure B.4: **Impulse Responses of a Positive Risk Shock**

This figure plots the impulse responses of key macro and finance variables in the model after a positive Risk shock. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^{\$}$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^{\$}$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

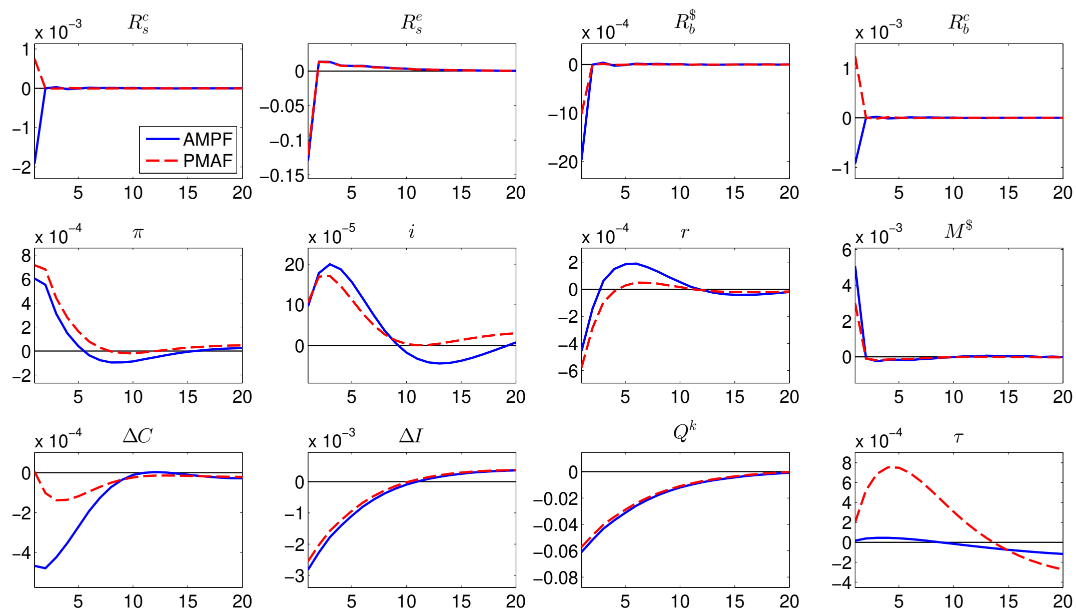


Figure B.5: **Impulse Responses of a Positive MP Shock**

This figure plots the impulse responses of key macro and finance variables in the model after a positive MP shock. These variables are return on stock (claim on consumption) (R_s^c), average return on entrepreneur wealth (R_s^e), return on long-term nominal bond ($R_b^\$$), return on long-term real bond (R_b^c), inflation (π), short-term nominal interest rate (i), short-term real interest rate (r), nominal pricing kernel ($M^\$$), growth rate of consumption (ΔC), growth rate of investment (ΔI), price of capital (Q^k), and lump-sum tax-to-output ratio (τ). The blue and red lines represent impulse responses under the AMPF and PMAF regimes, respectively. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.