Differences of Opinion of Public Information and Speculative Trading in Stocks and Options

H. Henry Cao
Cheung Kong Graduate School of Business (CKGSB)

Hui Ou-Yang
Lehman Brothers and CKGSB

We analyze the effects of differences of opinion on the dynamics of trading volume in stocks and options. We find that disagreements about the mean of the current- and next-period public information lead to trading in stocks in the current period but have no effect on options trading. Without options, we find that disagreements about the precision of all past and current public information affect trading in stocks in the current period. With options, only disagreements about the precisions of the next- and current-period information affect stocks and options trading in the current period. Our results suggest that options trading is concentrated around information events that are likely to cause disagreements among investors, whereas trading in stocks may be diffusive over many periods. (*JEL* G1, G11, G12)

Trading in exchange-listed securities, such as stocks and their options, is extremely active. In 2000, the average daily trading volume in the NYSE reached 1.04 billion shares for 43.9 billion US dollars. Trading volume in options is also huge. "Options trading is now the world's biggest business, with an estimated daily turnover of over 2.5 trillion US dollars and an annual growth rate of around 14%."[1] Given such a high trading volume, the following question arises naturally: What drives investors' trading in the securities market and the associated options market?

This article analyzes the effects of differences of opinion regarding the mean and the precision of public information on trading in stocks and options and the effects of options introduction on the trading volume of the underlying stock. In our model, investors have heterogeneous beliefs even if they observe the same

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public signal. Kandel and Pearson (1995) have provided compelling evidence that investors may interpret public information differently. The Milgrom-Stokey (1982) no-trade theorem does not apply when investors have differences of opinion regarding public information.

More specifically, in our model, investors have constant absolute risk aversion (CARA) utility and believe that the stock payoff distribution is normal. After the first round of trade, there are new public signals about the final stock payoff arriving at the market. Investors have the opportunity to trade again in the market. These new public signals create differences of opinion across investors because investors interpret them differently. Following Kandel and Pearson (1995); Daniel, Hirshleifer, and Subrahmanyam (1998); and Hong and Stein (2003), we assume that investors disagree on the mean and the precision of signals. Disagreements about the mean of a public signal capture the investors' conditional optimism and pessimism about the asset value, while disagreements about the precision of a public signal capture the heterogeneity of the investors' confidence level in the signal. For example, an investor, who has low expectations about the public signal in the next period, tends to be more bearish in the current period. However, this investor will be relatively more optimistic regarding the asset payoff in the next period after the public information is announced and will act more bullish in the next period. On the other hand, differences regarding the precision of the public information in the next period do not induce directional speculation in the current period and thus have no effect on stock trading. An investor who overweighs the precision of the public signal will update more than the average investor, because his posterior expectation of the stock payoff is the precision-weighted average of prior expectations and the signal. For a positive shock in the public information, such an investor will believe that the market has under-reacted to the public information and, as a result, he will purchase more of the stock when the news is good and sell the stock when the news is bad.

The trading volume in the stock can be divided into four components. The first comes from the differences in the interpretation of the mean of the current public signal and the second comes from the differences of opinion regarding the mean of the public signal in the next period. Consequently, differences of opinion regarding the mean of a public signal generate trading in stocks both in the previous period and in the current period. The third component arises from differences of opinion regarding the precision of the current public signal and the fourth component arises from the differences of opinion about the precisions of all past public signals. If investors disagree on the precision of a public signal once, they will continue to trade even if they agree on the signals in all future periods. Disagreements about the precision of future public information do not generate trading in the current stock market. We further establish the following trading volume dynamics and their relationship with the stock price change: (i) Trading volume and absolute price changes are positively serially correlated; (ii) The trading volume and absolute change of the precision-weighted average forecast are positively correlated; (iii) Trading volume is higher when a public signal is very informative; (iv) Trading volume increases with the dispersion of beliefs among investors.

When options are introduced, we show that investors who have higher conditional volatility (lower precision) about the stock payoff take long positions in options to synthesize convex payoffs, whereas investors who have lower conditional volatility take short positions in options to achieve concave payoffs.[2] Options trading volume can be decomposed into two components. The first part arises from the disagreements about the precision of the current public signal, whereas the second part comes from the disagreements about the precision of the next-period public signal. The trading volume in options exhibits very different temporal patterns than that in stocks. Disagreements about the mean and disagreements about the precision of past signals and signals after the next period do not generate trading in the options market. Consequently, options trading should be clustered before and during a big, rare news event that is likely to cause differences of opinion among investors, and trading volume should decline quickly afterward. On the contrary, when the market is incomplete, stock trading should be active at the trading session when important public information is announced and persist thereafter. We further show that in the presence of options, the trading volume of the stock is related not only to its price change but also to lagged price changes.

Our model has the following empirical implications regarding the trading volumes of the stock and options. First, trading volumes in options should be higher around the dates of public events, such as earnings announcements, mergers and acquisitions, and bond rating changes, because public information generates differences of opinion. Second, when there are more differences of opinion about a stock's payoff, trading volumes in both the stock and its options should be higher because investors' demands for options depend on their beliefs about the volatility of the stock payoff.[3] Third, trading volumes are higher for stocks with options because investors use underlying stocks to hedge their positions in the options market. Fourth, trading volume for optioned stocks should be more responsive to concurrent and lagged price changes than nonoptioned stocks, due to additional hedging demands associated with options.

[2] Leland (1980) shows that investors with higher expected returns buy portfolio insurance, but he does not consider the case of different volatilities.

[3] One proxy for differences of opinion is the dispersion of beliefs among financial analysts. It would be informative to determine whether trading volume in options is higher for stocks with more dispersion in financial analysts' forecasts. Another proxy is to use open interest in futures markets as a measure for differences of opinion. Bessembinder, Chan, and Seguin (1996) find that trading volume in stock index futures is correlated with open interest in the index futures market. While our paper has focused on trading volume in stocks and options, the model can also be used to analyze the relationship between stock returns and investor heterogeneity. Anderson, Ghysels, and Juergens (2005) estimate a consumption-based model that incorporates dispersion and biases in analysts' forecasts and demonstrate empirically that heterogeneity explains a portion of expected returns and volatility.

We find that the introduction of certain options can make the market complete.[4] Consequently, we show that the prices of all option claims on the underlying stock satisfy the "risk-neutral" pricing property of the Black-Scholes (1973) model and that the prices of all assets are determined as if there existed a representative agent. The representative agent's belief is equal to the precision-weighted average of all investors' beliefs.

We further extend the model to a multiple-stock setting and show that the trading volume of a stock depends not only on its own stock price change but also on the price changes of related stocks. It is also shown that even if there are no differences of opinion or no signals about a stock's payoff, there may still be trading in that stock due to differences of opinion about the payoffs of other related stocks. These results may shed light on the empirical findings of Kandel and Pearson (1995) and Huberman and Regev (2001). For example, the Kandel-Pearson result that the trading volume of a stock can be positive even if its price does not change arises in our multistock model, as well as in our one-stock model with options. Again, the equilibrium asset prices are equal to the prices that would arise in a representative-agent economy,[5] and the representative agent's belief is equal to the precision-weighted average belief of all investors. We show that the expected asset returns for the representative agent follow the Capital Asset Pricing Model (CAPM) and that this agent holds the market portfolio, which is on his efficient frontier. Suppose that the data represent the true distribution of the stock payoff. Our result suggests that the CAPM will not be the correct description of the data unless the average belief happens to be the correct one.

This article is related to Harris and Raviv (1993) and Kandel and Pearson (1995). Both papers use differences of opinion to generate trades for a stock in the absence of options. They show that with two types of investors, differences of opinion can generate trading patterns consistent with stylized empirical evidence. The main difference between our work and these studies is that we focus on the trading volume of options, as well as on the trading volume of a stock in a multiple-stock environment, which are not considered in those models. Our model also makes different predictions regarding the trading volume of the underlying stock. For example, Harris and Raviv predict that trading can occur only when investors have different interpretations about public signals in every period, whereas we require that investors interpret signals differently in only one period to generate sustained trading. The presence of many types of investors generates additional empirical implications. Kandel and Pearson consider a two-period model and do not analyze the dynamic changes of trading volume of the underlying stock. Other studies that employ differences of opinion to generate trades include Harrison and Kreps (1978); Varian (1989); Detemple

4 Yet trading occurs in every period as investors interpret new signals differently in every period.

5 Our aggregation result with differences of opinion is similar to those of DeMarzo and Skiadas (1998) and Biais, Bossaerts, and Spatt (2003) with asymmetric information.

and Murthy (1994); Kraus and Smith (1996); Morris (1996); Biais and Bossaerts (1998); Odean (1998); Zapatero (1998); Basak (2000); Viswanathan (2001); Brav and Heaton (2002); Duffie, Garleanu, and Pederson (2002); Kyle and Lin (2002); Buraschi and Jiltsov (2005); David (2003); Hong and Stein (2003); Qu, Starks, and Yan (2003); and Scheinkman and Xiong (2003).

Our article is also related to studies that employ noise traders/random endowments and asymmetric information to generate trades. These include Pfleiderer (1984); Kyle (1985); Admati and Pfleiderer (1988); Brown and Jennings (1989); Grundy and McNichols (1989); Kim and Verrecchia (1991); Holden and Subrahmanayam (1992); Spiegel and Subrahmanayam (1992); Back (1993); Foster and Viswanathan (1993); Shalen (1993); Biais and Hillion (1994); Wang (1994); He and Wang (1995); Brennan and Cao (1996); and Easley, O'Hara, and Srinivas (1998). In particular, Brennan and Cao consider an equilibrium model with options. They focus on the impact of introducing options on investors' welfare rather than on trading volume in the options market. They show that with the introduction of an appropriate option security, Pareto efficiency can be achieved in only a single round of trading,[6] and, as a result, investors will no longer trade in either the underlying stock or the option security in future rounds. To generate additional trading with the arrival of new public information, Brennan and Cao and all other works under asymmetric information rely on the introduction of additional noise/liquidity trading. A potential problem with this approach is that the argument to explain the trading volume is circular: it essentially requires new exogenous supply shocks to the stock to generate trading volume. In this sense, trading is imposed onto the economy rather than endogenously generated. For example, to generate trading around the earnings announcement dates, these studies need noise traders for the equilibrium to be partially revealing. However, Kandel and Pearson (1995) find no evidence that noise trading is particularly high around earnings announcements, and Pan and Poteshman (2006) find no asymmetrically informed trading in the index options market. On the other hand, trading in our model is driven endogenously by differential interpretation of public signals without the need to introduce exogenous noise traders.

Models using the asymmetric information paradigm make very different testable predictions regarding the interaction between options and the underlying stock. For example, in the absence of additional noise trading, Brennan and Cao (1996) predict that the introduction of options reduces the trading volume of the underlying stock to zero and that there will be no trading volume in options with the arrival of new public information in future periods.[7] On the contrary, our model predicts that trading volumes in the options market

[6] Back (1993), Biais and Hillion (1994), and Easley, O'Hara, and Srinivas (1998) also analyze the effects of options when information is asymmetric. Detemple and Selden (1991) analyze the effects of options under symmetric information.

[7] These predictions are not supported by empirical evidence. See, for example, Cao (1999) for a discussion of stylized empirical results regarding the impact of options on the trading volume of the underlying stock.

should be clustered before and around the dates of public announcements and are positively related to the degrees of dispersion of beliefs among investors. Moreover, our model predicts that options trading makes the trading volumes of the underlying stocks both higher and more sensitive to stock price changes.

The rest of this article is organized as follows. Section 1 considers the economic model. Section 2 analyzes trading volume in stocks and options. Section 3 develops a multistock equilibrium model, and Section 4 concludes the article. The Appendix contains technical proofs.

1. Economic Model

In this section, we consider trading in a stock due to differences of opinion in the absence of options. It is assumed that the financial market consists of a continuum of investors, each indexed by i where $i \in [0, 1]$. At time 0, each investor is endowed with x^i units of the stock and, to avoid unnecessary notation, we assume that individual endowments of the riskless bond are zero. Without loss of generality, the riskless interest rate is taken as zero. The risky stock pays off at time T an amount u, where u is normally distributed with mean $\bar{u}$ and precision h. The per capita supply of the stock is positive and denoted as x. Investor i has a negative exponential utility function defined over the time T wealth, $U(W^i) = -\exp(-W^i/\tau)$, where τ represents the risk tolerance of investors.

As an introduction, we first consider the basic single-period model.

1.1 A static one-period model

Let P_0 be the price of the stock and D_0^i be the demand of investor i for the stock. Investor i's time 1 wealth is given by $W^i = P_0 x^i + (u - P_0)D_0^i$. It is well known that in this setting (Sharpe, 1964) a linear equilibrium exists in which

$$P_0 = \bar{u} - \frac{x}{\tau h}, \tag{1}$$

$$D_0^i = \tau h[\bar{u} - P_0] = x. \tag{2}$$

Equation (1) expresses the equilibrium price as the expected payoff less a risk premium that depends on the per capita stock supply x. We assume that the values of $\bar{u}$, x, τ, and h are such that P_0 is positive.

The expected utility of investor i conditional on his endowment is given by

$$\mathrm{E}U^i = -\exp\left[\frac{-\bar{u}x^i}{\tau} + \frac{(x^i)^2}{2\tau^2 h} - \frac{(x^i - x)^2}{2\tau^2 h}\right]. \tag{3}$$

Investor i's wealth at time 1, W^i is a linear function of the stock payoff u and can be written as $W^i(u) = P_0 x^i + x(u - P_0)$. The marginal rate of substitution for investor i between wealth contingent on $u = u_l$ and $u = u_k$ is given by

$$M^i_{kl} = \frac{\exp\{-h(u_k - \bar{u})^2/2 - W^i(u_k)/\tau\}}{\exp\{-h(u_l - \bar{u})^2/2 - W^i(u_l)/\tau\}} = \exp\left\{-\frac{1}{2}h(u_k - u_l)(u_k + u_l - 2P_0)\right\}. \tag{4}$$

Because the marginal rate of substitution is the same for all investors, this equilibrium is Pareto efficient.

1.2 A dynamic model with differences of opinion

In this subsection, we extend the single-period model to allow for additional market sessions between time 0 and time T, at which point the stock payoff is realized and consumption occurs. Immediately before each market session, a public signal about the stock payoff arrives. Note that the one-period equilibrium allocation is Pareto efficient. According to Milgrom and Stokey (1982), there should be no more trading after the first round when new information about the final stock payoff becomes publicly available. However, the Milgrom and Stokey theorem holds only when investors have essentially concordant beliefs about the public information. When investors' beliefs are not essentially concordant, trading among investors can occur with the arrival of public information.

1.2.1 Equilibrium price and demand

Consider a setting in which information about the final payoff u is made available gradually by a series of public signals y_t at time $t = 1, \ldots, T-1$. To generate options trading in a tractable manner, we assume that investors disagree about how to interpret the relationship between y_t and u. In particular, investor i believes that

$$y_t = u + \eta_t, \quad u \sim N(\bar{u}, 1/h), \quad \eta_t \sim N\big(m^i_t, 1/n^i_t\big).$$

As a result, investors disagree about both the mean and the precision of y_t. Without loss of generality, we assume that $\int_i n^i_t m^i_t di = 0$.[8] Investor i believes that η_t has a mean m^i_t and a precision n^i_t. Let $n_t \equiv \int_0^1 n^i_t$ denote the average precision of the public signal. We define the concepts of high confidence and low confidence as follows.

Definition 1. *Let $\rho^i_t \equiv n^i_t/n_t$. When $\rho^i_t > 1$ ($\rho^i_t < 1$), we define that investor i has high (low) confidence about the public signal at time t.*

[8] If $\int_i n^i_t m^i_t di \neq 0$, then let $n_t = \int_i n^i_t di$, $\bar{m}_t = \int_i n^i_t m^i_t di/n_t$, We can redefine a new public signal $\hat{y}_t = y_t - \bar{m}_t = u + \hat{\eta}_t$, $\hat{\eta}_t \sim N(\hat{m}^i_t, 1/n^i_t)$, $\hat{m}^i_t \equiv m^i_t - \bar{m}_t$. We then have $\int_i n^i_t \hat{m}^i_t di = \int_i (n^i_t m^i_t - n^i_t \bar{m}_t) di = n_t \bar{m}_t - n_t \bar{m}_t = 0$.

After each signal the market opens for trading, and at time T, the payoff of the stock is realized and consumption occurs. Let P_t denote the price of the stock at time t. Trader i's optimal demand for the stock at time t is denoted by D_t^i. A dynamic equilibrium is described in the following theorem. Its proof and all other proofs are given in the Appendix.

Theorem 1. *In an economy with T trading sessions, there exists a dynamic equilibrium in which prices, and demands for the stock, are given by*

$$P_t = \mu_t - x/(\tau K_t), \tag{5}$$

$$\begin{aligned} D_t^i &= \tau K_t^i \left[\mu_t^i - P_t\right] + \frac{\tau K_t n_{t+1}^i m_{t+1}^i}{n_{t+1}} \\ &= \tau \left[h\bar{u} + \sum_{j=0}^{t} n_j^i y_j - K_t^i P_t + \frac{K_t n_{t+1}^i m_{t+1}^i}{n_{t+1}} \right], \end{aligned} \tag{6}$$

where

$$\mu_t \equiv \frac{h\bar{u} + \sum_{j=0}^{t} n_j y_j}{h + \sum_{j=0}^{t} n_j}, \quad \mu_t^i \equiv \frac{h\bar{u} + \sum_{j=0}^{t} n_j^i \left(y_j + m_j^i\right)}{h + \sum_{j=0}^{t} n_j^i},$$

$$K_t^i \equiv \left(\mathrm{Var}_t^i[u]\right)^{-1} = h + \sum_{j=0}^{t} n_j^i, \text{ and } \quad K_t \equiv \int_0^1 K_t^i di = h + \sum_{j=0}^{t} n_j.$$

Here μ_t and μ_t^i denote the conditional expectations of the stock final payoff u for the average investor and investor i at time t, respectively. K_t^i denotes the conditional precision of u for investor i and K_t denotes the average precision of all investors.

Because investor i is risk averse, his demand for the risky stock increases with his precision about the signal, as well as his conditional mean of the final payoff, as in a typical mean-variance framework. An interesting feature of the equilibrium is that the price depends on the average investor's conditional mean and conditional precision of the stock payoff. The average investor does not buy or sell in equilibrium. We next examine the equilibrium stock demands and prices when options are added to the market.

1.3 A dynamic model with options

The financial market is incomplete with one risky stock and one risk-free asset. Breeden and Litzenberger (1978) have shown that the market can be completed by the introduction of a complete set of options. We complete the market by introducing all call options with positive strike prices and all put options with negative strike prices. All options are in zero net supply. Any derivative asset with a twice differentiable price function $f(u)$ can be synthesized using a collection of options:

$$f(u) = f(0) + f'(0)u + \int_{-\infty}^{0} (Z-u)^{+} f''(Z)dZ + \int_{0}^{\infty} (u-Z)^{+} f''(Z)dZ, \tag{7}$$

where Z denotes the strike prices. Let P_{CZt} denote the price of a call option with strike price Z in trading session t. Let D^{i}_{CZt} denote investor i's demand density of options at strike price Z, that is, the holdings of call options with strike price Z to $Z + dZ$ is given by $D^{i}_{CZt}dZ$. Define the price and demand for the put options similarly as P_{PZt} and D^{i}_{PZt}.

We consider a set of bounded, continuous strategies for investor i, $\{D^{i}_{t}, D^{i}_{CZt}, D^{i}_{PZt}\}$. We show in the proof of Theorem 2 that the profits derived from the investors' equilibrium holdings in the stock and options are indeed bounded under the proposed equilibrium prices. Theoretically, we should also consider discrete holdings in options. For example, an investor may hold 100 contracts at the strike price of 20. It can be shown that the strategies with discrete holdings are not optimal in equilibrium. We can then ignore such strategies without loss of generality. Moreover, due to the put-call parity, we consider nonzero demands for calls with positive strike prices and nonzero demands for puts with negative strike prices.

The next theorem establishes the existence of an equilibrium with options and describes the demands and prices for both stock and options.

Theorem 2. *There exists a sequential dynamic equilibrium in which*

$$P_t = \mu_t - x/(\tau K_t), \tag{8}$$

$$D^{i}_{t} = \tau K^{i}_{t}\left[\mu^{i}_{t} - P_t\right] + \frac{\tau K_t n^{i}_{t+1} m^{i}_{t+1}}{n_{t+1}} - \tau\left[\left(1-\rho^{i}_{t+1}\right)\frac{K^{2}_{t}}{n_{t+1}} + K_t - K^{i}_{t}\right] P_t, \tag{9}$$

$$P_{CZt} = (P_t - Z)N((P_t - Z)\sqrt{K_t}) + \frac{1}{\sqrt{K_t}} n((P_t - Z)\sqrt{K_t}), \quad Z \geq 0, \tag{10}$$

$$P_{PZt} = (Z - P_t)N((Z - P_t)\sqrt{K_t}) + \frac{1}{\sqrt{K_t}} n((Z - P_t)\sqrt{K_t}), \quad Z < 0, \tag{11}$$

$$D^{i}_{CZt} = \tau\left[\left(1-\rho^{i}_{t+1}\right)\frac{K^{2}_{t}}{n_{t+1}} + K_t - K^{i}_{t}\right], \quad Z \geq 0, \tag{12}$$

$$D^{i}_{PZt} = \tau\left[\left(1-\rho^{i}_{t+1}\right)\frac{K^{2}_{t}}{n_{t+1}} + K_t - K^{i}_{t}\right], \quad Z < 0, \tag{13}$$

where

$$\mu_t \equiv \frac{h\bar{u} + \sum_{j=0}^{t} n_j y_j}{h + \sum_{j=0}^{t} n_j}, \quad \mu_t^i \equiv \frac{h\bar{u} + \sum_{j=0}^{t} n_j^i \left(y_j + m_j^i\right)}{h + \sum_{j=0}^{t} n_j^i},$$

$$K_t^i \equiv \left(\mathrm{Var}_t^i[u]\right)^{-1} = h + \sum_{j=0}^{t} n_j^i, \text{ and } \quad K_t \equiv \int_0^1 K_t^i di = h + \sum_{j=0}^{t} n_j.$$

Theorem 2 shows that options are not redundant securities. In this equilibrium, investors with high confidence take short positions in the options while investors with low confidence take long positions in the options.[9] Intuitively, investors with high confidence perceive a lower volatility for the stock, so they believe that options are overvalued. As a result, they take short positions on options. Similarly, investors with low confidence perceive options to be undervalued, so they take long positions. Although investors achieve the Pareto optimal allocation, those whose precision about the public signal is different from that of the average investor will trade in options in every period. In the presence of options, investors will trade in the underlying stock to hedge option positions even if the price of the underlying stock does not change. Indeed, it will be shown that the trading volume of the underlying stock is positive even if the stock price remains unchanged. Note that the average investor serves as the representative agent who prices both the stock and the options based on the average belief. This investor does not buy and sell the stock and holds no options in equilibrium.

With normal stock payoff distribution and CARA utility, the Pareto efficient allocation is a quadratic function of the final payoff of the stock u. In equilibrium, investors use options to synthesize the appropriate payoffs that are quadratic functions of the stock payoff. Consequently, it is not necessary to introduce a continuum of call and put options to complete the market. We next show that a derivative asset with a payoff of $Q(u) = u^2$ can complete the market, yielding the following result.[10]

Theorem 3. *Let P_{Qt} denote the price of the quadratic derivative asset and D_{Qt}^i denote the demand for the quadratic derivative asset by investor i in trading session t. Then there exists a dynamic equilibrium in which*

$$P_t = \mu_t - x/(\tau K_t), \tag{14}$$

$$D_t^i = \tau K_t^i\left[\mu_t^i - P_t\right] + \frac{\tau K_t n_{t+1}^i m_{t+1}^i}{n_{t+1}} - 2D_{Qt}^i P_t, \tag{15}$$

[9] Assuming that, for each investor, $\rho_t^i - 1$ has the same sign for all t.

[10] Under asymmetric information, Brennan and Cao (1996) also show that a quadratic option can complete the market.

$$P_{Qt} = K_t^{-1} + P_t^2, \tag{16}$$

$$D_{Qt}^i = \frac{\tau}{2}\left[\left(1 - \rho_{t+1}^i\right)\frac{K_t^2}{n_{t+1}} + K_t - K_t^i\right], \tag{17}$$

where K_t^i and K_t are the same as in Theorem 2.

Like Theorem 2, this theorem also shows that investors with high confidence take short positions in options and investors with low confidence take long positions in options. In addition, there will be trading volume in the underlying stock even if its price does not change.

Note that in one-period models, when markets are complete, there exists a representative agent who prices all assets according to his belief and marginal utility.[11] Interestingly, Theorems 2 and 3 show that there also exists a representative agent in our dynamic trading model with differential interpretation of public signals.[12] To make a connection with the one-period models, we next demonstrate the existence of a representative agent under differences of opinion by solving a social planning problem.

Let $W(T)$ denote the per capital aggregate wealth and $W^i(T)$ the wealth of investor i at the final date T. As in Rubinstein (1974) and Brennan and Kraus (1978), the social planning problem is to maximize the following objective function:

$$\max_{W^i(T)} \int_i \pi^i[-\exp(W^i(T)/\tau)]di, \tag{18}$$

subject to the wealth constraint

$$\int_i W^i(T)di = W(T), \tag{19}$$

where π^i is a positive coefficient. The first-order condition implies that

$$\pi^i \exp(-W^i(T)/\tau)/\tau = \lambda. \tag{20}$$

The Pareto optimal allocation is then given by

$$W^i(T) = -\tau[\ln(\tau\lambda/\pi^i)]. \tag{21}$$

Consequently, we have

$$W(T) = -\tau\left[\int_i \ln(\tau\lambda/\pi^i)di\right]. \tag{22}$$

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[11] See Rubinstein (1974), Brennan and Kraus (1978), and Brennan (1979).

[12] See also Jouini and Napp (2006) for the existence of a representative agent under differences of opinion among investors in a model with consumption in each period.

The social planner's utility function is thus given by

$$\int_i \pi^i[-\exp(W^i(T)/\tau)]di = -\tau\lambda = -\exp\left[\ln\left(\int_i \pi^i di\right)\right]\exp(-W(T)/\tau). \tag{23}$$

It can now be seen that the utility function of the representative investor is also exponential with a risk tolerance coefficient τ. To derive the representative agent's probability belief, we note that Pareto optimality implies that

$$\frac{\pi \exp(-W(T)/\tau)}{\pi_i \exp(-W^i(T)/\tau)} = \theta_i, \tag{24}$$

where θ^i is a constant. This leads to

$$W(T) = W^i(T) + \tau[\ln(\pi) - \ln(\pi^i) - \tau\ln(\theta_i)]. \tag{25}$$

Integrating both sides of Equation (25) with respect to i yields

$$\begin{aligned}
\ln(\pi) = \int_i \ln(\pi^i)di + \int_i \ln(\theta_i)di &= -T\ln(2\pi) + \frac{1}{2}\ln(h) - \frac{1}{2}hu^2 \\
&\quad + \frac{1}{2}\sum_{t=1}^{T-1}\int_i \ln\left(n_t^i\right)di - \frac{1}{2}\int_i n_t^i\left(\eta - m_t^i\right)^2 di + \int_i \ln(\theta_i)di \\
&= -T\ln(2\pi) + \frac{1}{2}\ln(h) - \frac{1}{2}hu^2 + \frac{1}{2}\sum_{t=1}^{T-1}\int_i \ln\left(n_t^i\right)di - \frac{1}{2}\ln(n_t) \\
&\quad - \int_i \frac{1}{2}n_t\eta^2 di - \frac{1}{2}\int_i n_t^i\left(m_t^i\right)^2 di + \int_i \ln(\theta_i)di \\
&= T\ln(2\pi) + \frac{1}{2}\ln(h) - \frac{1}{2}hu^2 + \frac{1}{2}\sum_{t=1}^{T-1}\ln(n_t) - \frac{1}{2}n_t\eta^2,
\end{aligned} \tag{26}$$

where

$$\int_i \ln(\theta_i)di = -\frac{1}{2}\sum_{t=1}^{T-1}\int_i \ln\left(n_t^i\right)di + \frac{1}{2}\ln(n_t) + \int_i \frac{1}{2}n_t^i\left(m_t^i\right)^2,$$

because π^i integrates to one. As a result, the representative investor has a belief that $\eta_t \sim N(0, n_t^{-1})$, and his conditional belief at time t about u is given by $\mathcal{N}(\mu_t, K_t^{-1})$, where $\mu_t = K_t^{-1}(h\bar{u} + \sum_{j=0}^{t} n_j y_j)$ and $K_t = h + \sum_{j=0}^{t} n_j$.[13] The next proposition summarizes the result.

[13] Given the construction of the representative agent, we can construct investor i's optimal demand using expression (25).

Proposition 1. *When markets are completed by adding either a continuum of call and put options or a single quadratic option, there exists a representative investor with risk tolerance* τ *and belief* $\mathcal{N}(\mu_t, K_t^{-1})$*, where* $\mu_t = K_t^{-1}(h\bar{u} + \sum_{j=0}^{t} n_j y_j)$.

It is indeed quite striking that markets are effectively complete with the introduction of a single derivative, and as a result, the prices of all contingent claims behave as if there existed a representative investor. Trading in options and the underlying stock, however, is active among investors at all trading sessions due to differences of opinion among investors in every period.

Interestingly, even in the absence of options, the representative investor pricing still works, that is, equilibrium prices depend only on K_t and μ_t. To understand this result, we start with a market being completed by a quadratic option. Consider the last trading period $t = T - 1$. In this complete market, Theorem 3 states that there exists a representative investor and his probability distribution of u is $\mathcal{N}(\mu_{T-1}, K_{T-1}{}^{-1})$.

The first-order condition for investor i with respect to his stock holdings is given by

$$E^i_{T-1}\left[(u - P_{T-1})\exp\left(-\frac{D^i_{T-1}(u - P_{T-1}) + D^i_{Q(T-1)}(u^2 - P_{Q(T-1)})}{\tau}\right)\right] = 0. \tag{27}$$

Note that we can rewrite the payoff of the quadratic asset as $u^2 = (u - P_{t-1})^2 + 2P_{T-1}(u - P_{T-1}) + P^2_{T-1}$. The first-order condition then reduces to

$$E^i_{T-1}\left[(u - P_{T-1})\exp\left(-\frac{\hat{D}^i_{T-1}(u - P_{T-1}) + D_{Q(T-1)}(u - P_{T-1})^2}{\tau}\right)\right] = 0, \tag{28}$$

where

$$\hat{D}^i_{T-1} = D^i_{T-1} + 2D^i_{Q(T-1)}P_{T-1},$$

and

$$\int_i \hat{D}^i_{t-1} = \int_i D^i_{T-1}di + 2P_{T-1}\int_i D^i_{Q(T-1)}di = x + 0 = x. \tag{29}$$

It means that holding D^i_{T-1} shares of the stock and $D^i_{Q(T-1)}$ shares of the quadratic asset u^2 is equivalent to holding $\hat{D}^i_{T-1}$ shares of the stock and $D^i_{Q(T-1)}$ shares of a new quadratic asset $(u - P_{T-1})^2$, plus a bond with a face value of $D^i_{Q(T-1)}P^2_{T-1}$.

Substituting the probability distribution of investor i into Equation (28), we have

$$\int_{-\infty}^{\infty}(u-P_{T-1})\frac{\exp\left(-\frac{\hat{D}^i_{T-1}-\tau K^i_{T-1}[\mu^i_{T-1}-P_{T-1}](u-P_{T-1})}{\tau}\right)}{\exp\left(\frac{D_{Q(T-1)}(u-P_{T-1})^2}{\tau}+\frac{(u-P_{T-1})^2K^i_{T-1}}{2}\right)}du=0. \tag{30}$$

In Equation (30), the integrand is $(u-P_{T-1})$ times a fraction. The denominator of the fraction is an even function of $(u-P_{T-1})$. At the optimum, the stock holding is given by $\hat{D}^i_{T-1}-\tau K^i_{T-1}[\mu^i_{T-1}-P_{T-1}]=0$ so that the numerator of the fraction is 1. As a result, the integrand is an odd function and the integral is thus zero. Note that $(u-P_{T-1})^2$ is also an even function of $(u-P_{T-1})$. Changing the shares of the quadratic asset $(u-P_{T-1})^2$ does not change the fact that the denominator of the fraction remains an even function of $(u-P_{T-1})$. In particular, when investors cannot trade in the quadratic asset, i.e., $D^i_{Q(T-1)}=0$, the Euler condition still holds at $\hat{D}^i_{T-1}$. As a result, even in an incomplete market without derivative assets, $\hat{D}^i_{T-1}$ is still the optimal demand for investor i at the representative-investor price P_{T-1}. In addition, $\hat{D}^i_{T-1}$ clears the market, which means that $\hat{D}^i_{T-1}$ and P_{T-1} constitute an equilibrium in the incomplete market without options. In other words, equilibrium prices depend only on K_t and μ_t.

2. Trading Volume in Stocks and Options

In this section, we use the results obtained in Theorems 1–3 to analyze the investors' trading strategies and trading volume in stocks and options. In Sections 3.1 through 3.4, we discuss trading strategies and price dynamics in the economy without options.

2.1 Trading strategies and price dynamics

Let $\Delta P_t \equiv P_t - P_{t-1}$ denote the price change and $\Delta D^i_t \equiv D^i_t - D^i_{t-1}$ denote the trading size of investor i in trading session t. In equilibrium, Equation (6) yields:

$$\begin{aligned}\Delta D^i_t &= \tau\left[\frac{K_t n^i_{t+1} m^i_{t+1}}{n_{t+1}} - \frac{K_{t-1} n^i_t m^i_t}{n_t} - n^i_t m^i_t\right]\\ &\quad + \tau\left(\frac{n^i_t}{K^i_{t-1}} - \frac{n_t}{K_{t-1}}\right)\frac{K_{t-1}K^i_{t-1}}{n_t}\Delta P_t\\ &= \tau K_t\left[\frac{K_t n^i_{t+1} m^i_{t+1}}{n_{t+1}} - \frac{K_t n^i_t m^i_t}{n_t}\right] + \frac{\tau}{n_t}[K_{t-1}(n^i_t - n_t)\\ &\quad - n_t(K^i_{t-1} - K_{t-1})]\Delta P_t.\end{aligned} \tag{31}$$

Equation (31) illustrates that the trading size of investor i can be divided into four components. The first two components come from disagreements about the mean of the next-period public signal and the current public signal. The remaining two components come from the disagreements about the precisions of the current signal and past cumulative signals. Trading can occur due to disagreements about past public information, current public information, and the next-period public information. However, disagreements about the precisions of future public information and disagreements about the means of past public information and future public information beyond the next period do not generate trading in stocks. We thus have the following result regarding how disagreements about the mean and precision of public information affect stock trading differently.

Proposition 2. *Disagreements about the mean of a public signal result in stock trading in only the current period and the previous period. Disagreements about the precision of a public signal result in stock trading in the current period and all future periods.*

This proposition indicates that to generate trading, investors need to disagree on the precision of public signals only once. Thereafter, they will trade even if they agree on all signals in future periods.

2.2 Trading volume and disagreements about mean

We first discuss trading volume due to disagreements about the mean of y_t alone. We assume in this subsection that $n_t^i = n_t$ for all t and i. Then the trading of investor i at period t reduces to

$$\Delta D_t^i = \tau\left[K_t m_{t+1}^i - K_{t-1} m_t^i - n_t m_t^i\right] = \tau K_t\left[m_{t+1}^i - m_t^i\right].$$

Note that at time $t-1$, when investor i has positive expectation (negative) about the mean of η_t, $m_t^i > 0 (< 0)$, he will be more bullish (bearish) about the next-period price change. As a result, the investor holds more stock relative to the average investor at time $t-1$, given by $\tau K_{t-1} m_t^i$. However, at time t, given the observation of y_t, investor i expects to be relatively pessimistic about future realizations of the asset payoff. Investor i will unwind part of his holdings at time t, in the amount of $\tau K_{t-1} m_t^i$. Moreover, as the investor is relatively pessimistic at time t, he further sells in the amount of $\tau n_t m_t^i$ due to the difference of conditional expectation of asset payoff. Thus trading at time t induced by m_t^i is to sell in the amount of $\tau K_t m_t^i$. In addition, if investor i also has positive expectation about η_{t+1}, $m_{t+1}^i > 0$, he will buy relatively more stock compared to the average investor in the amount of $\tau[K_t m_{t+1}^i]$. The combined effects of m_t^i and m_{t+1}^i yield the trading amount of $\tau K_t[m_{t+1}^i - m_t^i]$. The differences about the mean of public information affect trading only in two periods: the current period and the previous adjacent period.

2.3 Trading volume and disagreements about precision

To analyze trading due to differences in precision, we assume that $m_t^i = 0$, and ρ_t^i is the same for all t, that is, $\rho_t^i = \rho^i$ for this subsection and the next subsection. Using Equations (5) and (6), we have

$$D_{t+1}^i - D_t^i = \tau(\rho^i - 1)h(P_{t+1} - P_t). \tag{32}$$

Investors who have high confidence ($\rho^i > 1$) about a public signal put more weight on the signal and thus trade in the direction of the signal. If the public signal is very positive, the price will go up. But investors with high confidence still believe that the price has not fully incorporated the positive signal, due to the presence of investors with low confidence. Hence, investors with high confidence believe that the stock price will go up even further and demand more shares of the stock. On the other hand, investors who have low confidence ($\rho^i < 1$) about a public signal put less weight on it. When the stock price goes up, they believe that the price is overreacting to the public signal due to the presence of investors with high confidence. Hence, investors with low confidence sell the stock. The following proposition characterizes the relations between trades and price changes from different investors' perspectives.

Proposition 3.

(i) *The trades of investors with high confidence* ($\rho^i > 1$) *at time t are positively correlated with the price change at time t;*
(ii) *The trades of investors with low confidence* ($\rho^i < 1$) *at time t are negatively correlated with the price change at time t;*
(iii) *Investors with the average confidence level* ($\rho^i = 1$) *do not trade.*

The next proposition summarizes the price dynamics based on different investors' perspectives.

Proposition 4.

(i) *For an investor with high confidence* ($\rho^i > 1$)*, the price change at time $t+1$ is positively correlated with the price at time t and is positively correlated with the price change at time t, that is,*

$$\mathrm{Cov}^i(P_{t+1} - P_t, P_t) > 0, \quad \mathrm{Cov}^i(P_{t+1} - P_t, P_t - P_{t-1}) > 0. \tag{33}$$

(ii) *For an investor with low confidence* ($\rho^i < 1$)*, the price change at time $t+1$ is negatively correlated with the price at time t and is negatively correlated with the price change at time t, that is,*

$$\mathrm{Cov}^i(P_{t+1} - P_t, P_t) < 0, \quad \mathrm{Cov}^i(P_{t+1} - P_t, P_t - P_{t-1}) < 0. \tag{34}$$

(iii) *For the average investor* ($\rho^i = 1$)*, the price change at time $t+1$ is not correlated with the price at time t and is not correlated with the price change at time t, that is,*

$$\mathrm{Cov}^i(P_{t+1} - P_t, P_t) = 0, \quad \mathrm{Cov}^i(P_{t+1} - P_t, P_t - P_{t-1}) = 0. \tag{35}$$

We next apply Theorem 1 to study the equilibrium trading volume of the stock.

2.4 Trading volume and price changes

Many empirical studies have examined the contemporaneous behavior of volume and absolute price changes and found a positive correlation between the two (e.g., Karpoff, 1987). Since the dynamics of price volatility and trading volume can only be studied in a multiple trading economy, this subsection presents additional results on the autocorrelation properties of trading volume, as well as the relation between trading volume and the number of trading sessions between time 0 and time T.

Let $\Delta P_t = P_t - P_{t-1}$ denote the price change at time t, where $t = 1, \ldots, T-1$. Let trading volume at time t, V_t, be defined as one-half the sum of all purchases and sales, that is,

$$\begin{aligned} V_t = \frac{1}{2}\int_0^1 \left|D_t^i - D_{t-1}^i\right| di &= \frac{1}{2}\int_0^1 \tau K_t \left|\frac{K_t^i}{K_t} - \frac{n_t^i}{n_t}\right| |\Delta P_t|\, di \\ &= \frac{1}{2}\int_0^1 \tau h|\rho^i - 1||\Delta P_t| di. \end{aligned} \tag{36}$$

Note that there is no hedging demand for the stock in our model and that all trades are due to the differences of opinion about public signals. As a result, we obtain a simple result that the trading volume in each period is proportional to the product of the absolute price change and the dispersion of investors' precisions in public information. In Proposition 4, we have shown that price changes are serially correlated if the true precision is different from the average precision. Since the correlation coefficient between the absolute values of two normally distributed variables x and y with correlation $r(x, y)$ and means of zero is given by

$$\mathrm{Corr}(|x|, |y|) = \frac{2}{\pi - 2}\int_0^{r(x,y)} \arcsin t\, dt > 0,$$

the following lemma is immediate.[14]

[14] Harris and Raviv (1993) derive results (i), (ii), and (iii) based on differences of opinion with risk-neutral investors. Brennan and Cao (1996) also obtain predictions (i) and (v), using a partially revealing rational expectations model with differentially informed investors.

Lemma 1.

(i) *Trading volume and absolute price changes are positively correlated.*
(ii) *Trading volume and absolute change of the precision-weighted average forecast are positively correlated.*
(iii) *Trading volume is higher when the public signal is very informative (a high value of n_t).*
(iv) *Trading volume increases with the dispersion of beliefs among investors.*
(v) *For any investor whose precision is different from the average precision, the absolute price change and trading volume are positively serially correlated.*

The first three implications are consistent with the empirical evidence summarized in Karpoff (1987). Implication (iv) implies that trading volume may be related to the dispersion among financial analysts' forecasts. Empirically, Frankel and Froot (1990) examine foreign exchange data and find a positive relation between volume and dispersion, and Ajinkya, Atiase, and Gift (1991) also obtain a positive relation between stock volume of trading and the dispersion in financial analysts' earnings forecasts, both of which support our prediction (iv). Implication (v) may be tested using survey data of investors' beliefs about the stock price changes.

2.5 Trading volume in options

This subsection considers trading volume in options. Let ΔD^i_{CZt} denote investor i's amount of trading for a call option with strike price Z, which, according to Theorem 2, is given by

$$\Delta D^i_{CZt} = \tau\left[\left(n_t - n^i_t\right)\left(1 - \frac{K^2_{t-1}}{n^2_t}\right) + \left(n_{t+1} - n^i_{t+1}\right)\frac{K^2_t}{n^2_{t+1}}\right]. \tag{37}$$

Trading in options can be divided into two parts. The first part comes from the disagreements about current information while the second part comes from the disagreements about the next-period information. Suppose that investors disagree about the public information only at time t. Then investor i will trade in the options market with a holding of $\tau(n_t - n^i_t)K^2_{t-1}/n^2_t$ at trading session $t-1$. At trading session t, investor i will partially unwind the holdings in session $t-1$ to achieve a position of $\tau(n_t - n^i_t)$, and there will be no more trading in future periods. Past disagreements about precisions and disagreements about means do not generate trading in the options market, contrary to the results obtained in the stock market. The following proposition summarizes this result.

Proposition 5. *Disagreements about the precision of the current- and next-period information generate trading in the options market. Disagreements about precisions of past information and information after the next period do* *not generate trading in the options market. Moreover, disagreements about mean do not generate trading in the options market.*

It is interesting to contrast the results on trading strategies in stocks and options. We have shown that past disagreements and current agreements in precision generate trading in the stock market, as do the disagreements in the mean in the current period and next period. In contrast, trading in options depends only on current disagreements and next-period disagreements about precisions. In a big rare event, it is more likely to have differences of opinion. Our results suggest that we should observe more clustered trading in options market just before and during the event.

Let the trading volume in options, V_{CZt}, be defined as half of the sum of the absolute trades. Let O_{CZt} denote the open interest in options. We now analyze how dispersion of beliefs in precisions and average precision n_t affect options trading and open interest. To simplify the analysis, we assume that $m_t^i = 0$ and $\rho_t^i = \rho^i$ for all t in the rest of this section. We have

$$\begin{aligned} V_{CZt} &= \tau \left| \frac{K_t K_{t+1}}{n_{t+1}} - \frac{K_{t-1} K_t}{n_t} \right| \int_0^1 |1 - \rho^i| di \\ &= \tau \left| \frac{1}{\mathrm{Var}[\Delta P_t]} - \frac{1}{\mathrm{Var}[\Delta P_{t+1}]} \right| \int_0^1 |1 - \rho^i| di, \end{aligned} \tag{38}$$

$$O_{CZT} = \int_i \left| D^i_{CZt} \right| di = \tau \int_0^1 |1 - \rho^i| di \left[\frac{K_t^2}{n_{t+1}} + K_t - h \right]. \tag{39}$$

Because the average of ρ^i is one, $\int_0^1 |1 - \rho^i| di$ is a measure of investors' dispersion of beliefs. Equations (38) and (39) then lead to the following result regarding trading volume in options.[15]

Proposition 6. *Investors' open interest in options increases with the average perceived precision, n_t, of a public signal and investors' trading volume and open interest in options are higher when they have higher dispersion of beliefs.*

With a higher dispersion of beliefs, investors disagree on the volatility of the stock payoff more; equivalently, they disagree on the value of options more, and the trading volume for options naturally increases. When public information is very informative at time t, or n_t is so large that $\mathrm{Var}[\Delta P_t] - \mathrm{Var}[\Delta P_{t+1}] > 0$, the volume of trades in options increases with the average precision of public information in that session. Thus, trading in options will also be more active before and during informative public announcement dates. These predictions may be tested using event studies to analyze the behavior of open interest and trading volume in options around informative public events.

[15] The results for put options can be obtained similarly.

2.6 Stock trading volume in the presence of options

The literature has provided some empirical evidence that the introduction of options tends to increase the trading volume of the underlying stock. See, for example, Skinner (1990) and Kumar, Sarin, and Shastri (1998).[16]

In this subsection, we examine the effects of options on the trading volume of the underlying stock:

$$
\begin{aligned}
\Delta D_t^i &= \tau\left[\frac{K_t n_{t+1}^i m_{t+1}^i}{n_{t+1}} - \frac{K_t n_t^i m_t^i}{n_t}\right] + \tau\left[\frac{n_t^i}{K_{t-1}^i} - \frac{n_t}{K_{t-1}}\right]\frac{K_{t-1}K_{t-1}^i}{n_t}\Delta P_t \\
&\quad -\tau\left[\left(1-\rho_{t+1}^i\right)\frac{K_t^2}{n_{t+1}}P_t + \left(1-\rho_t^i\right)\frac{K_{t-1}^2}{n_t}P_{t-1}\right] \\
&= \tau\left[\frac{K_t n_{t+1}^i m_{t+1}^i}{n_{t+1}} - \frac{K_t n_t^i m_t^i}{n_t}\right] \\
&\quad - \tau\left[\left(1-\rho_{t+1}^i\right)\frac{K_t^2}{n_{t+1}}P_t + \left(1-\rho_t^i\right)K_t\left(P_t - \frac{K_{t-1}}{n_t}P_{t-1}\right)\right]. \qquad (40)
\end{aligned}
$$

Note that in the presence of options, stock trading no longer responds to past disagreements about precisions. Also note that the market is completed by the introduction of options. If there will be no more disagreements in the future, then investors would have traded to a Pareto optimal allocation after trading in the current period and there will be no more trading. Investors will trade only if there are disagreements in the future. We thus have the following results.

Proposition 7. *In the presence of options, disagreements about the mean and the precision of a public signal result in stock trading in only the current period and the previous period.*

To simplify our analysis further, we consider a special case in which $m_t^i = 0$, and ρ_t^i is the same for all t, that is, $\rho_t^i = \rho^i$.

Following the results of Theorem 2, we have

$$
\begin{aligned}
\Delta D_t^i &= \tau\left[\left(n_t^i y_t - K_t P_t + K_{t-1}\right)P_{t-1} + (\rho^i - 1)\left(\frac{K_t^2}{n_{t+1}}P_t - \frac{K_{t-1}^2}{n_t}P_{t-1}\right)\right] \\
&= \tau(\rho^i - 1)\left[\frac{K_t K_{t+1}}{n_{t+1}}P_t - \frac{K_{t-1}K_t}{n_t}P_{t-1}\right]. \qquad (41)
\end{aligned}
$$

[16] Mayhew and Mihov (2005), however, argue that the previous literature does not take into account the endogeneity issues in options listing appropriately. Using a matched control sample to avoid the endogeneity issue, they reconfirmed earlier results that options trading increases the trading volume of the underlying stock.

The trading volume of the underlying stock is then given by

$$V_t \equiv \frac{1}{2}\int_0^1 \left|\Delta D_t^i\right| di = \tau \left| \frac{K_t K_{t+1}}{n_{t+1}} P_t - \frac{K_{t-1} K_t}{n_t} P_{t-1} \right| \int |(\rho^i - 1)| di. \quad (42)$$

It is clear that the trading volume can be positive even if the stock price remains the same, that is, $\Delta P_t = 0$. The reason is that in the presence of options, investors trade in the stock to hedge options. Even if the stock price remains unchanged, there may still be a need to hedge options because option prices may change due to differences of opinion about public signals. Our result indicates that trading volume is related not only to the current price change but also to the past price changes. In addition, the coefficient on lagged price change should be larger for stocks with options than for stocks without options.

To offer sharper empirical predictions, we next assume that the volatility of the stock-price change is stationary across trading sessions for the average investor, that is, $\mathrm{Var}[\Delta P_t] = \mathrm{Var}[\Delta P_{t+1}]$ for all t. This serves as a sufficient condition for the result regarding the expected trading volume of the underlying stock given in the following proposition.

Proposition 8. *When* $\mathrm{Var}[\Delta P_t] = \mathrm{Var}[\Delta P_{t+1}]$, *the introduction of options increases the expected trading volume of the underlying stock. Moreover, the expected trading volume is more sensitive both to the price changes of the stock and to the dispersion of forecasts among investors.*

The results of this proposition are due to investors' hedging demands for options. For example, with options, a change in the stock price affects the properties of the options associated with the stock, which requires more hedging for options. As a result, the expected trading volume of the stock is more sensitive to stock price changes in the presence of options. To test the implications of Proposition 8, one may take two approaches. The first approach is to conduct an event study to analyze the amount of trading volume before and after the introduction of options. The second approach is to perform a cross-sectional study to compare trading volume and its sensitivity to price changes and the dispersion of forecasts among investors between stocks with options and those without options.

3. Multiple Stocks

We have shown that the trading volume is related to the price change in a single-stock model. Empirical studies have shown that the trading volume of a stock is related to the price changes of not only that stock but also those with related payoffs (Huberman and Regev, 2001). We next consider a multistock dynamic model and examine the relationship between trading volume and price changes. For tractability, we omit options in this model.

The payoffs of the M stocks are realized at time 1, and are represented by an $M \times 1$ normally distributed random vector $\tilde{U}$ with mean $\bar{U}$ and precision matrix H. Each investor $i, i \in [0, 1]$, is endowed at time 0 with risky assets denoted by the vector X^i; investors are characterized by negative exponential utility functions as defined earlier. The vector of the aggregate per capita supply of the risky assets is X.

Immediately prior to trading session t, a vector of public signals is released. The public signals are represented by the $M \times 1$ vector $\tilde{Y}_t$, where

$$\tilde{Y}_t = \tilde{U} + \tilde{\eta}_t.$$

Investors have differential interpretation about the public signals. For each investor, $\tilde{\eta}_t$ is normally distributed with mean M_t^i and precision matrix N_t^i.[17] The average precision matrix for the public signal is $N_t \equiv \int_i N_t^i di$.

Let P_t denote the vector of equilibrium risky asset prices, D_t^i the vector of investor's demands for the risky assets, and F_t the public information set including the prices P_t, all at trading session t. The following theorem describes the asset prices and the investors' asset demands at each market session in a sequential equilibrium.

Theorem 4. *There exists a sequential equilibrium. (i) The vectors of risky asset prices, P_t, and investor i's asset demands, D_t^i, are given by*

$$P_t = K_t^{-1}[K_t \mu_t - \tau X], \qquad D_t^i = \tau K_t^i[\mu_{ti} - P_t] + \tau K_t N_{t+1}^{-1} N_{t+1}^i M_{t+1}^i, \tag{43}$$

where

$$\mu_t^i \equiv \mathrm{E}_t^i[U] = K_{ti}^{-1}\left(H\bar{U} + \sum_{j=0}^{t} N_j \tilde{Y}\right)_j, \quad \mu_t \equiv K_t^{-1}\int_0^1 K_t^i \mu_t^i di,$$

$$K_t^i \equiv [\mathrm{Var}_t^i[\tilde{U}]]^{-1} = H + \sum_{j=1}^{t} N_j^i, \quad K_t \equiv \int_0^1 K_{ti} di = H + \sum_{j=0}^{t} N_j.$$

(ii) The optimal trading strategy of investor i, ΔD_t^i, and the trading volume of the stocks, V_t, are given by

$$\begin{aligned}\Delta D_t^i \equiv D_t^i - D_{t-1}^i &= \tau\big(K_t - K_t^i\big)[P_t - P_{t-1}] \\ &\quad + \tau\big[K_t N_{t+1}^{-1} N_{t+1}^i M_{t+1}^i - K_{t-1} N_t^{-1} N_t^i M_t^i - N_t^i M_t^i\big] \\ &= \tau(K_t - K_t^i)[P_t - P_{t-1}] + \tau K_t[N_{t+1}^{-1} N_{t+1}^i M_{t+1}^i - N_t^{-1} N_t^i M_t^i],\end{aligned} \tag{44}$$

[17] We assume that $N_{0i} = O$ and $N_{Ti}^{-1} = O$, where O denotes the zero matrix. This assumption is consistent with the earlier assumption that there is no public information at time 0 and that all risky asset returns are realized at session T.

$$V_t = \int_i \left|\Delta D_t^i\right| di = \int_i \tau\left(K_t - K_t^i\right)[P_t - P_{t-1}] \\ + K_t\left[N_{t+1}^{-1} N_{t+1}^i M_{t+1}^i - N_t^{-1} N_t^i M_t^i\right]|di. \tag{45}$$

Again, the equilibrium stock prices are determined by the average investor whose belief is equal to the average of the beliefs of all investors. In other words, this average investor serves as the representative agent in our economy with heterogeneous beliefs.

Note that in equilibrium, the stock price change at time $t + 1$ is given by

$$\Delta P_{t+1} \equiv P_{t+1} - P_t = K_{t+1}^{-1} N_{t+1}(Y_{t+1} - P_t). \tag{46}$$

Equation (45) shows that the trading volume of a stock depends not only on the price change of that stock but also on the price changes of other related stocks. For example, if stocks 1 and 2 are correlated, then the corresponding terms, $K_t(1, 2)$ and $K_t^i(1, 2)$, in the K_t and K_t^i matrices are nonzero. As a result, the price changes of stocks 1 and 2, $\Delta P_{1(t+1)}$ and $\Delta P_{2(t+1)}$, contribute to the trading volume of these stocks as given in Equation (45). Even if $\Delta P_{1(t+1)}$ is zero, the trading volume of stock 1 may not be zero.

Note that the trading volume of a stock is related to the sum of the price changes of the related stocks weighted by the absolute value of the precision difference and the differences about the means of signals. It shows that even if there are no differences of opinion or no signals about a stock's payoff, there may still be trading in that stock due to differences of opinion about the payoffs of other related stocks. These results may be used to explain the empirical findings of Kandel and Pearson (1995) and Huberman and Regev (2001).

We next show that the CAPM holds for the average investor a. The belief of the average investor a is represented by the expected payoff μ_t and precision matrix K_t. Using Equation (46), we obtain that the expected dollar return of investor a is given by

$$\mathrm{E}_t^a \Delta P_{t+1} = K_{t+1}^{-1} N_{t+1} K_t^{-1} X = \mathrm{Cov}_t^a[\Delta P_{t+1}, \Delta P_{M(t+1)}]X,$$

where $\Delta P_{M(t+1)}$ denotes the return of the market portfolio defined as $P_{Mt} = P_t^T X$, with the superscript T representing the transpose of a vector. This calculation indicates that from the average investor's point of view, the CAPM holds, yielding

$$\mathrm{E}_t^a[\Delta P_{k(t+1)}] = \frac{\mathrm{Cov}_t^a[\Delta P_{k(t+1)}, \Delta P_{M(t+1)}]}{\mathrm{Var}_t^a[\Delta P_{M(t+1)}]} \mathrm{E}_t^a[\Delta P_{M(t+1)}] \equiv \beta_{kt} \mathrm{E}_t^a[\Delta P_{M(t+1)}], \tag{47}$$

where $k = 1, 2, \ldots, N$ is the index for the number of stocks. Recall that the risk-free rate is assumed to be zero throughout this article. In our model, the average investor holds the market portfolio and the market portfolio is on his efficient frontier at all times. As a result, the average investor does not trade dynamically and the CAPM holds in every period from his point of view. Our result suggests that if the real data reflect the true distribution of the stock payoff, then the CAPM will not be confirmed by the data unless the average belief happens to be the correct one.

4. Conclusion

In this article we develop a model of trading based on differences of opinion regarding public information. Trading in a stock can be divided into four components: trading due to disagreements about the mean and the precision of the current public information, trading due to disagreements about the mean of the next-period public information, and trading due to disagreements about the precision of past public information. Trading may take place even when there are no differences of opinion about the current and future public signals, as long as there is a disagreement about precision in the past. Disagreements about the precision of future public information do not generate trading in the current period. Trading in the options market can be divided into two components: trading due to current disagreements and trading due to next-period disagreements. Disagreements about the mean of public signals do not generate trading in the options market. Our results indicate that stock trading and options trading respond differently to public information. Stock trading starts at the public event date and decays slowly, whereas options trading are clustered before and during the public event dates.

We show that the Pareto optimal allocation is achieved with the introduction of the options market. All assets are priced as if there existed a representative investor whose belief is equal to the precision-weighted average of all investors' beliefs. Trading volumes, however, reflect the underlying differences of opinion among investors. More specifically, all option contracts in our model are priced in accordance with the risk-neutral pricing principle of Black and Scholes (1973). With differences of opinion, the volatility implied by the option price may not be the same as the true volatility of the stock price.

In a multiple-stock model, we demonstrate that the trading volume of a stock is related to the investors' differences of opinion about that stock's payoff, as well as to those about the payoffs of other correlated stocks. Assets are priced according to a representative agent whose belief is equal to the precision-weighted average of all investors' beliefs. The expected asset returns based on this investor's belief are governed by the CAPM.

For tractability, we have adopted some simplifying assumptions. For example, because differences of opinion may affect investors' consumption decisions, a generalization would be to consider an intertemporal consumption model with multiple consumption dates. To focus on trading based only on differences of opinion, we have ignored the effects of private information acquisition. It would be of interest to develop a general model that incorporates both differences of opinion and asymmetric information in the presence of options. We leave them for future research.

Appendix: Proofs

Proof of Theorems 1 and 4. We present here the proof of Theorem 4, which includes Theorem 1 as a special case in which the number of stocks is one.

We prove this theorem using mathematical induction. It is straightforward to show that the price function stated in the theorem clears the market; then we only need to show that the investors' demands are optimal.

In the last trading session, since there is only one trading session left, it is well known that investor i's trading strategy is as described in the theorem:

$$D^i_{T-1} = \tau K^i_{T-1}\Big[\mu^i_{T-1} - P_{T-1}\Big]. \tag{A1}$$

Given this equilibrium trading strategy, investor i's expected utility in the last trading session is then given by

$$\mathrm{E}^i_{T-1}[U^i] = -\exp\left[-\frac{1}{\tau}W^i_{T-1} - \frac{1}{2}(b^i_{T-1})^T K^i_{T-1} b^i_{T-1}\right], \tag{A2}$$

where the superscript T denotes the transpose of a matrix and where $b^i_{T-1} \equiv \mu^i_{T-1} - P_{T-1}$ denotes investor i's excess return at session $(T-1)$. The first term in the exponential comes from the investor's wealth, and the second term represents the certainty equivalent gains in the expected utility from trading in the last session.

At trading session t, let a_t denote the price change, b^i_t denote the expected excess return for investor i, and W^i_t denote the wealth of investor i, that is,

$$a_t = P_t - P_{t-1},\ b^i_t = \mu^i_t - P_t,\ W^i_t = W^i_{t-1} + (D^i_t)^T(P_t - P_{t-1}).$$

Suppose that at trading session t, investor i's demand and expected utility are given by

$$D^i_t = \tau K^i_t\Big[\mu^i_t - P_t\Big] + \tau K_t N^{-1}_{t+1} N^i_{t+1} M^i_{t+1} \quad \text{and} \tag{A3}$$

$$\mathrm{E}^i_t[U^i] \propto -\exp\left[-\frac{1}{\tau}W^i_t - \frac{1}{2}(b^i_t)^T K^i_t b^i_t\right], \tag{A4}$$

respectively. If at session $(t-1)$, investor i's optimal trading strategy and expected utility are equal to

$$D^i_{t-1} = \tau K^i_{t-1}\Big[\mu^i_t - P_t\Big] + \tau K_{t-1} N^{-1}_t N^i_t M^i_t \quad \text{and}$$

$$\mathrm{E}^i_{t-1}[U^i] \propto -\exp\left[-\frac{1}{\tau}W^i_{t-1} - \frac{1}{2}(b^i_{t-1})^T K^i_{t-1} b^i_{t-1}\right], \tag{A5}$$

then the proposed equilibrium holds by induction.

To determine the optimal strategy of investor i at session t, we need to calculate the expected utility given any strategy D^i_t at session t, that is, $\mathrm{E}^i_t[U^i]$. This can be determined by the law of iterated expectations: $\mathrm{E}^i_t[U^i] = \mathrm{E}^i_t[\mathrm{E}^i_t[U^i|a_{t+1}]]$. From Equation (5), we have $K_{t+1}P_{t+1} - K_t P_t = N_{t+1}Y_{t+1}$, which implies that $K_{t+1}a_{t+1} = K_{t+1}(P_{t+1} - P_t) = N_{t+1}(Y_{t+1} - P_t)$. Thus we can rewrite a_{t+1} as $a_{t+1} = A_{t+1}(u - P_t + \eta_{t+1})$, where A_{t+1} is defined by $A_{t+1} = K^{-1}_{t+1}(K_{t+1} - K_t)$.

Define

$$\Lambda^i_t \equiv \Big(\mathrm{Var}^i_t[u|a_{t+1}]\Big)^{-1} = K^i_{t+1},$$

$$\begin{aligned}
\mu_{b^i_{t+1}} &\equiv \mathrm{E}^i_t\Big[b^i_{t+1}|a_{t+1}\Big] = B^i_t a_{t+1} + C^i_t b^i_t - (I - C^i_t)M^i_{t+1},\\
\mu^i_{a_{t+1}} &\equiv \mathrm{E}^i_t[a_{t+1}] = A_{t+1}\Big(b^i_t + M^i_{t+1}\Big), \quad B^i_t \equiv K^i_{t+1}K^i_t A^{-1}_{t+1} - I,\\
C^i_t &\equiv (K^i_{t+1})^{-1}K^i_t,\\
\Psi^i_t &\equiv \Big(\mathrm{Var}^i_t[a_{t+1}]\Big)^{-1} = ((A_{t+1})^{-1})^T\Big[\Big(K^i_t\Big)^{-1} + \Big(N^i_{t+1}\Big)^{-1}\Big]^{-1}(A_{t+1})^{-1}\\
&= \Big(B^i_t + I\Big)^T K^i_t A^{-1}_{t+1}.
\end{aligned}$$

We now calculate the expected utility conditional on a_{t+1}; dropping irrelevant terms, this is given by

$$\begin{aligned}
\mathrm{E}^i_t[U^i|a_{t+1}] &\propto -\exp\Big[-\frac{1}{\tau}\Big(D^i_t\Big)^T a_{t+1} - \frac{1}{2}\Big(b^i_{t+1}\Big)^T K^i_{t+1}b^i_{t+1}\Big]\\
&= -\exp\Big[-\frac{1}{\tau}\Big(D^i_t\Big)^T a_{t+1} - \frac{1}{2}\Big(B^i_t a_{t+1} + C^i_t b^i_t - \Big(I - C^i_t\Big)M^i_{t+1}\Big)^T\\
&\quad \times K^i_{t+1}\Big(B^i_t a_{t+1} + C^i_t b^i_t - \Big(I - C^i_t\Big)M^i_{t+1}\Big)\Big]. \qquad \text{(A6)}
\end{aligned}$$

Taking the expectation with respect to a_{t+1}, we get

$$\begin{aligned}
\mathrm{E}^i_t[U^i] &\propto -\int_R \exp\Big[-\frac{1}{\tau}\Big(D^i_t\Big)^T a_{t+1} - \frac{1}{2}\Big(B^i_t a_{t+1} + C^i_t b^i_t - \Big(I - C^i_t\Big)M^i_{t+1}\Big)^T\\
&\quad \times K^i_{t+1}\Big(B^i_t a_{t+1} + C^i_t b^i_t - \Big(I - C^i_t\Big)M^i_{t+1}\Big) - \frac{1}{2}\Big(a_{t+1} - \mu^i_{a_{t+1}}\Big)^T\\
&\quad \times \Psi^i_t\Big(a_{t+1} - \mu^i_{a_{t+1}}\Big)\Big]da_{t+1}\\
&\propto -\sqrt{\frac{1}{G^i_t}}\exp\Big[\frac{1}{2}\Big(F^i_t\Big)^T G^i_t\Big(F^i_t\Big) - \frac{1}{2}\Big(b^i_t\Big)^T H^i_t b^i_t\Big], \quad \text{where} \qquad \text{(A7)}\\
F^i_t &= D^i_t/\tau + \Big(B^i_t\Big)^T K^i_{t+1}C^i_t b^i_t - \Psi^i_t\mu^i_{a_{t+1}} - \Big(B^i_t\Big)^T\Big(I - C^i_t\Big)K^i_{t+1}M^i_{t+1}, \qquad \text{(A8)}\\
G^i_t &= \Big[\Big(B^i_t\Big)^T K^i_{t+1}B^i_t + \Psi^i_t\Big]^{-1}, \qquad \text{(A9)}\\
H^i_t &= \Big(C^i_t\Big)^T K^i_{t+1}C^i_t + A^T_{t+1}\Psi^i_t A_{t+1} = C^i_t K^i_t + \Big((B^i_t)^T + I\Big)A_{t+1}K^i_t\\
&= \Big[C^i_t + I - C^i_t\Big]K^i_t = K^i_t. \qquad \text{(A10)}
\end{aligned}$$

Because K^i_{t+1} and Ψ^i_t are positive definite, G^i_t is positive definite.

The first-order condition simplifies to

$$F^i_t = 0, \qquad \text{(A11)}$$

which implies that investor i's optimal demand for the tth trading session is given by

$$\begin{aligned}
D^i_t &= \tau\Big[\Psi^i_t\mu^i_{a_{t+1}} - \Big(B^i_t\Big)^T K^i_{t+1}C^i_t b^i_t - \Big(B^i_t\Big)^T(I - C^i_t)K^i_{t+1}M^i_{t+1}\Big]\\
&= \tau\Big\{\Big[\Big(B^i_t\Big)^T + I - \Big(B^i_t\Big)^T\Big]K^i_t\Big(b^i_t + M^i_{t+1}\Big) - \Big(B^i_t\Big)^T K^i_{t+1}M^i_{t+1}\Big\}\\
&= \tau K^i_t\Big[\mu^i_t - P_t\Big] + \tau K_t N_{t+1}N^{-1}_{t+1}M^i_{t+1}. \qquad \text{(A12)}
\end{aligned}$$

The optimal demand in Equation (A12) has the same form as in Equation (A3). Substituting Equations (A8), (A9), and (A10) into Equation (A7), we have

$$\mathrm{E}_t^i[U^i] \propto -\exp\left[-\frac{1}{\tau}W_t^i - \frac{1}{2}\left(b_t^i\right)^T K_t^i b_t^i\right]. \tag{A13}$$

Proof of Theorem 2. We first verify that the payoffs for the related trading profits are finite. Let M_t^i be an upper bound for the absolute value of holdings in the proposed strategy for investor i, that is, $M_t^i \geq \max\{|D^i|, |D_{CZt}^i|, |D_{PZt}^i|, Z \in \mathcal{R}\}$, where $\mathcal{R}$ is the real line. The absolute value of the payoff at time t is bounded by

$$M_t^i|P_t| + \int_{-\infty}^{0} P_{PZt}dZ + \int_{0}^{\infty} P_{CZt}dZ = M_t^i\left(|P_t| + 1/(2K_t) + P_t^2/2\right).$$

Thus the payoff for the proposed strategy and the associated trading profits are bounded. Note that the expected CARA utility function is concave, and we next show that it is Frechet differentiable with respect to holdings in option portfolios. We provide the proof for the last period. Let $\pi_Z(u)$ denote the call payoff $(u - Z)^+$ for $Z > 0$ and $(Z - u)^+$ for $Z < 0$, and let $P_{Z,T-1}$ denote the call or put price at date $T - 1$. We show that the Frechet derivative is given by

$$E_{T-1}^i\left[(\pi_Z(u) - P_{Z,T-1})U'\left(W_T^i\right)\right].$$

First, we show that $E_{T-1}^i[(\pi_Z(u) - P_{Z,T-1})U'(W_T^i)]$ is in L^1:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\left|E_{T-1}^i\left[(\pi_Z(u) - P_{Z,T-1})U'\left(W_T^i\right)\right]\right|dZ \\
&\quad\leq \int_{-\infty}^{\infty}\left|E_{T-1}^i\left[(\pi_Z(u) + P_{Z,T-1})U'\left(W_T^i\right)\right]\right| \\
&\quad= \left|E_{T-1}^i\left[\frac{u^2}{2}u'\left(W_T^i\right)\right]\right| \\
&\qquad+ \left|E_{T-1}^i\left[\left(\frac{P_{T-1}^2}{2} + \frac{1}{2K_{T-1}}\right)u'\left(W_T^i\right)\right]\right| < \infty.
\end{aligned} \tag{A14}$$

Let $x(Z)$ be a bounded continuous function for the strike prices of option portfolios, and $h(Z)$ denote a small change to $x(Z)$. We have

$$\begin{aligned}
&E_{T-1}^i\left[U\left(W_T^i(x(Z) + h(Z))\right)\right] - E_{T-1}^i[U(W_T^i(x(Z)))] \\
&\quad- \int_{-\infty}^{\infty} h(Z)E_{T-1}^i[(\pi_Z(u) - P_{Z,T-1})U'(W_T^i)]dZ \\
&= O\left(\int_{-\infty}^{\infty} h^2(Z)dZ\right) = o\left(\int_{-\infty}^{\infty} h(Z)dZ\right).
\end{aligned} \tag{A15}$$

Consequently, the expected utility is Frechet differentiable and its Frechet derivative is given by

$$E_{T-1}^i\left[(\pi_Z(u) - P_{Z,T-1})u'\left(W_T^i\right)\right].$$

To establish that the proposed prices and demands constitute an equilibrium, we only need to verify that the market clears and the Euler condition holds for the stock and options. It is easy to check that the market clears at the proposed equilibrium prices. We then show that the proposed equilibrium demands satisfy the first-order conditions. In the last trading session $(T - 1)$, investor

i's wealth in the next period is given by

$$W_T^i = W_{T-1}^i + D_{T-1}^i(u - P_{T-1}) + \int_{-\infty}^0 D_{PZ(T-1)}^i[(Z-u)^+ - P_{PZ(T-1)}]dZ$$
$$+ \int_0^\infty D_{CZ(T-1)}^i[(u-Z)^+ - P_{CZ(T-1)}]dZ.$$

Given the conjectured demands and prices for the stock and options, we have

$$W_T^i = W_{T-1}^i + \tau\left(\mu_{t-1}^i - P_t\right)(u - P_{T-1}) + \tau\left[K_{T-1} - K_{T-1}^i\right]$$
$$\times\left[(u - P_{T-1})^2 - \frac{1}{K_{T-1}}\right].$$

It can be shown that in this proposed equilibrium, investors achieve the Pareto optimal allocation and the Euler conditions for the stock and options are satisfied, that is,

$$\mathrm{E}_{T-1}^i\left[(u - P_{T-1})U'\left(W_T^i\right)\right] = 0,\ \mathrm{E}_{T-1}^i\left[((Z-u)^+ - P_{PZ(T-1)})U'\left(W_T^i\right)\right] = 0,$$
$$\mathrm{E}_{T-1}^i\left[((u-Z)^+ - P_{CZ(T-1)})U'\left(W_T^i\right)\right] = 0.$$

Thus the proposed equilibrium holds in the last period.

Suppose that the proposed equilibrium holds for period t. If we can show that the proposed demands and prices also constitute an equilibrium at trading session $(t-1)$, then the demands and prices form an equilibrium for all t. Let $U^i(W_t^i, P_t)$ denote the expected utility that investor i achieves conditional on his wealth and belief in trading session t. As shown in the proof of Theorem 3, it can be shown that

$$U^i\left(W_t^i, P_t\right) \propto -\exp\left[-\frac{W_t^i}{\tau} - \frac{K_t^i}{2}\left(\mu_t^i - P_t\right)^2\right].$$

To prove that the proposed demands are optimal, we need to show that the following Euler conditions hold:

$$\mathrm{E}_{t-1}^i\left[(P_t - P_{t-1})U'\left(W_t^i, P_t\right)\right] = 0,\ \mathrm{E}_{t-1}^i\left[(P_{PZt} - P_{PZ(t-1)})U'\left(W_t^i, P_t\right)\right] = 0,$$
$$\mathrm{E}_{t-1}^i\left[(P_{CZt} - P_{CZ(t-1)})U'\left(W_t^i, P_t\right)\right] = 0.$$

Next, we show that the marginal rates of substitution are equalized across all investors. In trading session t, investors observe a public signal y_t. Let f^i denote investor i's probability density of y_t. Considering the marginal rate of substitution M_{tkl}^i between two realizations of y_t: y_{tk} and y_{tl}, we have

$$M_{tkl}^i = \frac{f^i(y_{tk})U'\left(W_t^i(y_{tk}), P_t(y_{tk})\right)}{f^i(y_{tl})U'\left(W_t^i(y_{tl}), P_t(y_{tl})\right)}. \tag{A16}$$

We now determine the probability-weighted marginal utility $f^i(y_{tk})U'(W_t^i(y_{tk}), P_t(y_{tk}))$. Dropping terms not related to y_{tk}, we have

$$f^i(y_{tk})U'\left(W_t^i(y_{tk}), P_t(y_{tk})\right)$$
$$\Rightarrow \exp\left\{-\frac{W_{t-1}^i}{\tau} - \frac{D_{t-1}^i[P_t(y_{tk}) - P_{t-1}]}{\tau} - \frac{K_t^i}{2}\left[\mu_t^i(y_{tk}) - P_t(y_{tk})\right]^2\right.$$

$$
\begin{aligned}
& -\frac{1-\rho^i}{2}\left(\frac{K_{t-1}K_t}{n_t}-h\right)[P_t(y_{tk})-P_{t-1}]^2-\frac{n_t^iK_{t-1}^i}{2K_t^i}\left(y_{tk}-\mu_{t-1}^i\right)^2\Bigg\} \\
&= \exp\Bigg\{-K_{t-1}^i\left(\mu_{t-1}^i-P_{t-1}\right)\frac{n_t}{K_t}(y_{tk}-P_{t-1}) \\
&\quad -\frac{1}{2K_t^i}\left[\left(n_t^i-\frac{K_t^in_t}{K_t}\right)(y_{tk}-P_{t-1})+K_{t-1}^i\left(\mu_{t-1}^i-P_{t-1}\right)\right]^2 \\
&\quad -\frac{1-\rho^i}{2}\left(\frac{K_{t-1}K_t}{n_t}-h\right)\frac{n_t^2}{K_t^2}(y_{tk}-P_{t-1})^2-\frac{n_t^iK_{t-1}^i}{2K_t^i}(y_{tk}-\mu_{t-1}^i)^2\Bigg\} \\
&\Rightarrow \exp\Bigg\{-\left[\frac{K_{t-1}^in_t}{K_t}+\frac{K_{t-1}^i}{K_t^i}\left(n_t^i-\frac{n_tK_t^i}{K_t}\right)-\frac{K_{t-1}^in_t^i}{K_t^i}\right] \\
&\quad \times\left(\mu_{t-1}^i-P_{t-1}\right)(y_{tk}-P_{t-1})-\frac{1}{2}\left[\frac{K_{t-1}^in_t^i}{K_t^i}+\frac{1}{K_t^i}\left(n_t^i-\frac{n_tK_t^i}{K_t}\right)^2\right. \\
&\quad \left.+(1-\rho^i)\left(\frac{K_{t-1}K_t}{n_t}-h\right)\frac{n_t^2}{K_t^2}\right](y_{tk}-P_{t-1})^2\Bigg\} \\
&= \exp\left[\frac{K_{t-1}K_t}{n_t}(y_{tk}-P_{t-1})^2\right],
\end{aligned}
\tag{A17}
$$

where "$\Rightarrow$" means that a multiplier (a proportional factor) that is unrelated to y_{tk} has been omitted. Consequently, the marginal rates of substitution are unrelated to i and are equal for all investors.

Because $f^i(y_{tk})U'(W_t^i(y_{tk}), P_t(y_{tk}))$ is independent of investor i's information, the Euler equations for all investors differ by only a multiplier. As a result, if the Euler conditions for one investor (e.g., the average investor) are satisfied, then the Euler equations for all other investors will be satisfied. Consequently, we only need to show that the Euler equations hold for the average investor who does not trade in equilibrium.

Let a denote the average investor with $\rho^a = 1$. Following Brennan (1979), we have

$$
P_tU'\left(W_t^a, P_t\right)=\mathrm{E}_t^a\left[P_TU'\left(W_T^a, P_T\right)\right], \quad U'\left(W_t^a, P_t\right)=\mathrm{E}_t^a\left[U'\left(W_T^a, P_T\right)\right],
$$

which implies that

$$
\begin{aligned}
\mathrm{E}_{t-1}^a\left[(P_t-P_{t-1})U'\left(W_t^a, P_t\right)\right] &= \mathrm{E}_{t-1}^a\left[P_tU'\left(W_t^a, P_t\right)-P_{t-1}U'(W_t^a, P_t)\right] \\
&= \mathrm{E}_{t-1}^a\left[P_TU'\left(W_T^a, P_T\right)-P_{t-1}U'(W_{t-1}^a, P_{t-1})\right] \\
&= 0.
\end{aligned}
$$

Similarly, it can be shown that

$$
\begin{aligned}
\mathrm{E}_{t-1}^a\left[(P_{PZt}-P_{PZ(t-1)})U'\left(W_t^a, P_t\right)\right] &= 0, \\
\mathrm{E}_{t-1}^a\left[(P_{CZt}-P_{CZ(t-1)})U'\left(W_t^a, P_t\right)\right] &= 0.
\end{aligned}
$$

Thus, the Euler equations hold for investor a at session $(t-1)$, which further implies that the Euler equations hold for all investors. The proposed demands are optimal for all investors at session $(t-1)$. By mathematical induction, the proposed equilibrium demands are optimal in all periods.

Proof of Theorem 3. We prove this theorem using mathematical induction. It is straightforward to show that the price function stated in the theorem clears the market; then we only need to show that the investors' demands are optimal.

In the last trading session, substituting investor i's terminal wealth into the utility function, using the conjectured equilibrium price of the option, and taking expectations, we can write investor i's portfolio problem as

$$\begin{aligned}\max_{D^i_{Q(T-1)},D^i_{T-1}} \mathrm{E}^i_{T-1}[U^i] = &-\sqrt{\frac{1}{1+\frac{2D^i_{Q(T-1)}}{\tau K^i_{T-1}}}}\\ &\times \exp\left\{\frac{\left[\frac{D^i_{T-1}}{\tau} - K^i_{T-1}\left(\mu^i_{T-1} - P_{T-1}\right)\right]^2}{K^i_{T-1} + \frac{2D^i_{Q(T-1)}}{\tau}}\right\}\\ &\times \exp\left\{\frac{D^i_{Q(T-1)}P_{Q(T-1)}}{\tau} - \frac{K^i_{T-1}}{2}\left(\mu^i_{T-1} - P_{T-1}\right)^2\right\},\end{aligned}$$

where $D^i_{Q(T-1)}$ and D^i_{T-1} are the number of units of the quadratic option and the stock purchased by investor i, respectively. The optimal solutions are then given by

$$D^i_{T-1} = \tau K^i_{T-1}\left(\mu^i_{T-1} - P_{T-1}\right), \quad D^i_{Q(T-1)} = \frac{1}{2}\tau\left(\frac{1}{P_{Q(T-1)}} - K^i_{T-1}\right).$$

Given the equilibrium trading strategies in the last trading session, investor i's expected utility in the last trading session is

$$\mathrm{E}^i_{T-1}[U^i] \propto -\exp\left[-\frac{W^i_{T-1}}{\tau} - \frac{1}{2}\left(b^i_{T-1}\right)^2 K^i_{T-1}\right]. \tag{A18}$$

The first term in the exponential comes from the investor's wealth, the second term represents the gains from trading in the last session, and $b^i_{T-1} \equiv \mu^i_{T-1} - P_{T-1}$.

Suppose that at trading session $(t+1)$, we have

$$D^i_{t+1} = \tau K^i_{t+1}\left[\mu^i_{t+1} - P_{t+1}\right], \tag{A19}$$

$$\begin{aligned}D^i_{Q(t+1)} &= \frac{\tau}{2}\left[(1-\rho^i)\frac{K^2_{t+1}}{n_{t+2}} + \frac{1}{P_{Q(t+1)} - P^2_{t+1} - 1/K_{t+2}} - K^i_{t+1}\right]\\ &= \frac{\tau}{2}(1-\rho^i)\left[\frac{K_t K_{t+1}}{n_{t+1}} - h\right],\end{aligned} \tag{A20}$$

$$\mathrm{E}^i_{t+1}[U^i] \propto -\exp\left[-\frac{W^i_{t+1}}{\tau} - \frac{1}{2}\left(b^i_{t+1}\right)^2 K^i_{t+1}\right]. \tag{A21}$$

If at session t, we have

$$D^i_t = \tau K^i_t\left[\mu^i_t - P_t\right], \tag{A22}$$

$$D^i_{Qt} = \frac{\tau}{2}\left[(1-\rho^i)\frac{K^2_t}{n_{t+1}} + \frac{1}{P_{Qt} - P^2_t - 1/K_{t+1}} - K^i_t\right], \tag{A23}$$

$$\mathrm{E}_t^i[U^i] \propto -\exp\left[-\frac{W_t^i}{\tau} - \frac{1}{2}\left(b_t^i\right)^2 K_t^i\right], \tag{A24}$$

then the proposed equilibrium holds by induction.

We now calculate the expected utility conditional on a_{t+1}; dropping irrelevant terms, this is given by

$$\begin{aligned}
&\mathrm{E}_t^i[U^i|a_{t+1}] \propto \\
&\quad -\int_R \exp\left[-\frac{D_t^i a_{t+1} + D_{Qt}^i\left(P_t^2 + 2P_t a_{t+1} + a_{t+1}^2 + 1/K_{t+1} - P_{Qt}\right)}{\tau}\right] \\
&\quad \times \exp\left[-\frac{\left(b_{t+1}^i\right)^2 K_{t+1}^i}{2} - \frac{\left(b_{1+1}^i - \mu_{b_{t+1}^i}\right)^2 \Gamma_t^i}{2}\right] db_{t+1}^i \\
&= -\exp\left[-\frac{D_t^i a_{t+1} + D_{Qt}^i\left(P_t^2 + 2P_t a_{t+1} + a_{t+1}^2 + 1\big/K_{t+1} - P_{Qt}\right)}{\tau}\right] \\
&\quad \times \exp\left[-\frac{\left(B_t^i a_{t+1} + C_t^i b_t^i\right)^2 \Lambda_t^i}{2}\right].
\end{aligned} \tag{A25}$$

Taking the expectation with respect to a_{t+1}, we get

$$\begin{aligned}
&\mathrm{E}_t^i[U^i] \propto \\
&\quad -\int_R \exp\left[-\frac{D_t^i a_{t+1} + D_{Qt}^i\left(P_t^2 + 2P_t a_{t+1} + a_{t+1}^2 + 1\big/K_{t+1} - P_{Qt}\right)}{\tau}\right] \\
&\quad \times \exp\left[-\frac{\left(B_t^i a_{t+1} + C_t^i b_t^i\right)^2 \Lambda_t^i}{2} - \frac{\left(a_{t+1} - \mu_{a_{t+1}}^i\right)^2 \Psi_t^i}{2}\right] da_{t+1} \propto -\frac{1}{\sqrt{G_{tQ}^i}} \\
&\quad \times \exp\left[-\frac{D_{Qt}^i(P_t^2 + 1/K_{t+1} - P_{Qt})}{\tau} + \frac{(F_{tQ}^i)^2 G_{tQ}^i}{2} - \frac{(b_t^i)^2 H_{tQ}^i}{2}\right],
\end{aligned} \tag{A26}$$

where

$$F_{tQ}^i = \left(D_t^i + 2D_{Qt}^i P_t\right)/\tau + B_t^i \Lambda_t^i C_t^i b_t^i - \Psi_t^i \mu_{a_{t+1}}^i, \tag{A27}$$

$$G_{tQ}^i = \left[\left(B_t^i\right)^2 \Lambda_t^i + \Psi_t^i + 2D_{Qt}^i/\tau\right]^{-1}, \tag{A28}$$

$$\begin{aligned}
H_{tQ}^i &= (C_t^i)^2 \Lambda_t^i + A_{t+1}^2 \Psi_t^i = C_t^i K_t^i + (B_t^i + I) A_{t+1} K_t^i \\
&= [C_t^i + I - C_t^i]' K_t^i = K_t^i.
\end{aligned} \tag{A29}$$

Since Λ_t^i and Ψ_t^i are positive, G_t^i is positive.

The first-order conditions simplify to

$$F_{tQ}^i = 0, \tag{A30}$$

$$2G_{tQ}^i + P_t^2 + \tfrac{1}{K_{t+1}} - P_{Qt} = 0, \tag{A31}$$

which imply that investor i's optimal demands in the tth trading session are given by

$$D_t^i = \tau K_t^i \left[\mu_t^i - P_t\right] - 2D_{Qt}^i P_t, \tag{A32}$$

$$D_{Qt}^i = \frac{1}{2}\tau\left[(1-\rho^i)\frac{K_t^2}{n_{t+1}} + \frac{1}{P_{Qt} - P_t^2 - 1/K_{t+1}} - K_t^i\right]. \tag{A33}$$

These are indeed the proposed equilibrium strategies. Substituting Equations (A27), (A28), and (A29) into Equation (A26), we have

$$\mathrm{E}_t^i[U^i] \propto -\exp\left[-\frac{W_t^i}{\tau} - \frac{\left(b_t^i\right)^2 K_t^i}{2}\right]. \tag{A34}$$

Proof of Proposition 1. Note that the option price P_{Q_t} is independent of ρ^i. Hence, even if $\rho^i = 1$ for all investors, the option prices would remain the same. Because all state contingent claims can be synthesized using options, all option prices can be determined according to the average investor and thus all assets can be priced using the same principle.

Proof of Proposition 2. The first part is obvious. For the second part, assume that $K_{t-1}^i = K_{t-1}$, $n_t^i \neq n_t$, and $n_j^i = n_j$ for $j > t$. From investor i's demand function, we get

$$\Delta D_t^i = \tau\left(\frac{n_t^i}{n_t} - 1\right) K_{t-1}\Delta P_t \neq 0$$

when $\Delta P_t \neq 0$. In addition, because there is agreement on the public signals after session t, we have

$$\Delta D_j^i = \tau\left(K_j - K_j^i\right)\Delta P_j \neq 0, \quad \text{for} \quad \Delta P_j \neq 0.$$

Proof of Proposition 3. This proposition follows directly from Equation (32).

Proof of Proposition 4. From Equation (5), we have

$$\begin{aligned}\mathrm{Cov}^i(P_{t+1} - P_t, P_t) &= \frac{n_{t+1}}{K_{t+1}}\mathrm{Cov}^i(u - P_t, P_t)\\ &= \frac{n_{t+1}}{K_{t+1}}\mathrm{Cov}^i(u - \rho^i P_t + (\rho^i - 1)P_t, P_t) = \frac{n_{t+1}}{K_{t+1}}(\rho^i - 1)\mathrm{Var}^i(P_t),\end{aligned} \tag{A35}$$

where $(u - \rho^i P_t)$ is independent of P_t. Similarly, we have

$$\begin{aligned}\mathrm{Cov}^i(P_{t+1} - P_t, P_t - P_{t-1}) &= \frac{n_t n_{t+1}}{K_t K_{t+1}}\mathrm{Cov}^i(u - P_t, u + \eta_t - P_{t-1})\\ &= \frac{n_t n_{t+1}}{K_t K_{t+1}}\mathrm{Cov}^i(u - \rho^i P_t + (\rho^i - 1)P_t, P_t) = \frac{h n_t n_{t+1}}{K_{t-1}K_t^2 K_{t+1}}\frac{\rho^i - 1}{\rho^i}.\end{aligned} \tag{A36}$$

Proof of Proposition 6. The proposition follows directly from Equations (38) and (39).

Proof of Lemma 1. We provide the proof for the economy with two trading sessions. The proof for the general case is similar and is thus omitted.

Investor i's wealth at time 1 is given by

$$W_1^i = W_0^i + D_0^i(u - P_0) + \left(D_{1i} - D_0^i\right)(u - P_1).$$

Note that

$$P_1 - P_0 = \frac{n_1}{K_1}(u + \eta_1 - P_0).$$

The wealth function, ignoring some irrelevant terms, is then given by

$$\begin{aligned} W_1^i &= \frac{n_2}{K_2}\left[(u + \eta_1 - P_0)\left(u - P_0 - \frac{n_1}{K-1}(u + \eta_1 - P_0)\right)\right] \\ &\quad \times \left(K_0^i - \frac{n_{1i}}{n_1}K_0\right). \end{aligned}$$

Taking the expectation of investor i's utility function with respect to u and η_1, we have

$$\mathrm{E}U^i = -\sqrt{\frac{K_0^i n_{1i}}{\det|\Omega|}}\exp\left[\frac{-\bar{u}x^i}{\tau} + \frac{(x^i)^2}{2\tau^2 h} - \frac{(x^i - x)^2}{2\tau^2 h}\right] = \sqrt{\frac{K_0^i n_{1i}}{\det|\Omega|}}\mathrm{E}U^i(1),$$

where

$$\Omega = \left\{\begin{array}{cc} \frac{1}{2}K_{0i} + \frac{k_0 n_1}{k_1^2}\left(\frac{n_{1i}}{n_1}K_1 - K_{1i}\right), & \frac{n_1}{K_1}(\frac{K_0 - n_1}{K_1})\left(\frac{n_{1i}}{n_1}K_1 - K_{1i}\right) \\ \frac{n_1}{K_1}(\frac{K_0 - n_1}{K_1})\left(\frac{n_{1i}}{n_1}K_1 - K_{1i}\right), & \frac{1}{2}n_{1i} + \frac{n_1^2}{K_1^2}\left(\frac{n_{1i}}{n_1}K_1 - K_{1i}\right) \end{array}\right\}.$$

Further algebra gives the results in the lemma for the two-trading-session case.

Proof of Lemma 2. From the proof of Theorem 3 and Equations (A7) and (A26), the added value of options trading in period t is given by

$$\frac{\tau}{2}\ln\left(\frac{G_{tQ}^i}{G_t^i}\right) - D_{Qt}^i\frac{n_{t+1}}{K_t K_{t+1}},$$

which reduces to

$$\frac{\tau}{2}\left[x_t^i - \ln(1 + x_t^i)\right],$$

where

$$x_t^i = \frac{n_{(t+1)i}K_t}{n_{t+1}K_{t+1}} + \frac{n_{t+1}K_{ti}}{K_t K_{t+1}} - 1 = (\rho^i - 1)\left(1 - \frac{hn_{t+1}}{K_t K_{t+1}}\right).$$

Proof of Lemma 3. Parts (i), (iv), and (v) follow immediately from investors' trades, price changes, and Proposition 2. Let $\Delta\mu_t \equiv \mu_t - \mu_{t-1}$. Part (ii) follows because

$$\Delta P_t = \Delta u_t + \frac{xn_t}{\tau K_{t-1}K_t}.$$

Part (iii) follows because $\mathrm{E}^i[\Delta P_t]$ and $\mathrm{Var}^i[\Delta P_t]$ are both increasing in n_t and because $V(\mu, \sigma)$ is increasing in μ and σ for positive μ and σ.

Proof of Lemma 4. The total trading volume is defined as $V(T) = \sum_{t=1}^{T} V_t$, where $V_t = Q\tau h|\Delta P_t|$ is the trading volume at time t. Note that

$$\mathrm{Var}^i[\Delta P_t] = \frac{n_t}{\rho^i K_{t-1} K_t} + \frac{n_t^2 h}{K_{t-1}^2 K_t^2}\left(1 - \frac{1}{\rho^i}\right). \tag{A37}$$

The expected volume at time t, $\mathrm{E}^i[V_t]$, is obtained using a property of the normal distribution:

$$\mathrm{E}^i[V_t] = V\left(\mathrm{E}^i[\Delta P_t], \sqrt{\mathrm{Var}^i[\Delta P_t]}\right),$$

where $V(\cdot, \cdot)$ is the expected absolute value of a normally distributed variable with mean μ and variance σ^2 and where

$$V(\mu, \sigma) = 2\mu N\left(\frac{\mu}{\sigma}\right) - \mu + 2\sigma n\left(\frac{\mu}{\sigma}\right).$$

Note that $1/K_t$ represents the conditional variance of the average investor at time t. Let

$$s(t) = \frac{1/K_{t-1} - 1/K_t}{1/T}.$$

Here the variable $s(t)$ represents the rate of change of variance at trading session t and is assumed to be bounded from above. For a large T, $\mathrm{E}^i[\Delta P_t]$ and $\mathrm{Var}^i[\Delta P_t]$ go to zero in the order of $1/T$. We thus have $E^i[V_t] \approx \sqrt{\frac{2s(t)}{\pi T}}$, which implies that

$$\frac{V(T)}{\sqrt{T}} \approx Q \int_0^1 s(t)dt \sqrt{\frac{2h}{\rho^i \pi}}.$$

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