An Equilibrium Model of Asset Pricing and Moral Hazard

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This article develops an integrated model of asset pricing and moral hazard. It is demonstrated that the expected dollar return of a stock is independent of managerial incentives and idiosyncratic risk, but the equilibrium price of the stock depends on them. Thus, the expected rate of return is affected by managerial incentives and idiosyncratic risk. It is shown, however, that managerial incentives and idiosyncratic risk affect the expected rate of return through their influence on systematic risk rather than serve as independent risk factors. It is also shown that the risk aversion of the principal in the model leads to less emphasis on relative performance evaluation than in a model with a risk-neutral principal.

Principal–agent models are typically developed in the absence of a multi-asset equilibrium as well as under a risk-neutral principal. As a result, optimal contracts are based on firms' cash flows or accounting measures rather than their market values. For example, Holmström (1982) discusses relative performance evaluation (RPE), in which an agent's compensation depends not only on his own performance but also on the performance of others. He finds that RPE improves welfare, because it can be used to filter out common risk from agents' compensation. Baiman and Demski (1980) and Diamond and Verrecchia (1982) obtain similar results in more specialized settings.

The RPE with respect to the market portfolio has been the focus of extensive empirical testing, but the evidence has been mixed.[1] Specifically, the empirical studies have focused mainly on the implicit relation between Chief Executive Officer (CEO) compensation, firm performance, and market and/or industry performance. In the absence of an asset-pricing model, the hypothesis is that if the stock price of a firm is positively

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[1] For comprehensive reviews, see, for example, Abowd and Kaplan (1999), Murphy (1999), Prendergast (1999), and Core, Guay, and Larcker (2001).

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correlated with the market portfolio, then its executive compensation should be negatively related to the performance of the market portfolio. On the one hand, Antle and Smith (1986), Barro and Barro (1990), Jensen and Murphy (1990), Janakiraman, Lambert, and Larcker (1992), Aggarwal and Samwick (1999a), and Bertrand and Mullainathan (2001) find little evidence of RPE. Aggarwal and Samwick (1999b) even find positive RPE in some cases. On the other hand, Gibbons and Murphy (1990), Sloan (1993), and Himmelberg and Hubbard (2000) find empirical support for RPE. Summarizing the empirical findings, Abowd and Kaplan (1999) and Prendergast (1999) have identified the lack of widespread use of RPE in executive compensations as an unresolved puzzle.

Because a corporate executive's compensation is typically based on the market value rather than the accounting measures of the firm, the aforementioned empirical tests and other tests in the principal–agent literature often employ the market price of the firm, whereas theoretical predictions are based on the cash flow of the firm as well as a risk-neutral principal. Principals or investors are certainly risk averse with respect to the market-wide risk, because the market-risk premium has been nonzero. Because accounting measures are realized ones while market values are based on investors' expectations about the future cash flows or dividends conditional on the current information, theoretical predictions based on the accounting measures may differ from those based on the market values. In addition, the empirical tests often use the total risk of a firm's market value without distinguishing between idiosyncratic risk and systematic risk, which cannot be defined rigorously in the absence of an equilibrium asset-pricing model. It is known that a more volatile cash flow does not necessarily result in a more volatile market price and that a higher total risk does not necessarily mean a higher idiosyncratic risk, which may play different roles in the determination of managerial incentives. Consequently, without considering the distinction between idiosyncratic risk and systematic risk, the tests of agency results, such as the negative association between managerial incentives and risk, may be flawed.

To bridge the gap between theoretical modeling and empirical testing in the principal–agent literature as well as to provide more precise guidance of empirical testing, it is important to develop an integrated model of equilibrium asset pricing and moral hazard. In particular, it would be of interest to determine in an equilibrium multi-asset framework whether previous results under cash flows or accounting measures still hold under market prices and whether the previous result on RPE is robust with respect to risk-averse principals.

On the other hand, in many equilibrium asset-pricing models, the cash flow or dividend processes are specified exogenously by either normal or lognormal processes. For example, the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) shows that the expected excess return of a stock is linearly related to the expected excess return of the market portfolio. The slope coefficient in this relation, β, is defined by the ratio of the covariance between the asset and market returns to the variance of the market return.[2] Thus, managerial incentives and certain firm characteristics, such as idiosyncratic risk, play no role in the determination of expected asset returns.

In reality, however, a firm's cash flows are owned by its investors or shareholders but influenced by its manager, who cannot fully hedge against the firm's idiosyncratic risk. Because the interests of the manager (the agent) and the shareholders (the principal) may not be perfectly aligned, there exists a potential moral hazard problem. This conflict raises two key questions. First, does the linear relation between the expected excess return of an asset and the expected excess return of the market still hold? Second, do the manager's compensation and individual firm characteristics affect expected asset returns in the presence of moral hazard?

The objective of this article is to develop an integrated model of asset pricing and moral hazard.[3] In particular, using the CAPM as a benchmark, we develop an equilibrium asset-pricing model in the presence of moral hazard as well as a multi-agent moral hazard model by allowing principals to be risk averse and to trade in the financial market. Following Holmström and Milgrom (1987), we adopt a continuous-time framework for its convenience in the derivation of optimal contracts.[4] We explicitly characterize equilibrium asset prices, expected asset returns, and optimal contracts under certain assumptions. It is important to have closed-form solutions to determine whether the CAPM relation holds and to understand how both the risk aversion of a principal and the risk exposure of an asset affect RPE as well as how a manager's compensation affects the expected asset return.[5]

This article shows that the optimal sharing of systematic risk between risk-averse investors and risk-averse managers reduces the magnitude of RPE. On the one hand, because systematic risk can be inferred by investors in a large economy,[6] investors would like to reduce the risk-averse manager's exposure to this risk by the use of RPE (a negative coefficient in front of the market portfolio in the manager's compensation). On the

[2] See Ross (1976) for the Arbitrage Pricing Theory.

[3] See, for example, Brennan (1993), Dow and Gorton (1997), Allen (2001), Arora, Ju, and Ou-Yang (2001), Cuoco and Kaniel (2001), Mamaysky and Spiegel (2001), and Cornell and Roll (2004) for integrated models of asset pricing and delegated portfolio management, which may also involve moral hazard problems.

[4] Because the linear contract is typically not optimal in a one-period principal–agent model [see, e.g., Mirrlees (1974) for details], we deduce optimal contracts from a larger contract space.

[5] In the absence of closed-form solutions for the expected returns and optimal contracts, for example, it would be difficult to determine accurately the impact of idiosyncratic risk and managerial incentives on the expected profits and expected asset returns.

[6] We define a large economy as the one, in which the number of risky assets approaches infinity.

other hand, unlike risk-neutral investors who can bear all of the systematic risk, risk-averse investors must share it with risk-averse managers, which results in a positive coefficient in front of the market portfolio. Consequently, the magnitude of RPE in this case is smaller than the value obtained in previous models, in which the principal is risk neutral. Specifically, we show that RPE is determined by the risk aversion of both investors and managers as well as the ratio of the standard deviation of the firm's market value to that of the market portfolio. For example, the RPE can even be positive if the ratio of the two standard deviations is small enough or if the manager is not too risk averse. In general, RPE can be negative or positive. Therefore, if a cross-sectional regression is performed for the test of RPE, the coefficient in front of the performance of the market portfolio may well be insignificant. Furthermore, we show that, under certain conditions, the optimal contract is a linear combination of the stock price and the level of the market portfolio, justifying the use of market prices in empirical studies. We also show that the manager's pay-performance sensitivity (PPS), which is defined as the fraction of the firm owned by the manager, depends on the idiosyncratic risk rather than the total risk of the cash flow process and that RPE depends not only on the firm's systematic risk but also on its idiosyncratic risk.[7]

When both principal and agent have exponential utility functions and cash flow processes are normal, we find that the expected excess dollar return of a stock less the expected compensation to the manager is linearly related to the expected excess dollar return on the market portfolio. Similarly, β is given by the ratio of the covariance between a firm's stock return and the market return to the variance of the market return, with both the stock return and the market return adjusted for the compensation to managers. In particular, the expected dollar return of a stock is found to be independent of its manager's PPS and the firm-specific risk. This result suggests that, even under moral hazard, the notion of idiosyncratic risk in terms of the expected dollar returns remains the same as in the original CAPM.

When investors are risk neutral, the use of RPE completely filters out systematic risk from the manager's compensation. With risk-averse investors, the optimal contracts show that, in equilibrium, investors and managers share systematic risk optimally, as if there were no moral hazard, and the incentive part of the contract involves only firm-specific risk. In other words, the firm-specific risk and systematic risk are completely separate from each other in the manager's compensation. Because of the separation of these two types of risks, the PPS and firm-specific risk do not affect the expected dollar

[7] Almost all the empirical tests of the negative association between PPS and risk employ total risk. Two exceptions are Jin (2002) and Garvey and Milbourn (2003), but their approaches assume that in the presence of moral hazard, the CAPM beta holds for the expected rate of return. Their calculations of beta would have been justified by the current article if dollar returns had been used.

return, which depends on the covariance between the dollar return of the individual stock and that of the market portfolio.

In the exponential-normal case, we also show with closed-form solutions that when a firm's exposure to systematic risk hypothetically vanishes but its firm-specific risk remains, its expected excess dollar return is zero and its expected rate of return reduces to the risk-free rate. This means that idiosyncratic risk is not an independent risk factor, because moral hazard exists even in the absence of systematic risk. Furthermore, it implies that only investors' risk aversion toward undiversifiable risk matters for expected asset returns. Another example is when investors are risk neutral but managers are risk averse, there still exists moral hazard problem. We show that the expected asset returns again reduce to the risk-free rate, because risk-neutral investors are unconcerned about systematic risk. This result illustrates that, in the case of risk-neutral investors, managerial incentives do not affect the expected rate of return, because the expected excess dollar return is always zero. It clarifies the implications of the models of Diamond and Verrecchia (1982) and Holmström (1982), both of which consider risk-neutral principals but suggest that idiosyncratic risk affects expected asset returns. These two examples highlight the limitations of the previous literature both under a risk-neutral principal and in the absence of a multi-asset equilibrium.

The PPS and the firm-specific risk, however, do affect equilibrium stock prices. The higher the PPS, the higher the manager's effort, and thus, the stock price is higher in equilibrium. If we define the risk premium of a stock as the ratio of its expected excess dollar return to its current stock price, then the risk premium decreases with the manager's PPS. Similarly, after controlling for certain variables, the higher the idiosyncratic risk, the lower the equilibrium asset price, thus the risk premium is higher. Notice, however, that these results make sense only when the stock prices are positive.

We further argue that managerial incentives and idiosyncratic risk may affect expected asset returns under log-normal cash flow processes and general utility functions.[8] A key insight of this argument is that even if a cash flow follows a log-normal process, once a manager's compensation is introduced, the equilibrium stock price no longer follows a log-normal process.[9] In particular, the volatility (both systematic and idiosyncratic) of the stock price return process depends on both the manager's cash compensation and the level of the stock price itself. It is true that because investors can diversify across firms, firm-specific risks may not affect the expected return in a direct manner. However, the

[8] For simplicity, "expected return" means the expected rate of return.

[9] In general, the equilibrium stock price may not follow the same factor model and distribution as the original cash flow process. We thank a referee for this insight.

manager's compensation and firm-specific risk affect expected asset returns indirectly through the stock price, just as in the exponential-normal case.

Early developments in the valuation of assets under moral hazard include the one-period models of Diamond and Verrecchia (1982) and Ramakrishnan and Thakor (1984). Diamond and Verrecchia derive an optimal managerial contract in an equilibrium model in which identical risk-neutral investors trade between a riskless bond and a risky stock. They show that systematic risk is completely removed from the manager's compensation scheme. They also show that a risk-neutral investor can earn a higher expected dollar profit by adopting a project with a lower idiosyncratic risk. Because their model considers only one risky asset and risk-neutral investors, the role of idiosyncratic risk in expected asset returns cannot be rigorously addressed and a CAPM-type relation cannot be obtained. Ramakrishnan and Thakor (1984) extend the Diamond–Verrecchia model to incorporate a risk-averse investor in an environment with normally distributed asset returns.[10] They show that in the absence of moral hazard, managers are insured against idiosyncratic risks, but when moral hazard is included, contracts depend on both systematic and idiosyncratic risks. Ramakrishnan and Thakor demonstrate their results in a partial equilibrium that assumes that the Arbitrage Pricing Theory holds with moral hazard. Their model also assumes that the expected return on an asset increases with the agent's effort level.

Our model may be viewed as an extension of Holmström and Milgrom (1987) in the presence of both multiple agents and principal who can trade in a securities market as well as an extension of Diamond and Verrecchia (1982) in the presence of risk-averse investors and multiple risky assets. Our approach also generalizes the Ramakrishnan–Thakor framework by deriving an asset-pricing model in the presence of moral hazard; however, we assume that the cash flow process of an asset, rather than the expected return on an asset, increases with the agent's effort level.

In summary, this article develops the first equilibrium model of asset pricing and moral hazard in a large economy, in which both a CAPM-type linear relation for the expected asset returns and optimal contracts that involve RPE are explicitly characterized. Some of our equilibrium results suggest that partial equilibrium models may lead to incorrect conclusions. First, in the exponential-normal case, the equilibrium expected dollar returns are independent of idiosyncratic risk, whereas they increase with idiosyncratic risk in partial equilibrium models. Second, when investors are risk neutral or when systematic risk is hypothetically absent, the equilibrium expected rates of return for all risky assets reduce to the risk-free rate, whereas partial equilibrium models obtain that they

[10] In Diamond and Verrecchia, the uniform distribution for the systematic risk factor is adopted for tractability in the derivation of the optimal contract.

increase with idiosyncratic risk. Third, a striking result derived from our equilibrium is that managerial incentives and idiosyncratic risk do not serve as independent risk factors. Instead, they affect the expected rates of return through their influence on systematic risk. In addition, we consider a large economy in which the number of assets approaches infinity. Given such an economy, we can attribute the potential impact of idiosyncratic risk and managerial compensation on expected asset returns to agency problems rather than investors' lack of diversification. In models in which investors cannot hold fully diversified portfolios, idiosyncratic risk contributes to expected asset returns even in the absence of agency problems.

The rest of this article is organized as follows. Section 1 describes the model. As a benchmark case, Section 2 reviews the CAPM linear relation in the absence of moral hazard. The expressions for expected returns, equilibrium stock prices, and optimal contracts in the presence of moral hazard are derived in Section 3. Section 4 examines the robustness of the results under log-normal cash flow processes and general utility functions as well as under endogenous interest rates. Some concluding remarks are offered in Section 5. All proofs are given in the appendix.

1. The Model

For the tractability of derivation of optimal contracts, consider a continuous-time economy on a finite time horizon $[0, T]$. The following assumptions characterize our economy.

Assumption 1. *There are N risky assets and one riskless asset available for continuous trading. The cash flow process of firm i is given by*

$$\begin{aligned} dD_{it} &= (A_{it} + \Pi_i D_{it})dt + \sigma_{ic} dB_{ct} + \sigma_{ii} dB_{it} \\ &\equiv (A_{it} + \Pi_i D_{it})dt + b_{iD} dB_t, \quad i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

with the initial condition that $D_{i0} = d_{i0}$.

Here, the transpose of B_t is defined as $B_t^T = (B_{ct}, B_{1t}, B_{2t}, \cdots, B_{Nt})$, representing a $[1 \times (N+1)]$ vector of independent standard Brownian motion processes; b_{iD} is defined as $b_{iD} = (\sigma_{ic}, 0, \ldots, 0, \sigma_{ii}, \ldots, 0)$, which is a $[1 \times (N+1)]$ vector of constants, with the zero element and the ith element being nonzero. Thus, the cash flow processes across assets are correlated through the common Brownian motion B_{ct}. A_{it} denotes the effort or action taken by the manager of the ith firm, where $E_0 \int_0^T |A_{it}|^2 dt < \infty$. The manager influences the cash flow process through his effort A_{it} in the drift term.[11]

[11] The manager's control A_{it} must be measurable and adapted to his information, which will be described shortly.

$(A_{it}/D_{it}) + \Pi_i$ represents the expected growth rate of the cash flow process, where Π_i is a constant. We interpret Π_i as the intrinsic growth rate of the cash flow process: given the same level of effort, the cash flow of a firm may intrinsically grow faster than that of another firm. For example, the cash flow of a firm in a high-tech sector may grow faster than that of a firm in a more mature sector, partly because of the differences in the nature of their businesses.

For tractability, we have followed Holmström and Milgrom (1987) in assuming that the manager's action does not influence the diffusion rate of the cash flow process. In that model, the output process is given by $A_t dt + \sigma dB_t$, with A_t being the agent's control and σ being a constant.[12] Because we shall solve for optimal effort A_{it}^* in closed form, in equilibrium, it shall become clear that the above D_{it} processes are well-defined semimartingales and that there exist unique solutions to them. An undesirable feature of this cash flow process is that it can take negative values. As a result, the equilibrium stock price can take negative values. One can make the probability of the cash flow reaching negative small by choosing appropriate parameter values, so that the mean is large, and the variance is small in equilibrium.

The riskless asset yields a positive constant rate of return r.

Assumption 2. *The manager of firm i incurs a cumulative cost of* $\int_0^T c_i(t, A_{it})dt$ *associated with managing the firm from time 0 to T. The cost rate is given by a convex function:*

$$c_i(t, A_{it}) = \frac{1}{2}k_i(t)A_{it}^2, \tag{2}$$

where $k_i(t)$ is a deterministic function of time t and where the marginal cost rate increases with the level of effort, which helps guarantee a solution to the manager's maximization problem. $k_i(t)$ may serve as a proxy for a manager's skill, that is, the higher a manager's skill, the lower the value of $k_i(t)$.

At time T, manager i receives a salary of S_T^i from the investors or principals.[13] Managers expend efforts for their respective firms and are barred from trading securities for their own accounts. In other words, the only source of income for managers is from the compensation paid by investors. If we assume, as in Leland and Pyle (1977), Kihlstrom and Matthews (1990), Kocherlakota (1998), Magill and Quinzii (2000),

[12] See also Schättler and Sung (1993). In the absence of equilibrium asset prices, Amihud and Lev (1981), Sung (1995), Guo and Ou-Yang (2003), Ou-Yang (2003), Cadenillas, Cvitanić, and Zapatero (2004), and Dybvig, Farnsworth, and Carpenter (2004) allow the agent's action to affect the volatility of the output process.

[13] We shall use agent and manager, and principal and investor interchangeably.

DeMarzo and Urosevic (2001), Urosevic (2001), and Acharya and Bisin (2003), that the managers' trades are observable to investors, then this assumption can be enforced by a clause in the managerial contract. In other words, investors can solve the trading strategies for managers and enforce them through contracting. It shall be seen that even though managers cannot diversify on their own, the optimal compensation schemes provide each manager with a holding of his own firm for incentive purposes and a fully diversified market portfolio for risk-sharing purposes.

Assumption 3. *There are N managers, one for each of the N firms. Each manager has a negative exponential utility function with a constant risk-aversion coefficient R_a. The manager may be interpreted as a representative of the management team including all top executives of the firm. The exponential utility functions are assumed for tractability of the derivation of optimal contracts. Notice that the N managers are not the same because of different $k_i(t)$'s in their cost functions. It is not essential to assume the same risk-aversion coefficient for managers.*

Given the investor's contract S_T^i, the manager expends effort so as to maximize his own expected utility:

$$\sup_{\{A_{it}\}} E_0\left(-\frac{1}{R_a}\exp\left\{-R_a\left[S_T^i-\int_0^T c_i(t,A_{it})dt\right]\right\}\right), \tag{3}$$

where $\{A_{it}\} \equiv \{A_{it}: 0 \le t \le T\} \in \mathrm{A}_{0,T}$, with $\mathrm{A}_{0,T}$ being the set of measurable processes on $[0, T]$ adapted to the manager's information satisfying $E_0 \int_0^T |A_{it}|^2 dt < \infty$. Following Holmström and Milgrom (1987), we assume that there are no intertemporal consumptions by either investors or managers, that managers receive their compensation at the terminal date only, and that the manager's budget constraint compels him to consume $S_T^i - \int_0^T c_i(t, A_{it})dt$.

Accordingly, we assume that there are no intertemporal dividend payments. Only at the terminal date T do investors receive cash flows, D_{iT}, and simultaneously compensate their managers S_{iT} based on the cash flows generated. Theoretically, we can omit the discount factor for the manager's cost function. Because $k_i(t)$ defined in Equation (2) is a general function of time t, one can always write that $k_i(t) \equiv \mathrm{e}^{-rt}k_i(t)$, with e^{-rt} being the discount factor.

It is also assumed that each firm has one share available for trading and that there is an unlimited supply of the bond.[14] Denote by μ_{ij} the number

[14] Assuming that the bond is in zero-net supply, Section 5 determines the risk-free interest rate endogenously in a one-period model.

of shares for firm i demanded by investor j, μ_{ij} then represents the fraction of firm i owned by investor j.

Assumption 4. *There are N identical investors in our economy. Because the market must clear at all times, in equilibrium, the N identical investors will hold shares in all firms in the economy, that is,* $\mu_{ij}=1/N$, *or* $\sum_{j=1}^{N}\mu_{ij}=1, i=1,2,\ldots,N$.

This assumption is for tractability. Because investors can invest in every stock in the economy, if they are heterogenous, they may want to induce different effort levels for the manager of the same firm, or there may not exist a proper objective function for a manager to follow. Here, we focus on the conflict between investors and managers, thus ignoring the potential disagreement between shareholders themselves. See, for example, DeMarzo (1993) for such a discussion.

All investors have a negative exponential utility function with a constant risk-aversion coefficient R_p. Each investor decides on the number of shares to invest in the riskless bond and the risky stocks and designs incentive contracts for managers to maximize the expected utility over her terminal wealth:

$$\sup_{\{\mu_t,A_t\},S_T^i} E_0\left(-\frac{1}{R_p}\exp\left\{-R_p\left[W_T-\sum_{i=1}^{N}S_T^i(\{\mu_t\})\right]\right\}\right), \tag{4}$$

subject to the managers' participation and incentive compatibility constraints as well as the investor's budget constraint to be stated shortly. Here, $A_t=(A_{1t}, A_{2t}, \ldots, A_{Nt})$, $\mu_t=(\mu_{1j}, \mu_{2j}, \ldots, \mu_{Nj})$, and $S_T^i(\{\mu_t\})$ represents the investor's share of payment to the manager for holding $\{\mu_t\}$shares of the firm. In other words, an investor's payment to a manager depends on her ownership of the firm, to be specified in Assumption 6. A manager's participation constraint (PC) is that given S_T^i and the equilibrium effort vector A_{it}^*,

$$E_0\left(-\frac{1}{R_a}\exp\left\{-R_a\left[S_T^i-\int_0^T c_i\left(t, A_{it}^*\right)dt\right]\right\}\right)\geq-\frac{1}{R_a}\exp(-R_a\varepsilon_{i0}), \tag{5}$$

where ε_{i0} denotes manager i's certainty equivalent wealth at time 0. This constraint affords the manager a minimum level of expected utility that the manager would otherwise achieve elsewhere in the labor market. The manager's incentive compatibility constraint is that given S_T^i, the investor's equilibrium effort vector A_{it}^* satisfies every manager's dynamic maximization problem. That is, A_{it}^* maximizes

$$E_0\left(-\frac{1}{R_a}\exp\left\{-R_a\left[S_T^i-\int_0^T c_i(t,A_{it})dt\right]\right\}\right),\ \forall i. \tag{6}$$

The investor's budget constraint consists of her wealth process W_t and the condition that her terminal consumption is given by the terminal net wealth, $W_T - \sum_{i=1}^N S_T^i$. The investor's wealth process W_t shall be given in later sections where we solve for the investor's and the managers' maximization problems.

Note that, in the Homström–Milgrom (1987) model and many other principal–agent models as well as in almost all of the asset-pricing models under asymmetric information,[15] the consumptions of all agents can be negative. Following those models, we assume that the managers and investors are required to consume their terminal wealth, positive or negative, without the possibility of renegotiation. In the absence of this assumption, managers and investors would prefer to consume nothing if their terminal wealth is negative.[16]

Assumption 5. *Following the traditional asset-pricing literature where exponential utility functions and normal cash flow processes are employed,*[17] *we assume that the equilibrium-pricing function is of the following linear form*:

$$P_{it} = \lambda_{i0}(t) + \sum_{j=1}^N \lambda_{ij}(t)D_{jt}, \tag{7}$$

where the coefficients $\lambda(t)$'s are time-dependent deterministic continuous functions, which shall be verified by the market clearing condition.

It shall become clear that the boundary conditions are given by $\lambda_{i0}(T)=0$, $\lambda_{ii}(T)=1$, and $\lambda_{ij}(T)=0$ when $i\neq j$. Investors pay P_{it} for asset i at time t, with the understanding that they will compensate the manager at time T. When computing asset returns, one must take the manager's compensation into account.

For ease of exposition, we express the stock price vector in a compact form as

$$dP_t \equiv a_P dt + b_P dB_t, \tag{8}$$

[15] There is also asymmetric information regarding the agent's effort in a principal–agent problem, that is, the agent's effort is not observed by the principal.

[16] A more realistic setup would be to impose a nonnegativity constraint on the wealth and price processes, but this constraint would render the solutions for the optimal contract and equilibrium price intractable.

[17] See, for example, Campbell and Kyle (1993), Wang (1993), and He and Wang (1995). As in these articles, the uniqueness of equilibrium will not be discussed.

where a_p is an $(N \times 1)$ column vector with the ith element given by

$$a_{P_i} = \dot{\lambda}_{i0}(t) + \sum_{j=1}^{N} \left\{ \lambda_{ij}(t) A_{jt} + \left[\dot{\lambda}_{ij}(t) + \Pi_j \lambda_{ij}(t) \right] D_{jt} \right\}, \quad \dot{\lambda}(t) \equiv d\lambda/dt,$$

and where b_p represents an $[N \times (N+1)]$ matrix with the ith row given by

$$b_{P_i} = \sum_{j=1}^{N} \lambda_{ij}(t) b_{jD}.$$

Recall that b_{jD} is a row vector with $(N+1)$ elements defined in Equation (1).

Assumption 6. *The compensations to the managers are described by*

$$S_T^i = q_i(T, P_T) + \int_0^T g_i(t, P)dt + \int_0^T h_i(t, P_t)dP_t \equiv \int_0^T dS_t^i. \tag{9}$$

The functions $q_i(T, P_T)$, $g_i(t, P_t)$, and $h_i(t, P_t)$ describe the contract space. We assume that $q_i(T, P_t)$ is a continuous function on $\Re$ and that $g_i(t, P_t)$ and $h_i(t, P_t)$ are continuous functions on $[0, T] \times \Re$. The continuity of these functions ensures that $q_i(T, P_T)$, $g_i(t, P_t)$, and $h_i(t, P_t)$ are appropriately measurable and adapted to the investor's information set at time t given by $\mathrm{F}_t = \{D_s, P_s : s = t\}$.[18] Note that $h_i(t, P_t)$ and P_t denote a row and a column vector, respectively. Within the linear pricing function (7), D_t and P_t are interchangeable in the contract form. Though the pricing function is constrained to be linear, this contract space includes both linear and nonlinear functions.[19]

We interpret dS_t^i in Equation (9) as the investors' share of payment to manager i for holding one share of the stock within time interval $[t, t + dt]$. One may interpret dS_t^i as a negative-dividend payment to the investors for holding the asset within time interval dt. $dP_{it}^{adj} \equiv dP_{it} - \mathrm{e}^{-r(T-t)} dS_t^i$ may then be understood as the investors' capital gains within dt. Consequently, the current stock price will reflect the investors' costs of compensating the manager. This interpretation avoids a price jump right before the terminal

[18] This contract space is adapted from Schättler and Sung (1993) and Ou-Yang (2003). We assume that the standard conditions ensuring the existence of stochastic integrals described in those articles are met.

[19] This contract space excludes certain path-dependent functions of P_t in the coefficients $g_i(t, P_t)$ and $h_i(t, P_t)$, because we shall use the dynamic programming approach to solve both the investor's and the manager's maximization problems.

date, that is, the price of stock i will be D_{iT} rather than $(D_{iT} - S_T^i)$ at time T. For example, if an investor decides to hold a stock for the time interval $[T-dt, T]$ only, she should not be responsible for the entire manager's compensation S_T^i, which rewards the manager for work within the entire contract period $[0, T]$. In equilibrium, however, one will have the condition that $\sum_{j=1}^{N} \mu_{ij}(t) = 1$ or that investors own the firm at all times. Therefore, the investors as a whole will be responsible for the entire compensation to the manager of every firm.

We refer to the model setup as the exponential-normal case.

2. An Exchange Economy in the Absence of Managers

First, consider a benchmark case, in which the A_{it}'s in the cash flow processes are exogenously given or in which there are no managers. The asset prices and expected returns are completely determined by both the cash flow processes and the investors' demand for risky assets.

The excess dollar return for holding asset i within time interval dt is defined as

$$dQ_{it} = dP_{it} - rP_{it}dt.$$

Substituting the linear pricing function (7) for P_{it} into the above expression, one gets

$$dQ_{it} = \left\{ \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \sum_{j=1}^{N} \left[\dot{\lambda}_{ij}(t)\ A_{jt} - \left(r - \Pi_j \right) \lambda_{ij}(t) \right] D_{jt} \right\} dt + \sum_{j=1}^{N} \lambda_{ij}(t) b_{jD} dB_t.$$

For simplicity of notation, we write

$$a_{i0}(t) = \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \sum_{j=1}^{N} \lambda_{ij}(t) A_{jt}; \quad a_{ij}(t) = \dot{\lambda}_{ij}(t) - \left(r - \Pi_j \right) \lambda_{ij}(t).$$

The excess return then takes the following form:

$$dQ_{it} = \left[a_{i0}(t) + \sum_{j=1}^{N} a_{ij}(t) D_{jt} \right] dt + \sum_{j=1}^{N} \lambda_{ij}(t) b_{jD} dB_t \equiv a_{Q_t}^i dt + b_{Q_t}^i dB_t.$$

For convenience, we shall denote by dQ_t the $(\mathrm{N} \times 1)$ column vector of excess returns composed of dQ_{it} and write $dQ_t \equiv a_{Q_t} dt + b_{Q_t} dB_t$. By definition, one has

$$a_{Q_t} = \begin{bmatrix} a^1_{Q_t} \\ a^2_{Q_t} \\ \cdot \\ \cdot \\ \cdot \\ a^N_{Q_t} \end{bmatrix}, \quad b_{Q_t} = \begin{bmatrix} b^1_{Q_t} \\ b^2_{Q_t} \\ \cdot \\ \cdot \\ \cdot \\ b^N_{Q_t} \end{bmatrix}.$$

It is well known that the wealth process of investor n's portfolio is given by

$$dW_{nt} = rW_{nt}dt + \mu_{nt}dQ_t, \tag{10}$$

where μ_{nt} denotes a row vector defined as $\mu_{nt} = [\mu_{1n}, \mu_{2n}, \ldots, \mu_{Nn}]$. In equilibrium, $\mu_{nt} = [1/N, 1/N, \ldots, 1/N]$. It shall be shown that all of the elements in b_{Q_t} are bounded; hence, we have that $E_0\left[\int_0^T |\mu_{nt} b_{Q_t}|^2 dt\right] < \infty$. For simplicity, we omit the subscript n in the expressions for μ_{nt} and W_{nt}. The investor's problem is to choose the number of shares to invest in the risky assets to maximize her expected utility over the terminal wealth:

$$\sup_{\{\mu_t\}} E_0\left[-\frac{1}{R_p}\exp\left(-R_p W_T\right)\right] \qquad \text{s.t. } dW_t = rW_t dt + \mu_t dQ_t.$$

For comparison with the moral hazard case to be solved later, we present a CAPM-type equation for the expected excess return on the stock in the following lemma.

Lemma 1. *In equilibrium, the expected excess dollar return $a^i_{Q_t}$ on stock i is a linear function of the expected excess dollar return $a^M_{Q_t}$ on the market portfolio*:

$$a^i_{Q_t} = \frac{\text{cov}(dP_{it}, dP_{Mt})}{\text{var}(dP_{Mt})} a^M_{Q_t},$$

where the market portfolio is defined as $P_{Mt} = P_{1t} + P_{2t} + \ldots + P_{Nt}$. This is the CAPM relation in terms of the expected excess dollar returns. The expected excess rates of returns on stock i and the market portfolio, $R^i_{Q_t} = (1/P_{it})a^i_{Q_t}$ and $R^M_{Q_t} = \frac{1}{P_{Mt}} a^M_{Q_t}$, also satisfy the CAPM equation:

$$R^i_{Q_t} = \frac{\text{cov}(R_{it}, R_{Mt})}{\text{var}(R_{Mt})} R^M_{Q_t} \equiv \beta R^M_{Q_t},$$

where $R_{it} = dP_{it}/P_{it}$ and $R_{Mt} = dP_{Mt}/P_{Mt}$.[20]

[20] Note that the rate of return is well defined only when prices are positive.

3. A Principal–Agent Economy

We now consider the case in which the drift rate of each cash flow process is controlled by a manager at a cost. We extend the current asset-pricing literature by partially endogenizing the cash flow process. To solve for an equilibrium, we must determine the equilibrium-pricing functions and the optimal contracts simultaneously under the conditions that markets clear and that managers adopt optimal controls for both themselves and investors.

Recall that the investor's maximization problem is subject to the agent's PC and his incentive compatibility constraint. The strategy is to first impose these constraints on the original contract form (9) and then solve the unconstrained investor's problem.

3.1 An expression for the manager's equilibrium compensation

This subsection transforms the original contract form using the manager's PC. In doing so, we provide an expression for manager i's equilibrium compensation to be denoted by $\bar{S}_T^i$ in terms of both the coefficient vectors in the contract form, h, and the manager's value function to be defined shortly. It shall be seen that the manager's value function is defined at the optimal effort A_{it}^*; this equilibrium compensation is well defined and equals the original contract only at A_{it}^*. The inclusion of the manager's value function in this equilibrium compensation is natural, because the manager's PC is satisfied only at the optimal effort level and, at time 0, the manager's value function coincides with his expected reservation utility in equilibrium. In the absence of the manager's incentive compatibility constraint to be imposed in the next subsection, $\bar{S}_T^i$ may not implement the investors' optimal action.

Given the manager's compensation S_T^i in Equation (9), we define a *value function process* $V^i(t, P_t)$ for manager i's maximization problem as

$$V^i(t, P_t) = \sup_{\{A_{iu}\}} E_t\left[-\frac{1}{R_a}\exp\left(-R_a\left\{q_i(T, P_T)+\int_t^T [g_i(u, P_u)-c_i(u, A_{iu})]du + \int_t^T h_i(u, P_u)dP_u\right\}\right)\right],$$

where the conditional expectation is on the manager's information set $\mathrm{F}(t) = \{P_s, s \leq t\}$. Because of the linear relationship between P_t and D_t, D_t is redundant given P_t. Note that

$$-\frac{1}{R_a}\exp\left(-R_a\left\{\int_0^t [g_i(u, P_u) - c_i(u, A_{iu})]du + \int_0^t h_i(u, P_u)dP_u\right\}\right)$$
$$\times V^i(t, P_t) = \sup_{\{A_{iu}\}} E_t\left[-\frac{1}{R_a}\exp(-R_a\{q_i(T, P_T)\right.$$
$$\left.+\int_0^T\left[g_i(u, P_u) - c_i(u, A_{iu})\right]du + \int_0^T h_i(u, P_u)dP_u\right\}\bigg)\bigg]$$

represents manager i's expected utility at time t over the terminal net payoff. Though the manager maximizes the expected utility over the entire contract period, we use the dynamic programming approach, in which we solve the manager's problem backward, namely, we take the optimal solutions between t and T as given when both the optimal equations for $V^i(t, P_t)$ and the optimal manager's effort at t are determined. The key point is that the value function at time 0 corresponds to the manager's expected utility over his total net payoff, which is our original problem. It is assumed that $V^i(t, P)$ is continuously differentiable in t and twice continuously differentiable in P, where $P \in \Re^N$. The solution to $V^i(t, P)$ shall be obtained in closed form, which illustrates that these conditions are met.

The Bellman-type equation for the manager's maximization problem is then given by

$$\begin{aligned} 0 = \sup_{A_{it}} -\Big\{ & R_a V^i(t, P)[g_i(t, P) + h_i(t, P)a_P - c_i(t, A_{it}) \\ & -\frac{1}{2}R_a h_i(t, P)b_P b_P^T h^T(t, P)] \\ & + V_t^i + V_P^i\left[a_P - R_a b_P b_P^T h_i^T(t, P)\right] + \frac{1}{2}tr\left(V_{PP}^i b_P b_P^T\right)\Big\} \end{aligned} \tag{11}$$

Here, the subscripts under $V^i(t, P)$ denote the relevant partial derivatives of $V^i(t, P)$. Using the manager's PC at time 0, the above PDE, Ito's lemma, and a transformation, one can arrive at an expression for the equilibrium compensation S_T^i, which is presented in the next lemma.

Lemma 2. *The manager's equilibrium compensation is given by*

$$S_T^i = \varepsilon_{i0} + \int_0^T c_i(t, A_{it})dt + \frac{R_a}{2}\int_0^T \bar{h}_i b_P b_P^T \bar{h}_i^T dt + \int_0^T \bar{h}_i b_P dB_t, \tag{12}$$

where ε_{i0} denotes manager i's certainty equivalent wealth at time 0, and where $c_i(t, A_{it}) = \frac{1}{2}k_i(t)A_{it}^2$ and $\bar{h}_i \equiv \bar{h}_i(t, P_t) = h_i(t, P_t) - \frac{V_P^T}{R_a V}$.

This equilibrium compensation has an intuitive interpretation. At the terminal date, investors pay the manager a reservation wage, reimburse the manager's effort costs, and compensate the manager in the third term for the risk borne in the fourth term; the fourth term involves Brownian motion processes and represents the risk in the manager's compensation scheme. Note that, if the manager is risk neutral, the third term vanishes, because a risk-neutral manager does not require compensation for bearing risk.[21]

Substituting this equilibrium compensation into the manager's PC (5), we obtain under suitable regularity conditions that[22]

$$E_0\left\{-\frac{1}{R_a}\exp\left[-R_a\left(\varepsilon_{i0} + \frac{R_a}{2}\int_0^T \bar{h}_i b_P b_P^T \bar{h}_i^T\,dt + \int_0^T \bar{h}_i b_P dB_t\right)\right]\right\}$$
$$= -\frac{1}{R_a}\exp(-R_a\varepsilon_{i0}).$$

Therefore, the manager's PC is satisfied, which eliminates the $g_i(t, P_t)$ coefficient from the expression for S_T^i. On the other hand, if the manager has the bargaining power, one can always adjust the constant term ε_{i0}, so that the investor achieves her reservation utility, and the risk-sharing part of the contract would remain intact. Consequently, the optimal effort would be the same, which must satisfy both the investor's and the manager's maximization problems.

Using the equilibrium compensation in Equation (12), investors can now ignore the manager's PC when solving their maximization problems to determine the optimal effort level and trading strategies. This compensation, however, has not incorporated the manager's incentive compatibility constraint, that is, the first-order condition (FOC) of the manager's Bellman equation has yet to be imposed. In the first-best case, because the manager's effort is observed by the investor, one can ignore the manager's maximization problem and simply solve the investor's maximization problem using the equilibrium compensation. It can be shown that the equilibrium is Pareto optimal and that firm-specific risks play no role in the determination of asset prices. As a

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21 See also Holmström and Milgrom (1987), Schättler and Sung (1993), and Ou-Yang (2003).

22 One such regularity condition is Novikov's condition. See, for example, Karatzas and Shreve (1991) and Duffie (2001). It shall become clear that Novikov's condition is satisfied because the equilibrium solutions for the components in vectors $\bar{h}_i$ and b_P are bounded, deterministic functions.

result, the cash flow processes or the production decisions can simply be given exogenously.[23]

We next solve for the optimal contracts, equilibrium prices, and expected excess returns in the second-best case. In this case, we must take the FOC of the manager's Bellman equation into account. It must also be verified that the manager's Bellman equation is satisfied. The verification results for the Bellman equation are similar to those in Schättler and Sung (1993) and Ou-Yang (2003), and shall be discussed in the appendix.

3.2 The valuation relation and relative performance evaluation

This subsection examines the impact of moral hazard on expected asset returns and RPE. We show that in the current exponential-normal case, moral hazard does not alter the expected dollar returns on individual assets. In equilibrium, when the PPS of a manager is higher, investors price the asset higher while demanding the same expected dollar return. In addition, we show that the risk aversion of the investor leads to less emphasis on RPE in the manager's compensation scheme than in a model with a risk-neutral investor.

In this second-best case, we must take managers' Bellman equations and their FOCs into account when solving investors' maximization problems. More specifically, we use the FOCs of the managers' Bellman equations as constraints in the investors' maximization problems and choose the g coefficients in the contract form to ensure that managers' Bellman equations are satisfied.

Imposing the FOCs of the managers' Bellman equations (11), we obtain

$$k_i(t)A_{it} = \bar{h}_{ii}\lambda_{ii}(t) + \sum_{j\neq i}^{N} \bar{h}_{ij}\lambda_{ji}(t) \quad \text{or} \quad \bar{h}_{ii} = \frac{k_i(t)A_{it} - \sum\limits_{j\neq i}^{N} \bar{h}_{ij}\lambda_{ji}(t)}{\lambda_{ii}(t)},$$

where A_{it} and $\bar{h}_{ii}$ shall be determined from the investor's maximization problem. By definition, the $\bar{h}_i$ vector is given by

$$\bar{h}_i = \left[\bar{h}_{i1}\ \bar{h}_{i2}\ \cdots\ \bar{h}_{ii} = \frac{k_i(t)A_{it} - \sum\limits_{j\neq i}^{N} \bar{h}_{ij}\lambda_{ji}(t)}{\lambda_{ii}(t)}\ \cdots\ \bar{h}_{iN} \right].$$

For example, we have that $\bar{h}_1 = \left[\frac{k_1(t)A_{1t} - \Sigma_{j=2}^{N} \bar{h}_{1j}\lambda_{j1}(t)}{\lambda_{11}(t)}\ \bar{h}_{12} \cdots \bar{h}_{1N} \right].$

Once we substitute the $\bar{h}_i$ vector into the expression for the manager's equilibrium compensation (12), the manager's PC as well as his FOC are

[23] We thank a referee for this insight. The detailed results for the first-best case are available from the author on request.

satisfied. Consequently, we can solve the investor's maximization problem for the optimal A_{it} and $\bar{h}_{ij}$ $(i \neq j)$ without any additional constraints. We can then construct optimal contracts in terms of the variables (such as stock prices) that are observed by both investors and managers. Define $\{A_{it}, \bar{h}_{ij}, \mu_t\} \equiv \{A_{it}, \bar{h}_{ij}, \mu_t: 0 \le t \le T\} \in A^I_{0,T}$, with $A^I_{0,T}$ being the set of measurable processes on [0, T] adapted to the investor's information set satisfying $E_0 \int_0^T |A_{it}|^2 dt < \infty$, $E_0 \int_0^T |\bar{h}_{ij}|^2 dt < \infty$, and $E_0 \int_0^T |\mu_t|^2 dt < \infty$.[24]

The investor's problem is now given by

$$\sup_{\{A_{it}, \bar{h}_{ij}, \mu_t\}, i \neq j} E_0 \left[-\frac{1}{R_p} e^{-R_p \left(W_T - \int_0^T \mu_t dS_t \right)} \right],$$

subject to the wealth process, where $\mu_t dS_t$ is defined as

$$\begin{aligned} \mu_t dS_t &= \sum_{i=1}^N \mu_{it} dS_t^i \\ &\equiv \sum_{i=1}^N \mu_{it} \left[\frac{\varepsilon_{i0}}{T} dt + \frac{1}{2} k_i(t) A_{it}^2 dt + \frac{R_a}{2} \bar{h}_i b_P b_P^T \bar{h}_i^T dt + \bar{h}_i b_P dB_t \right]. \end{aligned}$$

We interpret $\mu_{it} dS_t^i = \mu_{it} \left[\left(\frac{\varepsilon_{i0}}{T} + \frac{1}{2} k_i(t) A_{it}^2 + \frac{1}{2} R_a \bar{h}_i b_P b_P^T \bar{h}_i^T \right) dt + \bar{h}_i b_P dB_t \right]$ as investor i's share of payment to manager i for holding μ_{it} share of the stock within time interval $[t, t + dt]$, which represents the investor's cost for holding μ_{it} percent of the firm. This specification implies that the investor is responsible for the manager's compensation, depending on her portion of the ownership in the firm throughout the contract period.

Notice that in the Holmström–Milgrom (1987) model, in which there is only one agent, $\bar{h}_{ij} = 0$, and the only control variable in the investor's problem is A_{1t}. Also notice that in an asset-pricing model in which there are no managers, that is, $\bar{h}_{ij} = 0$ and A_{it} is exogenously given, the only control variables are μ_t. In the current model, however, one must solve a system of N^2 nonlinear equations for A_{it} and $\bar{h}_{ij}$, where $\bar{h}_{ij} \neq \bar{h}_{ji}$ in general. It shall be seen that the RPE term h_{ij} is not entirely determined by the correlation between the two assets.

The next theorem summarizes the key results of this article.

Theorem 1. *When the number of firms in the economy approaches infinity, the optimal effort level and the optimal contract are given, respectively, by*

$$A_{it}^* = \frac{1}{k_i(t) + R_a k_i^2(t) \sigma_{ii}^2} e^{\Pi_i (T-t)}, \tag{13}$$

[24] It shall be seen that these regularity conditions are satisfied by the equilibrium solutions.

and

$$S_T^i = \varepsilon_{i0} + \frac{1}{2}\int_0^T k_i(t)A_{it}^{*2}dt + \frac{R_a}{2}\int_0^T \bar{h}_i b_P b_P^T \bar{h}_i^T dt - \int_0^T \bar{h}_i a_P dt + \int_0^T \frac{k_i(t)A_{it}^*}{\lambda_{ii}(t)} dP_{it} + \sum_{j\neq i}^N \int_0^T \bar{h}_{ij} dP_{jt}, \tag{14}$$

where a_p *and* b_p *are defined in Equation* (8),

$$\begin{aligned} &f(t) = \mathrm{e}^{r(T-t)}, \quad \lambda_{ii}(t) = e^{(\Pi_i - r)(T-t)}, \quad \lambda_{ij}(t) = 0, \quad i \neq j, \\ &\bar{h}_{ij} = \frac{1}{N-1}\left[\frac{R_p}{R_a+R_p}f(t) - \frac{k_i(t)A_{it}^*}{\lambda_{jj}(t)}\frac{\sigma_{ic}}{\sigma_{jc}}\right] \\ &\quad = \frac{\mathrm{e}^{r(T-t)}}{N-1}\left[\frac{R_p}{R_a+R_p} - \left(\frac{1}{1+R_a k_i(t)\sigma_{ii}^2}\right)\frac{\lambda_{ii}\sigma_{ic}}{\lambda_{jj}\sigma_{jc}}\right], \quad i \neq j, \\ &\bar{h}_i = (\bar{h}_{i1}, \bar{h}_{i2}, \cdots, \tfrac{k_i(t)A_{it}^*}{\lambda_{ii}(t)}, \cdots, \bar{h}_{iN}), \quad \forall i. \end{aligned}$$

The expected excess dollar returns on asset i and the market portfolio, adjusted for managers' expected compensation, are given by

$$\begin{aligned} E^i(t) &= \frac{1}{N} R_p f(t) \mathbf{1}_i H b_{Q_t} b_{Q_t}^T H^T \mu_M^T = \frac{\mathrm{cov}\left(dP_{it}^{adj}, dP_{Mt}^{adj}\right)}{\mathrm{var}\left(dP_{Mt}^{adj}\right)} E^M(t) \\ &\equiv \beta^{adj} E^M(t). \end{aligned} \tag{15}$$

Here, $dP_{it}^{adj} = dP_{it} - [1/f(t)]dS_{it}$, $dP_{Mt}^{adj} = \Sigma_{i=1}^N \{dP_{it} - [1/f(t)]dS_{it}\}$, $E^i(t) \equiv a_{Q_t}^i - \frac{1}{f(t)}\left[(\varepsilon_{i0}/T) + (1/2)k_i(t)A_{it}^{*2} + (R_a/2)\bar{h}_i b_{Q_t} b_{Q_t}^T \bar{h}_i^T\right]$, $E^M(t) \equiv a_{Q_t}^M - \frac{1}{f(t)}\Sigma_{i=1}^N\left[(\varepsilon_{i0}/T) + (1/2)k_i(t)A_{it}^{*2} + (R_a/2)\bar{h}_i b_{Q_t} b_{Q_t}^T \bar{h}_i^T\right]$, $\mathbf{1}_i$ *denotes a row vector with the* ith *element being 1 and all other elements being 0, and H is given by*

$$H = \left[I_N - \frac{1}{f(t)} \begin{bmatrix} \bar{h}_1^T \\ \bar{h}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N^T \end{bmatrix} \right], \tag{16}$$

where I_N denotes the identity matrix.

We can also express the expected excess dollar return as

$$E^i(t) = R_p f(t)\frac{1}{N}\mathbf{1}_i H b_{Q_t} b_{Q_t}^T H^T \mu_M^T = R_p f(t)\frac{1}{N}\mathrm{cov}\left[dP_{it}^{adj}, dP_{Mt}^{adj}\right]$$
$$= \frac{R_a R_p}{R_a + R_p} f(t)\frac{1}{N}\left[\lambda_{ii}(t)\sigma_{ic} - \frac{R_p}{N(R_a + R_p)}\sum_{j=1}^{N}\lambda_{jj}(t)\sigma_{jc}\right]\sum_{i=1}^{N}\lambda_{ii}(t)\sigma_{ic}. \tag{17}$$

In a special case in which $r = 0$, $\Pi_i = 0$, and $k_i(t)$ is a constant, we have $f(t) = \lambda_{ii}(t) = 1$. The optimal contract then becomes linear and path independent:

$$\begin{aligned} S_T^i &= \text{constant} + k_i A_i^*(P_{iT} - P_{i0}) \\ &\quad + \frac{1}{N-1}\sum_{j\neq i}^{N}\left[\frac{R_p}{R_a + R_p} - k_i A_i^*\frac{\sigma_{ic}}{\sigma_{jc}}\right](P_{jT} - P_{j0}) \\ &= \text{constant} + k_i A_i^*(P_{iT} - P_{i0}) \\ &\quad + \frac{1}{N}\left[\frac{R_p}{R_a + R_p} - k_i A_i^*\frac{\sigma_{ic}}{\sigma_M}\right](P_{MT} - P_{M0}), \end{aligned} \tag{18}$$

where the optimal effort level A_i^* is a constant given by $A_i^* = (1/k_i)/(1 + R_a k_i \sigma_{ii}^2)$ and where *$N\sigma_M = \sigma_M$ denotes the standard deviation of the market portfolio. In addition, the equilibrium asset price P_{it} increases with the manager's ownership in his own firm, $k_i A_i^*$ (after controlling for k_i) and decreases with the firm-specific risk σ_{ii} (after controlling for k_i and R_a).*

Notice that the PPS, $\bar{h}_{ii} = [k_i(t)A_{it}^*/\lambda_{ii}(t)] = [e^{r(T-t)}/(1 + R_a k_i(t)\sigma_{ii}^2)]$, depends on σ_{ii}, the firm-specific risk of the cash flow, rather than on that of the stock price $\lambda_{ii}(t)\sigma_{ii}$. On the other hand, the RPE, $\bar{h}_{ij} = [e^{r(T-t)}/(N-1)]\{[R_p/(R_a + R_p)] - [1/(1 + R_a k_i \sigma_{ii}^2)](\lambda_{ii}\sigma_{ic}/\lambda_{jj}\sigma_{jc})]\}$, $i \neq j$, depends on the common risk of the stock prices as well as the firm-specific risk of the firm's cash flow. A rigorous empirical test of these results requires a careful specification of idiosyncratic and systematic risks in terms of both the cash flow process and the stock price process.

For simplicity, we use the results for the special case of the theorem in the remainder of this section.

3.2.1 Optimal contracts and RPE

The optimal contract (18) is a linear combination of the asset price and the level of the market portfolio. When investors are risk neutral, that is, $R_p = 0$, the RPE with respect to the market portfolio reduces to $-k_i A_i^*(\sigma_{ic}/\sigma_M) = -[1/(1 + R_a k_i \sigma_{ii}^2)](\sigma_{ic}/\sigma_M)$. To filter out the common risk from a risk-averse manager's compensation, a negative (positive) RPE must be used if the cash flows of a firm are positively (negatively) correlated with the market portfolio. Notice that the magnitude of RPE is determined by both the firm's exposure to systematic risk and its idiosyncratic risk rather than by only the firm's correlation with the market portfolio. A firm may be more highly correlated with the market portfolio, but the RPE for the manager of this firm can still be lower than that for the manager of another firm that has a lower correlation with the market portfolio, if the first firm has a higher idiosyncratic risk. In addition, the magnitude of RPE decreases with the standard deviation of the market portfolio.

When investors are risk averse, however, they would like to share the common risk with managers, resulting in a positive component in the RPE term. This risk-sharing need reduces the magnitude of RPE in managers' compensation, which can even be positive if the firm has a sufficiently high-idiosyncratic risk or a sufficiently low-systematic risk.

In equilibrium, the manager's compensation (18) is equal to

$$\begin{aligned} S_T^i &= \text{constant} + k_i A_i^* P_{iT} + \sum_{j\neq i}^{N} \frac{1}{N-1}\left[\frac{R_p}{R_a+R_p} - k_i A_i^* \frac{\sigma_{ic}}{\sigma_{jc}}\right] P_{jT} \\ &= \text{constant} + k_i A_i^* (\sigma_{ic} B_{cT} + \sigma_{ii} B_{iT}) + \frac{1}{N-1}\frac{R_p}{R_a+R_p}\sum_{j\neq i}^{N} P_{jT} \\ &\quad - k_i A_i^* \sigma_{ic} B_{cT} \\ &= \text{constant} + k_i A_i^* \sigma_{ii} B_{iT} + \frac{1}{N}\frac{R_p}{R_a+R_p}\sum_{j=1}^{N} P_{jT}. \end{aligned} \tag{19}$$

Here, we have used the relation $P_{iT} = \text{constant} + \sigma_{ic}B_{cT} + \sigma_{ii}B_{iT}$[25] and $[1/(N-1)]\sum_{j\neq i}^{N} k_i A_i^*(\sigma_{ic}/\sigma_{jc})P_{jT} = \text{constant} + k_i A_i^* \sigma_{ic} B_{cT} + [1/(N-1)] \sum_{j\neq i}^{N} k_i A_i^*(\sigma_{ic}/\sigma_{jc})\sigma_{jj}B_{jT} \to \text{constant} + k_i A_i^* \sigma_{ic} B_{cT}$, where, according to the Law of Large Numbers, the third term vanishes when $N \to \infty$. Note that B_{jT}'s are independent Brownian motions and that $k_i A_i^*(\sigma_{ic}/\sigma_{jc})\sigma_{jj}$ is bounded. Recall that B_{cT} and B_{iT} denote the common and firm-specific Brownian motions at time T.[26] This equilibrium compensation illustrates that investors and managers share the market portfolio optimally according to their risk-bearing capacity.

Investors provide managers with incentives through the firm-specific Brownian motion term. Without this incentive, the manager would not

[25] Note that, in this case, in which $r = \Pi = 0$, both the drift and the diffusion terms of the cash flow process are constant in equilibrium.

[26] Note that this equilibrium compensation is equal to the optimal contract only at the optimal effort level A_i^* and that it is not an enforceable contract, because B_{iT} is not observed by investors.

exert costly effort. Because of the optimal sharing of the common risk, the equilibrium effort level does not depend on it.[27] Also, investors behave as if they were risk neutral in inducing the equilibrium effort, because they can fully diversify away firm-specific risks.[28] This is the reason that the investor's risk-aversion coefficient does not appear in the expression for A_i^*.

Empirical studies have shown that the sensitivity of CEO compensation to the total firm value is growing but typically very small. Jensen and Murphy (1990) find the sensitivity to be about 0.3%. Hall and Liebman (1998) find its median and mean to be 0.5% and 2.5%, respectively. The low sensitivity of CEO compensation does not mean, however, that CEOs would simply shirk. Though a CEO's compensation may be small compared with the firm value, it typically represents a sizable portion of his total wealth, which imposes a large consumption risk on the CEO. Through calibration of agency models, Haubrich (1994) demonstrates that small sensitivity of an agent's compensation can induce the agent to exert proper effort under reasonable risk aversion parameter values for the agent. See also Hall and Liebman (1998) for qualitative arguments about this point.

In the model, $k_i A_i^*$ denotes a manager's PPS, where A_i^* denotes the manager's equilibrium effort. We next illustrate that, given a small PPS, the manager's effort can still be very large. Suppose that $k_i A_i^* = 1.0\%$, $R_a = 0.001$,[29] and $\sigma_{ii}^2 = 10^{17}$.[30] From $k_i A_i^* = [1/(1 + R_a k_i \sigma_{ii}^2)] = 0.01$, we obtain that k_i is approximately given by $k_i = 10^{-12}$. Therefore, we can back out the manager's effort from $k_i A_i^* = 0.01$, yielding $A_i^* = 10^{10}$, which is indeed a very large number.[31] Notice that this simple example should not be construed as a rigorous calibration of the model. To do so, one must determine R_a and k_i from the empirical data directly. Here, the point is that a small PPS can induce a high level of effort.

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[27] This result is the same as in Holmström (1982) for a risk-neutral principal. Jin (2002) develops a one-period model, in which there is one riskless asset and one risky asset. Assuming that the CAPM holds and that the contract is of a linear form without RPE, he shows that, if the manager is allowed to invest in the market portfolio, then the manager's effort is a function of the firm-specific risk alone. See also Garvey and Milbourn (2003).

[28] As long as the manager is risk averse, there always exists an agency problem even if the investor is risk neutral.

[29] Although there is no consensus on a realistic value of the risk-aversion coefficient, it should be a very small number. For example, if $R_a = 1$, then the utility function of $-(1/R_a)e^{-R_a C}$ would fast approach the maximum of 0 even for a consumption of $1000, where C denotes the consumption. It shall become clear that a larger value of R_a strengthens our argument.

[30] Table 2 of Baker and Hall (2002) summarizes a median variance of market value of 1.4×10^{17} for the 1996 ExecuComp sample. Using the CAPM as the benchmark model, Ed Fang finds the median idiosyncratic risk to be of order of 10^{17} for the CRSP sample from 1992 to 2000. We thank him for the information.

[31] Given a large effort, the manager's cost is high, but this cost is covered by investors in equilibrium.

3.2.2 Expected returns, idiosyncratic risk, and managerial incentives

The CAPM Equation (15) in Theorem 1 holds for a finite number of assets, N, where markets are incomplete.[32] When N is finite, however, idiosyncratic shocks affect asset prices and expected asset returns, because investors hold undiversified portfolios. To isolate the effect of moral hazard, we take the limiting case in which $N \to \infty$ in the derivation of asset prices and optimal contracts. Mathematically, we omit all individual terms that involve $1/N$ in the derivation.

According to Theorem 1, the expected excess dollar return depends only on the covariance between the firm value and the market portfolio, both of which are adjusted for the expected compensation to managers. Because A_i^*, which is a function of the firm-specific risk σ_{ii}, does not appear in front of the common risk term, the expected excess return is independent of σ_{ii}. In other words, because the incentive part, $k_i A_i^* B_{iT}$, is completely separate from the term that involves systematic risk, the firm-specific risk does not contribute to the value of the covariance.

For incentive purposes, the manager is required to bear idiosyncratic risk in the amount of $k_i A_i^* B_{iT}$. Its certainty equivalent wealth for the risk-averse manager is given by $(1/2)R_a\left(k_i A_i^* \sigma_{ii}\right)^2$, which is the cost to the investor for providing incentives. The investor's marginal cost with respect to the manager's effort is then given by $R_a \sigma_{ii}^2 k_i^2 A_i^* = R_a k_i \sigma_{ii}^2 / 1 + k_i R_a \sigma_{ii}^2$, which increases with σ_{ii}. It seems puzzling at first that the firm-specific risk has no impact on the expected excess dollar return given that the investor's cost of providing incentives increases with it. In equilibrium, however, investors simply lower the asset price in anticipation of a higher cost of providing incentives when the firm-specific risk is higher. This inverse relationship between size and idiosyncratic risk is consistent with the empirical finding of Malkiel and Xu (1997) that idiosyncratic volatility for individual assets is strongly (negatively) related to the size of the firm.[33]

If we define the risk premium on an asset as the ratio of its expected excess dollar return to its price, that is, $E^i(t)/P_{it}$ where we consider only the situation when $P_{it} > 0$, then the risk premium decreases with the sensitivity of the manager's compensation $k_i A_i^*$. For example, an investor purchases an asset for P_{i0} at time 0 and holds it until time T. She expects to receive a net profit of $P_{iT} - S_T^i$ after manager's compensation. E^i is the expected excess dollar return on the asset adjusted for both the investor's risk aversion and the manager's expected compensation, which is independent of $k_i A_i^*$. Because P_{i0} increases with $k_i A_i^*$, the firm's exposure to systematic risk in terms of the percentage return, σ_{ic}/P_{i0}, decreases. As a result, the risk premium on the asset, E^i/P_{i0}, decreases with $k_i A_i^*$. The key

[32] There are $N+1$ Brownian motions, but there are only N risky assets available for trading. Markets are always incomplete from a manager's perspective, because he cannot trade his own stock continuously.

[33] Recall that there is one share of asset outstanding, the asset price of a firm represents its total market value.

point here is that managerial incentives lead to changes in the exposure of percentage returns to systematic risks. In addition, the expected rate of return on the asset is given by $(rP_{it} + E^i)/P_{it} = r + (E^i/P_{it})$, which again decreases with its manager's PPS. This result arises, because the expected excess dollar return remains the same, but the asset price increases with $k_i A_i^*$.

Notice that idiosyncratic risk σ_{ii} may affect expected asset returns through the sensitivity of the manager's compensation, $k_i A_i^*$. After controlling for k_i and R_a, there exists a positive relationship between an asset's return and its idiosyncratic risk. Empirically, Campbell et al. (2001) find that there has been a noticeable increase in firm-level volatility relative to market volatility, and Malkiel and Xu (2000) and Goyal and Santa-Clara (2003) demonstrate that idiosyncratic risk may contribute to expected asset returns. Our theoretical model provides a potential mechanism for incorporating idiosyncratic risk into expected asset returns and may help explain the findings of Malkiel and Xu and Goyal and Santa-Clara.[34]

If we use k_i in the manager's cost function as a proxy for the manager's skill or experience, then its impact on expected asset returns is ambiguous. On the one hand, the higher the manager's skill, the lower the value of k_i; hence, the investor's marginal cost of providing incentives, $R_a k_i \sigma_{ii}^2/(1 + k_i R_a \sigma_{ii}^2)$, is lower. On the other hand, the higher the manager's skill, the higher the manager's reservation wage ε_{i0}. If the manager has the bargaining power, then the investor may end up with fewer dividends from a firm managed by a more-skilled manager than those from a firm managed by a less-skilled manager. If this occurs, the equilibrium price of the former firm may be lower than that of the latter firm. Consequently, the asset return on the former firm may be higher or lower than that on the latter firm, depending on who has the bargaining power. The impact of idiosyncratic risk on expected asset returns, however, does not depend on the bargaining power of the parties involved, because the manager's reservation wage is independent of it.

If there is no systematic risk, that is, $\sigma_{ic} = 0 \ \forall i$, then the expected excess dollar return reduces to 0. Because the investor is fully diversified, she behaves as if she were risk neutral toward firm-specific risk. Consequently, both the expected excess dollar return adjusted for the manager's expected compensation and the risk premium are zero for all assets. Moral hazard in this case decreases the asset price but has no impact on the expected rate of return. This example illustrates that firm-specific risk affects expected asset returns only through systematic risk. In other words, individual firm characteristics such as idiosyncratic risk associated with agency considerations do not serve as independent risk factors. As a result, if shareholders are risk neutral and do not discount systematic risk,

[34] O'Hara (2003) and Easley and O'Hara (2004) show that information risk may provide an explanation for why idiosyncratic risks matter for asset pricing.

then managers' compensation and firm-specific risks do not affect expected asset returns, even though the moral hazard problem still exists. Theorem 1 shows that when investors are risk neutral, the expected excess dollar returns for all risky assets are zero. Consequently, their expected rates of return are given by the risk-free rate. This point highlights the importance of generalizing the previous literature represented by Diamond and Verrecchia (1982) and Holmström (1982) in which principals are risk neutral. In the absence of a multi-asset equilibrium model with risk-averse principals, this literature suggests that moral hazard may affect expected asset returns.

Because the empirical evidence for RPE is mixed and some firms do compensate managers based only on the performance of their own firms, we have also examined the impact of this contracting restriction on expected asset returns. In this case, we remove the RPE term from the original contract space (9), that is, $\bar{h}_{ij}=0$ when $i \neq j$ and solve for expected asset returns and optimal contracts in equilibrium. We find that the basic result that moral hazard affects expected asset returns still holds. The detailed calculations are available on request.

4. Further Discussion

4.1 Log-normal cash flow processes and general utility functions

For tractability of the derivation of optimal contracts and equilibrium prices, we have so far introduced two assumptions, namely, the normality of cash flow processes and the exponential utility functions for both managers and investors. These assumptions are widely adopted in the principal–agent models as well as the asset-pricing models under asymmetric information.

Another set of assumptions commonly used in the asset-pricing literature is that investors have power utility functions and cash flow processes are log-normal. Naturally, one may wonder whether the result that managerial incentives affect expected asset returns would change under these assumptions. It is well known that the optimal contracting problem under these assumptions becomes intractable. Consequently, we cannot address this issue rigorously.

Even though a closed-form solution for the optimal contract is unavailable under these new conditions, the result regarding RPE is unlikely to change. As long as investors are risk averse, it is not optimal for them to bear all of the systematic risk, and they would like to share it with the managers, just as in the exponential-normal case. The same trade-off between sharing systematic risk with managers and removing it from managers' compensation exists, because systematic risk is inferable and beyond the control of the managers. Consequently, the magnitude of

RPE will be smaller than that under a risk-neutral investor, and it may still even be positive. We next argue heuristically with a simple example that even if the agent's effort affects the rate (as opposed to the mean) of a log-normal cash flow process, managerial incentives may still affect expected asset returns.

Suppose that there are many risky assets and one risk-free bond, that the time horizon is finite, [0, T], and that the cash flow generating processes are log-normal:

$$\frac{dD_{it}}{D_{it}} = A_{it}dt + \sigma_{ii}dB_{it} + \sigma_{ic}dB_{ct} \equiv A_{it}dt + \sigma_i dB_t, \quad i = 1, 2, \cdots, N, \tag{20}$$

where A_{it} denotes the manager's effort, $\sigma_i \equiv (\sigma_{ii}, \sigma_{ic})$ is a constant vector, and the transpose of B_t is defined as $B_t^T = (B_{it}, B_{ct})$, with the two Brownian motions being independent of each other. We interpret σ_{ii} and σ_{ic} as the exposure of the percentage return of firm i's cash flow to idiosyncratic risk and systematic risk, respectively. Notice that the volatility of the percentage return of the cash flow as defined in Equation (20) is a constant, whereas in the normal case as specified in Equation (1), the volatility of the level (as opposed to the percentage return) of the cash flow is a constant. For simplicity, we assume that manager i and the investors share the terminal cash flow D_{iT} at time T without intertemporal payments or consumption, where D_{iT} is generated by Equation (20). We further assume that investors and managers posses power utility functions, $X_l^{\gamma_l}/\gamma_l$, $l = p, a$, where X_p and X_a denote the investors' and the managers' terminal wealth or consumption, and γ_p and γ_a are their constant relative risk-aversion coefficients, respectively. For notational convenience, we omit the subscripts i and l in the following argument.

For comparison, first, consider the case in the absence of managers. Denote by δ_t the cost of capital or the risk-adjusted discount rate for the cash flow of a firm. Assume that δ_t is a deterministic function of time t. The stock price $P_t \equiv P(t, D_t)$ at time t is then defined as follows:

$$\begin{aligned} P_t &= E_t\left[\exp\left(-\int_t^T \delta_u du\right) D_T\right] = \exp\left(-\int_t^T \delta_u du\right) D_t \\ &\quad \times E_t\left\{\exp\left[\int_t^T A_u du - \frac{|\sigma|^2}{2}(T-t) + \sigma(B_T - B_t)\right]\right\} \\ &= \exp\left[\int_t^T (A_u - \delta_u) du\right] D_t, \end{aligned} \tag{21}$$

where E_t denotes investors' conditional expectation based on their information set at time t, and $|\sigma|^2 = \sigma_{ii}^2 + \sigma_{ic}^2$. Using Ito's lemma, the stock price process is then given by

$$\frac{dP_t}{P_t} = \delta_t dt + \sigma dB_t. \tag{22}$$

By definition, δ_t is the expected rate of return of the stock at time t. The key points of this exercise are that the volatility σ of the percentage change in the stock price, dP_t/P_t, is the same as the volatility of the percentage change in the original cash flow process, dD_t/D_t; that the drift rate A_t does not affect the volatility of the stock price return process; and that the stock price also follows a log-normal process. As a result, the risk premium of the stock is determined by σ, and it is independent of both A_t and the level of the stock price.

When we introduce moral hazard or separation of ownership and control into the above asset-pricing problem, both the optimal contract and the equilibrium price become intractable to obtain under the log-normal cash flow process. Even though the total cash flow follows a log-normal process, the portion that belongs to investors, which is the total cash flow less the compensation to the manager, is no longer log-normal. In general, the stock price may not follow the same factor model and distribution as the original cash flow process. To illustrate this point, consider a simple linear contract form. That is, the manager receives a fixed cash payment plus a fraction of the cumulative cash flow at the terminal date, $a + bD_T$, where a and b ($0 \leq \mathrm{b} \leq 1$) are assumed to be constants. For simplicity, we also assume that the manager's optimal effort A_t is a deterministic function of time t. In other words, the manager is confined to choose his effort only from this narrow effort space to maximize the expected utility. Consequently, the cash flow still follows a log-normal process.

Even under this simplified contract form and effort space, both the investor's and the manager's problems are still intractable. For example, the manager's maximization problem is given by

$$\sup_{\{A_t\}} \frac{1}{\gamma_a} E_0 \left\{ \left[a + bD_T - \frac{1}{2} \int_0^T k(t) A_t^2 dt \right]^{\gamma_a} \right\},$$

where $(1/2)\int_0^T k(t)A_t^2 dt$ represents the total cost to the manager associated with his continuous effort, and $a + bD_T - (1/2)\int_0^T k(t)A_t^2 dt$ thus represents the manager's net wealth or consumption at the terminal date. The investor faces a similar dynamic maximization problem that involves the determination of both the optimal linear contract and the equilibrium stock price.[35] It is unclear how a Bellman-type equation can be derived for the dynamic maximization problem. Even if one can somehow derive the partial differential equations (PDEs) for investor and manager, it may still be intractable analytically or numerically to solve for the optimal constants a and b in the contract form and the equilibrium stock price. To do so, one must ensure that the contract and price satisfy the investor's and the manager's PDEs, their FOCs, and the manager's PC. One would then have to extend the one-firm solutions to a large number of firms to address the issue whether diversifiable risks affect expected asset returns. But fortunately, the arguments that follow do not require a specific set of solutions to the contracting-pricing problem and will apply to any constants a and b including the optimal ones.

Suppose again that δ_t is the risk-adjusted discount rate of return and that it is a time-dependent function. The stock price is then defined as follows:[36]

$$\begin{aligned} P_t &= E_t\left\{\exp\left(-\int_t^T \delta_u du\right)[(1-b)D_T - a]\right\} \\ &= \exp\left(-\int_t^T \delta_u du\right)\left[(1-b)\exp\left(\int_t^T A_u du\right)D_t - a\right], \end{aligned} \tag{23}$$

or

$$P_t + \bar{a}_t = \exp\left[\int_t^T (A_u - \delta_u)du\right](1-b)D_t,$$

where $\bar{a}_t \equiv \exp\left(-\int_t^T \delta_u du\right)a$ represents the present value of the manager's cash compensation at time t to be payable at the terminal date. Applying Ito's lemma, we obtain the rate of return process as

$$\frac{dP_t}{P_t} = \delta_t dt + \left(1 + \frac{a_t}{P_t}\right)\sigma dB_t, \tag{24}$$

which, unlike the cash flow process, no longer follows a log-normal process. Recall that $\sigma dB_t = \sigma_{ii} dB_{it} + \sigma_{ic} dB_{ct}$. Even though we do not know how to deduce the exact formula for the expected rate of return δ_t, it must depend at least on the firm's exposure to the systematic shock, $[1 + (a_t/P_{it})]\sigma_{ic} dB_{ct}$, which cannot be diversified away by holding the market portfolio. For example, σ_{ic} affects both the expected dollar return

[35] Even under the linear contract space, the equilibrium stock price is most likely nonlinear.

[36] The cash flow that goes to investors at time T is given by $D_T - a - bD_T = (1-b)D_T - a$. Because investors trade in the market, they determine the stock price, which is given by the discounted value of the terminal payoff at time t.

and the expected rate of return in the exponential-normal case. Similarly, $[1 + (a_t/P_{it})]\sigma_{ic}$ should affect the expected rate of return in the current setting. In a typical principal–agent model, the constant compensation, a, consists of both the agent's cash salary and his costs associated with managing the firm, $1/2 \int_0^T k(t) A_t^{*2} dt$. If we interpret the agent as the representative of all top executives of the firm, the costs can be substantial. For example, in a rough calibration of the current model presented in Subsection 3.2.1, we obtained that $k = 10^{-12}$ and $A^* = 10^{10}$. The cost is then given by $0.5 \times 10^{-12} \times 10^{-12} = 0.5 \times 10^8$. Here the point is that $\bar{a}/P_t$ in Equation (24) may be significant.

Because the manager's effort influences the level of the cash flow, which in turn affects the level of the stock price, managerial incentives must matter for the expected asset returns, as in the exponential-normal case solved in detail in the previous section. Even though idiosyncratic risk can be diversified away by holding the market portfolio, it affects the manager's incentives and thus the level of the stock price. Therefore, idiosyncratic risk affects the expected asset returns through the level of the stock price, as in the exponential-normal case. In addition, the volatility of the stock-return process decreases with the level of the stock price, and it is more volatile than the return process of the cash flow. The phenomenon that the variance of percentage returns and dividend growth rates increases as the level of prices and dividends falls has been noted in both aggregate U.S. market data and individual stock data. See, for example, Black (1976), Schwert (1989), Nelson (1991), and Cho and Engle (1999). See also Campbell and Kyle (1993) for a detailed discussion about this point.

Intuitively, the result that managerial incentives affect expected asset returns should still hold even if we introduce nonlinear terms into the linear contract form. There will be more terms in the expression for the stock price P_t, which is related to the level of the cash flow D_t in a nonlinear manner. As a result, there will be more nonlinear functions of P_t in the diffusion term of the stock price process, and these additional terms in the volatility of the stock-return process will also be functions of P_t. In general, suppose the equilibrium stock price is given by a general polynomial function of D_t (as opposed to the linear form in the example), that is, $P_t = a(t) + b(t)D_t + c(t)D_t^2 + d(t)D_t^3 + \cdots$, where the coefficients are functions of time t. It is easy to see that the stock price does not follow a log-normal process and that the volatility of the stock return process, dP_t/P_t, is a function of the level of P_t. As long as the volatility of the stock return process depends on P_t, which is partially determined by the managerial contract, incentives would matter for expected asset returns. Also, for tractability of taking the expectation in Equation (23), we have restricted the manager's effort to be deterministic. Even if the effort space includes stochastic functions, it can be seen that the stock price P_t in Equation (23) will not be log-normal, because with a stochastic A_t in the drift term, the cash flow process is no longer log-normal. Furthermore, as long as the manager receives a cash compensation a, the stock price will not be log-normal, and its volatility will depend on the stock price itself.

Our arguments that incentives affect expected asset returns are clearly not dependent on any specific utility functions for investors and managers. We used the power utility function merely to demonstrate the difficulty of solving optimal contracts and equilibrium stock prices. It should be apparent that the result that the volatility of the stock-return process depends on the level of the stock price is independent of the utility functions involved. For example, under the linear contract form, different utility functions would lead to different values of the constants a and b, but our arguments are independent of the specific values of a and b. Furthermore, the result would still hold when the number of risky assets in the economy approaches infinity. In this case, investors who hold the entire market in equilibrium can diversify away individual firm-specific risks, but managers cannot fully hedge against their own firm risks. Managerial incentives that depend on firm-specific risk affect managerial effort and thus affect the level of stock prices. Because the level of stock prices affects stocks' exposure to the common risk that cannot be diversified away, managerial incentives and firm-specific risks affect expected asset returns. In other words, managerial incentives affect expected asset returns through their influence on common risk rather than serve as independent risk factors.

Although the arguments seem to be general and plausible, we must stress that they do not constitute a rigorous proof. It would be of great importance and challenge to formalize them in a systematic framework, such as the one developed for the exponential-normal case. The best strategy is perhaps to first extend the current principal–agent literature to incorporate more general utility functions, such as the power utility function, as well as more general output processes, such as the log-normal process. One can then extend a dynamic CAPM to incorporate moral hazard, using log-normal cash flow processes and power utility functions.[37]

4.2 Endogenous interest rates

For tractability, it has been assumed that the risk-free interest rate is exogenous to the model and that the risk-free bond has an unlimited supply. Although these assumptions are widely adopted in financial economics, it would enhance the generality of the current model to

[37] See, for example, Merton (1973), Rubinstein (1976), Lucas (1978), and Breeden (1979) for dynamic asset-pricing models.

discuss the impact of these assumptions on our main results. Qualitatively, it is perhaps easy to argue that relaxing these assumptions will not significantly alter the effect of managerial incentives or idiosyncratic risks on a stock's expected rate of return as well as the magnitude of RPE. In our model, the stock price itself affects the exposure of the firm's cash flow to the systematic shock, and thus affects the expected rate of return. It appears to be intuitive that managerial incentives and idiosyncratic risks always affect stock prices, whether the interest rate is endogenous or exogenous. In addition, because the result on RPE is due to the investor's risk aversion, that is, a risk-averse investor would want to share systematic risk with the manager, the result that the magnitude of RPE is smaller or may even be positive is unlikely to change when the interest rate is endogenized. We are unable to verify our intuition in a continuous-time framework.[38] Instead, this subsection shows that, in a one-period model, in which the contract space is confined to be linear, our results still hold under the conditions that the interest rate is endogenously determined and that the bond is in zero-net supply.

Assume that the cash flow of firm i is given by

$$D_i = A_i + \sigma_{ii}\varepsilon_i + \sigma_{ic}\varepsilon_c; \quad i = 1,2,\cdots,N, \tag{25}$$

where A_i denotes manager i's one-time effort and where ε_i and ε_c denote the idiosyncratic shock and systematic shock, respectively. ε_i and ε_c are independent, standard normal variables. Also assume that the contract for manager i is of the linear form given as

$$S_i = g_i + \sum_{j=1}^{N} h_{ij} D_j, \tag{26}$$

where g_i and h_{ij} are constant coefficients to be determined in equilibrium. The risk-free bond is in zero-net supply. All other assumptions are similar to those in the continuous-time model as specified in Section 1.

The following theorem summarizes the main results in this one-period economy.

Theorem 2. *When the number of firms in the economy approaches infinity, the optimal effort level and the optimal contract are given, respectively, by*

$$A_i^* = \frac{1}{k_i}\left(\frac{1}{1 + R_a k_i \sigma_{ii}^2}\right), \quad h_{ii} = k_i A_i^*,$$

[38] The techniques developed in Liu (2001) may be useful in future extensions of our model to incorporate more general utility functions and stochastic interest rates.

$$g_i = \varepsilon_{i0} - \sum_{j=1}^{N} h_{ij} A_j^* + \frac{1}{2} k_i A_i^{*2} + \frac{1}{2} R_a \sum_{j=1}^{N} h_{ij}^2 \sigma_{ji}^2 + \frac{1}{2} R_a \left(\sum_{j=1}^{N} h_{ij} \sigma_{jc} \right)^2,$$

$$h_{ij} = \frac{1}{N} \left(\frac{R_p}{R_a + R_p} - \frac{k_i A_i^* \sigma_{ic}}{\sigma_{jc}} \right), \quad i \neq j.$$

The equilibrium asset price and interest rate are given, respectively, by

$$P_{i0} = \frac{1}{1+r} \left(\mathrm{F} + \frac{1}{2} A_i^* \right); \quad r = \frac{1}{W_0} \left[\mathrm{F} + \frac{1}{2N} \sum_{i=1}^{N} A_i^* \right] - 1,$$

where W_0 denotes the investor's initial wealth and where F is a constant defined in the Appendix, which is independent of idiosyncratic shock σ_{ii}.

Denote by $E[\Delta P_i^{adj}]$ and $E[\Delta P_M^{adj}]$ the expected excess dollar returns on asset i and the market portfolio, adjusted for managers' expected compensation. The CAPM-type linear relation is given by

$$E\left[\Delta P_i^{adj}\right] = \beta^{adj} E\left[\Delta P_M^{adj}\right],$$

where β^{adj} is given by $\beta^{\mathrm{adj}} = \mathrm{cov}\left(\Delta \mathrm{P}_{\mathrm{M}}^{\mathrm{adj}}, \Delta \mathrm{P}_{\mathrm{i}}^{\mathrm{adj}}\right) / \mathrm{var}\left(\Delta \mathrm{P}_{\mathrm{M}}^{\mathrm{adj}}\right)$.

It can be seen that all of the results obtained in the continuous-time model remain essentially unchanged. For example, an asset's expected dollar return is still independent of its idiosyncratic risk, and its equilibrium price decreases with it. As a result, the risk premium or the expected rate of return of the asset decreases with the manager's pay-performance incentive and increases with its idiosyncratic risk. Because of the risk aversion of the investor, the magnitude of RPE (h_{ij}) is smaller than that under a risk-neutral investor, and it may even be positive. Although the current one-period model endogenizes the interest rate in equilibrium, an important limitation of this model is that the contract space is confined to be linear. Some of the results such as the CAPM-type relation may not be valid under a more general contract space. While taking the interest rate to be exogenous, the continuous-time model accommodates a more general contract space. Therefore, the two models should be viewed as complementing each other.

4.3 Agency models and their empirical tests

In the absence of equilibrium asset pricing, previous agency models establish various results, such as the negative relationship between pay-performance sensitivities (incentives) and the risk of outputs (cash flows) as well as the existence of RPEs, in terms of the properties of the cash flows. Empirical testing of these results, however, has typically employed firms' market values and their volatilities. Also, empirical testing has often employed the total risk of a firm's market value rather than the idiosyncratic risk of it. It is well known that a higher total risk of a firm does not mean a higher idiosyncratic risk. Our model predicts that PPS depends on the exposure of a firm's cash flow rather than its market value to the firm-specific risk and that RPE depends on the exposure of firms' market values to both systematic risk and firm-specific risk. This article demonstrates that a more volatile cash flow process does not necessarily result in a more volatile stock price process. For example, in the exponential-normal case, the idiosyncratic risk part of the volatility of the *i*th stock price is given by $\lambda_{ii}(t)\sigma_{ii} = \mathrm{e}^{(\Pi_i - r)(T-t)}\sigma_{ii}$, where Π_i is part of the growth rate of the cash flow process and σ_{ii} is the idiosyncratic risk of the cash flow process. Thus, the idiosyncratic risk of the stock price depends on this growth rate, and a higher σ_{ii} does not necessarily mean a higher $\lambda_{ii}(t)\sigma_{ii}$ without controlling for π_i. In the case of a log-normal cash flow process, the idiosyncratic risk of the *i*th stock price return process is given by $[1 + (a_{it}/P_{it})]\sigma_{ii}$, where σ_{ii} is the idiosyncratic risk for the return process of the cash flow. Again, a higher σ_{ii} does not necessarily lead to a higher idiosyncratic risk for the stock return process.

Consequently, any empirical test of agency models is perhaps flawed in the absence of an asset-pricing consideration and without distinguishing between the properties of cash flows and those of market prices. A thorough empirical test must incorporate an asset-pricing model that clearly defines systematic risk and idiosyncratic risk in terms of both market prices and cash flows.

5. Conclusion

This article develops an integrated model of asset pricing and moral hazard. It extends the asset pricing literature to incorporate a moral hazard problem. It also extends previous multi-agent principal–agent models by allowing risk-averse principals who can trade in a securities market. We show that in the exponential-normal case, the CAPM linear relation still holds in the presence of moral hazard, with the returns being adjusted for managers' expected compensation. In particular, the coefficient β^{adj} is still defined as the ratio of the covariance between the adjusted return on an asset and that on the market portfolio to the variance of the adjusted return on the market portfolio. We study the impact of managers' compensation on both asset prices and expected returns in equilibrium. We also examine the magnitude of RPE in the presence of risk-averse principals.

We show that the risk aversion of the principal reduces the magnitude of RPE in managers' compensation. Unlike in the previous models, where the principal is risk neutral, the risk-averse principal in this model does not want to remove systematic risk entirely from managers' compensation.

Under certain conditions, the manager's compensation consists of a fixed salary, a fraction of his own firm's performance, plus a fraction of the performance of the market portfolio, the latter of which allows the manager and investor to share the market-wide risk optimally based on their risk-bearing capacity. The coefficient in front of the market portfolio, which measures RPE, does not have to be negative as predicted for a risk-neutral principal. It can be either positive or negative. Thus, when cross-sectional regressions for the test of RPE are performed, negative, positive, or insignificant results may arise. The model establishes a theoretical framework that justifies empirical tests using market data, thus bridging the gap between previous theoretical modeling and its empirical testing. The model also implies that the previous empirical tests of agency models, which employ the properties of firms' market values, may be flawed in the absence of an equilibrium asset-pricing model that defines systematic risk and idiosyncratic risk in terms of both market prices and cash flows.

In addition, we find that the expected dollar return on a firm is unaffected by PPS because of optimal contracting, in which systematic risk and idiosyncratic risk are separate in the manager's compensation. Mathematically, an isolated term that involves idiosyncratic risk does not contribute to the covariance term in the β^{adj} coefficient. On the other hand, the manager's effort increases with the PPS of his compensation, and, consequently, the equilibrium price of the firm increases with it. Therefore, the risk premium of the firm, which is defined as the ratio of the expected dollar return to its equilibrium price, decreases with the PPS of the manager's compensation. Similarly, after certain controls, the risk premium of a firm increases with its idiosyncratic risk. We also argue that even when firms' cash flow processes are log-normal and investors and managers posses general utility functions, managerial incentives can still affect expected asset returns. It is demonstrated that the volatility of the stock return process depends on the stock price itself. For example, even if a cash flow follows a log-normal process, the stock price does not follow a log-normal process. Consequently, the stock price affects the expected stock return. We stress, however, that managerial incentives and idiosyncratic risk do not serve as independent risk factors. Instead, they affect the risk premia through their influence on the stock prices. We further show that, in a one-period model, our results are robust with respect to endogenous interest rates.

Appendix: Proofs

Proof of Lemma 1. Define the investor's value function as

$$J(w,d) = \sup_{\{\mu\}} E_0\left[-\frac{1}{R_p}\exp\left(-R_p W_T\right)\right],$$

where w and d are the initial wealth and cash flow values. It is useful to consider the value function process at time t: $J(t,W_t,D_t) = \sup_{\{\mu\}} E_t\left[-(1/R_p)\exp(-R_p W_T)\right]$. The investor's Bellman equation is then given by

$$\sup_{\mu}\left[J_t + J_W\left(rW + \mu a_Q\right) + \frac{1}{2}J_{WW}\mu b_Q b_Q^T \mu^T + J_D^T a_D + \frac{1}{2}tr\left(J_{DD} b_D b_D^T\right) + \mu b_Q b_D^T J_{WD}\right] = 0.$$

The FOC of this Bellman equation yields

$$J_W a_Q + J_{WW} b_Q b_Q^T \mu^T + b_Q b_D^T J_{WD} = 0,$$

from which, we obtain

$$\mu^T = -\frac{1}{R_p f_2}\left(b_Q b_Q^T\right)^{-1}\left(J_W a_Q + b_Q b_D^T J_{WD}\right).$$

Conjecture that $J(t,\ W_t,\ D_t)$ is given by

$$J(t,W_t,D_t) = -\frac{1}{R_p}\exp\left\{-R_p\left[f_2(t)W_t + f_3(t)\right]\right\},$$

where $f_2(t)$ and $f_3(t)$ are continuous, deterministic functions with boundary conditions $f_2(T) = 1$ and $f_3(T) = 0$. Substituting the conjectured form for J into the FOC gives

$$\mu^T = -\frac{1}{R_p f_2(t)}\left(b_Q b_Q^T\right)^{-1}\left(J_W a_Q + b_Q b_D^T J_{WD}\right).$$

Substituting the above equation for a_Q into the Bellman equation, we have

$$\left[\dot{f}_2(t) + rf_2(t)\right]W_t + \dot{f}_3(t) + \mu a_Q - \frac{1}{2}R_p f_2^2(t)\mu b_Q b_Q^T \mu^T = 0.$$

To satisfy the above Bellman equation, we must have the following conditions:

$$\dot{f}_2(t) + rf_2(t) = 0, \quad \dot{\lambda} - \left(r - \Pi_j\right)\lambda_{ii} = 0$$

$$\dot{\lambda}_{ij} - \left(r - \Pi_j\right)\lambda_{ij} = 0, \quad i \neq j \in \{1,2,\cdots,N\},$$

with boundary conditions $f_2(T) = 1$ and $\lambda_{ii}(T) = 0, \forall i$ and j. The solutions to the odinary differential equations (ODEs) are given by

$$f_2(t) = e^{r(T-t)}, \quad \lambda_{ij} = 0, \quad \forall i \neq j, \quad \lambda_{ii} = e^{(r-\Pi_i)(t-T)}.$$

It can be shown that $f_3(t)$ and $\lambda_{i0}(t)$ are given, respectively, by $f_3(t) = R_p \int_t^T f_2(t)\left[1 - (1/2)f_1(t)\right]\mu b_Q b_Q^T \mu^T dt$ and $\lambda_{i0}(t) = \int_t^T e^{r(t-T)}\left[\lambda_{ii}(u)A_{iu} - R_p f_2(t)\frac{1}{N}\mathbf{1}_i b_Q b_Q^T \mu^T\right]du$, where $\mathbf{1}_i$ denotes a vector with the ith element being one and all other elements being zero.

Note that $\mathrm{cov}(dP) = b_Q b_Q^T$ and that in equilibrium, $\mu = (1/N)\mu_M = (1/N)(1, 1,\ldots,1)$, where μ_M denotes the vector of the total number of shares of each stock in the market portfolio. Therefore, the variance of the market index, $\mathrm{Var}(dP_M) = \mu_M \mathrm{cov}(dP)\mu_M^T = \mu_M\left(b_Q b_Q^T\right)\mu_M^T$.[39] By definition,

$$a_Q^M = \mu_M a_Q = \frac{1}{N}\mu_M R_p f(t)\left(b_Q b_Q^T\right)\mu_M^T = \frac{1}{N}R_p f(t)\mathrm{var}(dP_M),$$

[39] Recall that the market portfolio is defined as $P_M = \mu_M P = P_1 + P_2 + \ldots + P_N$

yielding

$$\frac{1}{N}R_p f(t) = \frac{1}{\mathrm{var}(dP^M)} a_Q^M.$$

Similarly, the expected excess dollar return for stock i is given by

$$a_Q^i = \mathbf{1}_i a_Q = \frac{1}{N}\mathbf{1}_i R_p f(t)\left(b_Q b_Q^T\right)\mu_M^T = \frac{1}{N}R_p f(t)\mathrm{cov}(dP_i,\, dP_M).$$

From the expression for $R_p f(t)$, we get

$$a_Q^i = \frac{\mathrm{cov}(dP_i, dP_M)}{\mathrm{var}(dP_M)} a_Q^M.$$

Define the relevant expected rates of return as $R_Q^i = (1/P_i)a_Q^i$ and $R_Q^M = (1/P_M)a_Q^M$, we obtain

$$R_Q^i = \frac{\mathrm{cov}(R_i, R_M)}{\mathrm{var}(R_M)} R_Q^M \equiv \beta R_Q^M.\text{Q.E.D.}$$

Proof of Lemma 2. This lemma is an application of Theorem 6 of Holmström and Milgrom (1987), Corollary 4.1 of Schättler and Sung (1993), and Corollary B of Ou-Yang (2003). For completeness, we present its proof here.

By Ito's lemma, $dV^i(t,\, P_t)$ is given by

$$dV^i(t,\, P_t) = \left[V_t^i + V_P^i a_{P_t} + \frac{1}{2}tr\left(V_{P_tP_t}^i b_{P_t} b_{P_t}^T\right)\right]dt + V_{P_t}^i b_{P_t} dB_t.$$

Combining the above equation with manager i's Bellman Equation (11) yields

$$dV^i(t,\, P_t) = V^i\left[R_a(g_i + h_i a_{P_t} - c_i) - \frac{1}{2}R_a^2 h_i b_{P_t} b_{P_t}^T h_i^T\right]dt + V_{P_t}^i R_a h_i b_{P_t} b_{P_t}^T dt + V_{P_t}^i b_{P_t} dB_t.$$

Define an ε_{it} process as $R_a\varepsilon_{it} = -\log[-R_a V^i(t,\, P_t)]$, where $\varepsilon_{iT} = q_i(T,\, P_t)$ as defined in Equation (9). Using the expression for $dV^i(t,\, P_t)$ and with some manipulations, we obtain

$$\begin{aligned} R_a d\varepsilon_{it} &= -\frac{dV^i}{V^i} + \frac{1}{2}\left(\frac{dV^i}{V^i}\right)^2 \\ &= -R_a\left[g_i + h_i a_{P_t} - c_i - \frac{1}{2}R_a \bar{h}_i b_{P_t} b_{P_t}^T \bar{h}_i^T\right]dt + R_a(\bar{h}_i - h_i) b_{P_t} dB_t, \end{aligned}$$

where $\bar{h}_i \equiv h_i - \frac{V_P^i}{R_a V^i}$. Therefore, we have

$$d\varepsilon_{it} + g_i dt + h_i dP_t = c_i dt + \frac{R_a}{2}\bar{h}_i b_{P_t} b_{P_t}^T \bar{h}_i dt + \bar{h}_i b_{P_t} dB_t.$$

Integrating the above expression between 0 and T yields

$$S_T^i = \varepsilon_{i0} + \int_0^T c_i(t, A_{it})dt + \frac{R_a}{2}\int_0^T \bar{h}_i b_{P_t} b_{P_t}^T \bar{h}_i^T dt + \int_0^T \bar{h}_i b_{P_t} dB_t.$$

Imposing the manager's PC yields that ε_{i0} equals the manager's reservation wage at time 0. Q.E.D.

Proof of Theorem 1. For notational simplicity, we shall omit the time-dependent variable "t" wherever there is no ambiguity. For example, it should be understood that $A_i \equiv A_{it}$, $\lambda_{ii} \equiv \lambda_{ii}(t)$, $k_i \equiv k_i(t)$, etc. The investor's value function is given by

$$J(w,p) = \sup_{\{A_i,\bar{h}_{ij},\mu\},i\neq j} E_0\left[-\frac{1}{R_p}e^{-R_p\left(W_T-\int_0^T \mu d\bar{S}_t\right)}\right],$$

where w and p are the initial wealth and price values. It is useful to consider the value function process at time t: $J(t,W_t,P_t) = \sup_{\{A_i,\bar{h}_{ij},\mu\},i\neq j} E_t\left[-(1/R_p)e^{-R_p\left(W_T-\int_t^T \mu d\bar{S}_u\right)}\right]$, with $J(T,W_T,P_T) = -\frac{1}{R_p}\mathrm{e}^{-R_pW_T}$. Using the fact that $b_P = b_Q$, we obtain the investor's Bellman equation:

$$\begin{aligned}
&\sup_{\{A_i,\bar{h}_{ij},\mu\},i\neq j} J\left[R_p\sum_{i=1}^N \mu_i c_i + \frac{R_aR_p}{2}\sum_{i=1}^N \mu_i\bar{h}_i b_Q b_Q^T \bar{h}_i^T\right] + \frac{R_p^2}{2}\left[\left[\sum_{i=1}^N \mu_i\bar{h}_i\right] b_Q b_Q^T \left[\sum_{i=1}^N \mu_i\bar{h}_i\right]^T\right] + J_t \\
&+ J_W\left[rW + \mu a_Q + R_p\sum_{i=1}^N \mu_i\bar{h}_i b_Q b_Q^T \mu^T\right] + \frac{1}{2}J_{WW}\mu b_Q b_Q^T\mu^T \qquad (27)\\
&+ J_P^T\left[a_P + R_p b_Q b_Q^T\left[\sum_{i=1}^N \mu_i\bar{h}\right]^T\right] + tr\left[\frac{1}{2}J_{PP}b_Q b_Q^T\right] + J_{WP}^T\left(b_Q b_Q^T\mu^T\right) = 0.
\end{aligned}$$

Conjecture that the investor's value function is given by

$$J(t,W_t,P_t) = -\frac{1}{R_p}\exp\left[-R_p(f(t)W_t + f_1(t))\right],$$

where $f(t)$ and $f_1(t)$ are continuous, deterministic functions with boundary conditions $f(T)=1$ and $f_1(T)=0$. As $f_3(t)$ in the proof of Lemma 1, $f_1(t)$ is to ensure that the Bellman equation is satisfied.[40] The Bellman equation then becomes

$$\begin{aligned}
&\sup_{\{A_i,\bar{h}_{ij},\mu\},i\neq j}\left[\sum_{i=1}^N \mu_i c_i + \frac{R_a}{2}\sum_{i=1}^N \mu_i\bar{h}_i b_Q b_Q^T\bar{h}_i^T + \frac{R_p}{2}\sum_{i=1}^N \mu_i\bar{h}_i b_Q b_Q^T\left(\sum_{i=1}^N \mu_i\bar{h}_i\right)^T - \dot{f}(t)W - \dot{f}_1(t)\right.\\
&\left.-f(t)\left(rW + \mu a_Q + R_p\sum_{i=1}^N \mu_i\bar{h}_i b_Q b_Q^T\mu^T\right) + \frac{R_p}{2}f^2(t)\mu b_Q b_Q^T\mu^T\right] = 0 \qquad (28)
\end{aligned}$$

Following similar arguments as in the proof of Lemma 1, we obtain that $f(t) = \mathrm{e}^{r(T-t)}$, $\lambda_{ij} = 0$, and $\lambda_{ii} = e^{(r-\Pi_i)(t-T)}$. The solution for λ_{i0} requires the solutions for both the contract and the stock price and shall be discussed later.

The FOCs now reduce to with respect to A_i:

$$\begin{aligned}
&\mu_i k_i A_i + R_a\frac{\mu_i}{\lambda_{ii}}k_i\mathbf{1}_i b_Q b_Q^T\bar{h}_i^T + R_p k_i\frac{\mu_i}{\lambda_{ii}}\mathbf{1}_i b_Q b_Q^T\left[\sum_{i=1}^N \mu_i\bar{h}\right]^T\\
&-f(t)\left[\mu_i\lambda_{ii} + R_p\frac{\mu_i}{\lambda_{ii}}k_i\mathbf{1}_i b_Q b_Q^T\mu^T\right] = 0;
\end{aligned}$$

[40] With the exponential utility function, $f_1(t)$ does not play any role in the determination of both the optimal contract and the equilibrium stock price.

with respect to μ^T:

$$\begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ C_N \end{bmatrix} + \frac{R_a}{2} \begin{bmatrix} \bar{h}_1 b_Q b_Q^T \bar{h}_1^T \\ \bar{h}_2 b_Q b_Q^T \bar{h}_1^T \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N b_Q b_Q^T \bar{h}_N^T \end{bmatrix} + R_p \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N \end{bmatrix} b_Q b_Q^T \left[\bar{h}_1^T \ \bar{h}_2^T \cdots \bar{h}_N^T \right] \mu^T$$

$$- f(t) \left[a_Q + R_p \left(b_Q b_Q^T \left[\bar{h}_1^T \ \bar{h}_2^T \cdots \bar{h}_N^T \right] + \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N \end{bmatrix} b_Q b_Q^T \right) \mu^T \right] + R_p f^2(t) b_Q b_Q^T \mu^T = 0;$$

with respect to $\bar{h}_{ij}$:

$$R_a \mu_i \mathbf{1}_j b_Q b_Q^T \bar{h}_i^T + R_p \mu_i \mathbf{1}_j b_Q b_Q^T \sum_{i=1}^{N} \mu_i \bar{h}_i^T - f(t) R_p \mu_i \mathbf{1}_j b_Q b_Q^T \mu^T = 0.$$

In equilibrium, $\mu_i = 1/N$. We next solve the FOCs with respect to A_i and $\bar{h}_{ij}$ for both the optimal effort and the optimal contract in closed form. Start with the expression for $b_Q b_Q^T$:

$$b_Q b_Q^T = \begin{pmatrix} \lambda_{11}^2(\sigma_{1c}^2 + \sigma_{1i}^2) & \lambda_{11}\lambda_{22}\sigma_{1c}\sigma_{2c} & \cdots & \lambda_{11}\lambda_{NN}\sigma_{1c}\sigma_{Nc} \\ \lambda_{11}\lambda_{22}\sigma_{1c}\sigma_{2c} & \lambda_{22}^2(\sigma_{2c}^2 + \sigma_{2i}^2) & \cdots & \lambda_{22}\lambda_{NN}\sigma_{2c}\sigma_{Nc} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \lambda_{11}\lambda_{NN}\sigma_{1c}\sigma_{Nc} & \lambda_{22}\lambda_{NN}\sigma_{2c}\sigma_{Nc} & \cdots & \lambda_{NN}^2(\sigma_{Nc}^2 + \sigma_{Ni}^2) \end{pmatrix},$$

we obtain

$$\mathbf{1}_i b_Q b_Q^T \bar{h}_i^T = \lambda_{ii} \left[\sigma_{ic} \sum_{j \neq i}^{N} \lambda_{jj} \sigma_{jc} \bar{h}_{ij} + \left(\sigma_{ic}^2 + \sigma_{ii}^2 \right) k_i A_i \right],$$

$$\mathbf{1}_j b_Q b_Q^T \bar{h}_i^T = \lambda_{jj} \left(\sigma_{jc} \sum_{l \neq i}^{N} \lambda_{ll} \sigma_{lc} \bar{h}_{il} + \sigma_{ic}\sigma_{jc} k_i A_i + \lambda_{jj} \sigma_{ji}^2 \bar{h}_{ij} \right),$$

$$\mathbf{1}_j b_Q b_Q^T \bar{h}_j^T = \lambda_{jj} \left[\sigma_{jc} \sum_{i \neq j}^{N} \sum_{i \neq j}^{N} \sigma_{ic} \lambda_{ii} \bar{h}_{ji} + \left(\sigma_{jc}^2 + \sigma_{ji}^2 \right) k_i A_i \right],$$

$$\mathbf{1}_i b_Q b_Q^T \bar{h}_j^T = \lambda_{ii} \left(\sigma_{ic} \sum_{i \neq j}^{N} \lambda_{ll} \sigma_{lc} \bar{h}_{jc} + \lambda_{ii} \sigma_{ii}^2 \bar{h}_{ji} + \sigma_{jc}\sigma_{ic} k_j A_j \right),$$

$$\mathbf{1}_i b_Q b_Q^T \mu^T = \lambda_{ii} \left(\sigma_{ic} \sum_{j=1}^{N} \lambda_{jj} \sigma_{jc} + \lambda_{ii} \sigma_{ii}^2 \right), \quad \mathbf{1}_j b_Q b_Q^T \mu^T = \lambda_{jj} \left[\sigma_{jc} \sum_{i=1}^{N} \lambda_{ii} \sigma_{ic} + \lambda_{jj} \sigma_{ji}^2 \right].$$

The FOCs with respect to A_i and $\bar{h}_{ij}$ are now reduced to

$$
\begin{aligned}
& k_i A_i + R_a k_i \left[\sigma_{ic} \sum_{j\neq i}^{N} \lambda_{jj}\sigma_{jc}\bar{h}_{ij} + (\sigma_{ic}^2 + \sigma_{ii}^2) k_i A_i \right] \\
& \quad + R_p k_i \frac{1}{N} \left[\sigma_{ic} \sum_{j\neq i}^{N} \lambda_{jj}\sigma_{jc}\bar{h}_{ij} + (\sigma_{ic}^2 + \sigma_{ii}^2) k_i A_i \right. \\
& \quad \left. + \sum_{j\neq i}^{N} \left(\sigma_{ic} \sum_{l\neq j}^{N} \lambda_{ll}\sigma_{lc}\bar{h}_{jl} + \lambda_{ii}\sigma_{ii}\bar{h}_{ji} + \sigma_{jc}\sigma_{ic}k_j A_j \right) \right] \\
& \quad - f(t) \left[\lambda_{ii} + R_p k_i \frac{1}{N} \left(\sigma_{ic} \sum_{j=1}^{N} \lambda_{jj}\sigma_{jc} + \lambda_{ii}\sigma_{ii}^2 \right) \right] = 0
\end{aligned}
$$

and

$$
\begin{aligned}
& R_a \left(\sigma_{jc} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc}\bar{h}_{il} + \sigma_{ic}\sigma_{jc}k_i A_i + \lambda_{jj}\sigma_{ji}^2\bar{h}_{ij} \right) \\
& \quad + R_p \frac{1}{N} \left[\sigma_{jc} \sum_{i\neq j}^{N} \lambda_{ii}\bar{h}_{ji} + \left(\sigma_{jc}^2 + \sigma_{ji}^2 \right) k_j A_j \right. \\
& \quad \left. + \sum_{i\neq j}^{N} \left(\sigma_{jc} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc}\bar{h}_{il} + \sigma_{ic}\sigma_{jc}k_a A_i + \lambda_{jj}\sigma_{jc}^2\bar{h}_{ij} \right) \right] \\
& \quad - f(t) R_p \frac{1}{N} \left(\sigma_{jc} \sum_{i=1}^{N} \lambda_{ii}\sigma_{ic} + \lambda_{jj}\sigma_{ji}^2 \right) = 0,
\end{aligned}
$$

respectively.

Conjecture that $\bar{h}_{ij}, i \neq j$, is given by

$$
\bar{h}_{ij} = \frac{1}{N} \left[\frac{R_p}{R_a + R_p} f(t) - \frac{k_i A_i}{\lambda_{jj}} \frac{\sigma_{ic}}{\sigma_{jc}} \right],
$$

where σ_{ic} and σ_{jc} are nonzero. In the following derivations, we shall take the limiting case in which $N \to \infty$. For example, we shall treat $\bar{h}_{ij}$ as zero if it appears in an individual term or take the last term in the FOCs with respect to $\bar{h}_{ij}$, $f(t)R_p(1/\mathrm{N})\lambda_{jj}{\sigma_{ji}}^2$, to be zero. We shall also not distinguish between $N-1$ and N. We now verify that this choice of $\bar{h}_{ij}$ satisfies the FOCs with respect to $\bar{h}_{ij}$:

$$
\begin{aligned}
& R_a \left[\sigma_{jc} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc} \frac{R_a}{R_a + R_p} f(t) \frac{1}{N} - \sigma_{jc} \sum_{l\neq i}^{N} k_i A_i \sigma_{ic} \frac{1}{N} + \sigma_{jc}\sigma_{ic}k_i A_i \right] \\
& + R_p \frac{1}{N} \sum_{i\neq j}^{N} \left[\sigma_{jc} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc} \frac{R_p}{R_a + R_p} f(t) \frac{1}{N} - \sigma_{jc} \sum_{l\neq i}^{N} k_i A_i \sigma_{ic} \frac{1}{N} + \sigma_{ic}\sigma_{jc}k_i A_i \right] - f(t) R_p \frac{1}{N} \sigma_{jc} \sum_{i=1}^{N} \lambda_{ii}\sigma_{ic} \\
& = R_a \sigma_{jc} \frac{R_p}{R_a + R_p} f(t) \frac{1}{N} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc} + \frac{R_p^2}{R_a + R_p} f(t) \frac{1}{N} \sigma_{jc} \sum_{l\neq i}^{N} \lambda_{ll}\sigma_{lc} - f(t) R_p \frac{1}{N} \sigma_{jc} \sum_{i=1}^{N} \lambda_{ii}\sigma_{ic} \\
& = f(t) \frac{1}{N} \left[\frac{R_a R_p}{R_a + R_p} + \frac{R_p^2}{R_a + R_p} - R_p \right] \sigma_{jc} \sum_{l\neq i}^{N} \sigma_{ll}\sigma_{lc} = 0.
\end{aligned}
$$

Notice that the expression for A_i is not required in the derivation of $\bar{h}_{ij}$. Substituting $\bar{h}_{ij}$ into the FOC with respect to A_i, we have

$$
\begin{aligned}
&k_iA_i + R_ak_i\left[\sigma_{ic}\frac{R_p}{R_a+R_p}f(t)\frac{1}{N}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc} - \sigma_{ic}k_iA_i\sigma_{ic} + \left(\sigma_{ic}^2+\sigma_{ii}^2\right)k_iA_i\right]\\
&\quad + R_pk_i\frac{1}{N}\left(\sum_{j\neq i}^{N}\left\{\sigma_{ic}\sum_{l\neq j}^{N}\lambda_{ll}\sigma_{lc}\left[\frac{R_p}{R_a+R_p}f(t) - \frac{k_jA_j}{\lambda_{ll}}\frac{\sigma_{jc}}{\sigma_{lc}}\right]\frac{1}{N} + \sigma_{jc}\sigma_{ic}k_jA_j\right\}\right)\\
&\quad - f(t)\left(\lambda_{ii} + R_pk_i\frac{1}{N}\sigma_{ic}\sum_{j=1}^{N}\lambda_{jj}\sigma_{jc}\right) = k_iA_i + R_ak_i\left[\frac{R_p}{R_a+R_p}f(t)\frac{1}{N}\sigma_{ic}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc} + \sigma_{ii}^2k_iA_i\right]\\
&\quad + R_pk_i\frac{1}{N}\sum_{j\neq i}^{N}\sigma_{ic}\sum_{l\neq j}^{N}\lambda_{ll}\sigma_{lc}\frac{R_p}{R_a+R_p}f(t)\frac{1}{N} - f(t)\left(\lambda_{ii} + R_pk_i\frac{1}{N}\sigma_{ic}\sum_{j=1}^{N}\lambda_{jj}\sigma_{jc}\right) = 0,
\end{aligned}
$$

where

$$
\begin{aligned}
&R_ak_i\frac{R_p}{R_a+R_p}f(t)\frac{1}{N-1}\sigma_{ic}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc} + R_pk_i\frac{1}{N}\sigma_{ic}\frac{R_p}{R_a+R_p}f(t)\sum_{l\neq j}^{N}\lambda_{ll}\sigma_{lc}\\
&\quad - f(t)R_pk_i\frac{1}{N}\sigma_{ic}\sum_{j=1}^{N}\lambda_{jj}\sigma_{jc} = 0.
\end{aligned}
$$

We thus arrive at

$$
A_{it}^* = \frac{f(t)\lambda_{ii}(t)}{k_i(t) + R_ak_i^2(t)\sigma_{ii}^2} = \frac{1}{k_i(t) + R_ak_i^2(t)\sigma_{ii}^2}\mathrm{e}^{-\Pi_i(t-T)}.
$$

The verification theorems for the Bellman-type Equation (27) or (28) have been provided by Schättler and Sung (1993) and Ou-Yang (2003). For example, Schättler and Sung show that the technical conditions are that $E_0\left[\int_0^T J(t,W_t)^m dt\right] < \infty$ and $E_0\left\{\int_0^T [\partial J(t,W_t)/\partial W\sigma(t,W_t)]^m dt\right\} < \infty$, where $m > 2$ and $\sigma(t, W_t)$ denotes the diffusion term of the dW_t process. Our closed-form solutions show that A_{it} and λ_{ij} are deterministic, bounded, functions and that the investor's value function is given by $J(t,W_t) = -1/R_p\exp\{-R_p[f(t) + f_1(t)W_t]\}$ where $f(t)$ and $f_1(t)$ are bounded and W_t is normally distributed with both a bounded mean and a bounded variance (due to bounded solutions for A_{it} and λ_{ij}). It can be seen that the technical conditions are satisfied.

Note that as in the Holmström–Milgrom (1987) model, the manager's value function is not required in the derivation of the optimal solutions. Following Holmström and Milgrom and Schättler and Sung, we obtain optimal contract (14) by substituting $b_pdP_t = dP_t - a_pd_t$ into the equilibrium compensation (12). Similarly, we can show that the optimal contract and a value function given by $-1/R_a\exp\{-R_a[f_5(t)P_t + f_6(t)]\}$, where $f_5(t)$ and $f_6(t)$ are deterministic, bounded functions, satisfy the manager's Bellman equation (11). The verification results are essentially the same as for the investor's problem or for the Holmström–Milgrom and Schättler–Sung problems, in which the exponential-normal setup affords well-behaved solutions in closed form.[41]

[41] See also Bolton and Harris (1997), Detemple, Govindaraj, and Loewenstein (2001), and Cadenillas, Cvitanić, and Zapatero (2003) for other dynamic principal–agent models in the absence of asset pricing.

We now derive the modified CAPM equation for the expected asset returns. Using the FOCs with respect to μ^T, we get

$$\left(a_Q - \frac{1}{f(t)}\left(\begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_N \end{bmatrix} + \frac{R_a}{2}\begin{bmatrix} \bar{h}_1 b_Q b_Q^T \bar{h}_1^T \\ \bar{h}_2 b_Q b_Q^T \bar{h}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N b_Q b_Q^T \bar{h}_N^T \end{bmatrix}\right)\right)$$

$$= \frac{1}{N} R_p \left[f(t) b_Q b_Q^T \mu_M^T + \frac{1}{f(t)} \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N \end{bmatrix} b_Q b_Q^T \begin{bmatrix} \bar{h}_1^T & \bar{h}_2^T & \cdots & \bar{h}_N^T \end{bmatrix} \mu_M^T \right. \tag{29}$$

$$\left. - \left(b_Q b_Q^T \begin{bmatrix} \bar{h}_1^T & \bar{h}_2^T & \cdots & \bar{h}_N^T \end{bmatrix} + \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N \end{bmatrix} b_Q b_Q^T \right) \mu_M^T \right].$$

Multiplying both sides by $\mu_M = (1, 1, \ldots, 1)$, we have

$$\left[a_Q^M - \frac{1}{f(t)}\left(\sum_{i=1}^{N} c_i + \frac{R_a}{2}\sum_{i=1}^{N} \bar{h}_i b_Q b_Q^T \bar{h}_i^T\right)\right] = \frac{1}{N} R_p \mu_M \left[f(t) b_Q b_Q^T + \frac{1}{f(t)} \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N \end{bmatrix} b_Q b_Q^T \begin{bmatrix} \bar{h}_1^T \bar{h}_2^T \cdots \bar{h}_N^T \end{bmatrix} \right.$$

$$\left. - b_Q b_Q^T \begin{bmatrix} \bar{h}_1^T \bar{h}_2^T \cdots \bar{h}_N^T \end{bmatrix} - \begin{bmatrix} \bar{h}_1^T \\ \bar{h}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \bar{h}_N^T \end{bmatrix} b_Q b_Q^T \right] \mu_M^T. \tag{30}$$

Equation (30) can also be expressed as:

$$\left[a_Q^M - \frac{1}{f(t)}\sum_{i=1}^{N}\left(c_i + \frac{R_a}{2}\bar{h}_i b_Q b_Q^T \bar{h}_i^T\right)\right]$$
$$= R_p f(t)\mu_M \left[I_N - \frac{1}{f(t)}\begin{bmatrix}\bar{h}_1^T\\ \bar{h}_2^T\\ \cdot\\ \cdot\\ \cdot\\ \bar{h}_N^T\end{bmatrix}\right] b_Q b_Q^T \left[I_N - \frac{1}{f(t)}\begin{bmatrix}\bar{h}_1^T\\ \bar{h}_2^T\\ \cdot\\ \cdot\\ \cdot\\ \bar{h}_N^T\end{bmatrix}\right]^T \mu_M^T,$$

where I_N denotes the identity matrix.

To find the expected excess dollar return on stock i, adjusted for the manager's expected compensation, we multiply both sides of (29) by the $\mathbf{1}_i$ vector and have

$$E^i(t) \equiv \left\{a_Q^i - \frac{1}{f(t)}\left[c_i(t,\, A_{it}) + \frac{R_a}{2}\bar{h}_i b_Q b_Q^T \bar{h}_i^T\right]\right\}$$

$$= \frac{1}{N} R_p f(t)\mathbf{1}_i \left[I_N - \frac{1}{f(t)}\begin{bmatrix}\bar{h}_1^T\\ \bar{h}_2^T\\ \cdot\\ \cdot\\ \cdot\\ \bar{h}_N^T\end{bmatrix}\right] b_Q b_Q^T \left[I_N - \frac{1}{f(t)}\begin{bmatrix}\bar{h}_1^T\\ \bar{h}_2^T\\ \cdot\\ \cdot\\ \cdot\\ \bar{h}_N^T\end{bmatrix}\right]^T \mu_M^T$$
$$\equiv R_p f(t)\frac{1}{N}\mathbf{1}_i H b_Q b_Q^T H^T \mu_M^T$$
$$= R_p f(t)\frac{1}{N}\mathrm{cov}\left\{dP_{it} - \frac{1}{f(t)}dS_{it}, \sum_{i=1}^{N}\left[dP_{it} - \frac{1}{f(t)}dS_{it}\right]\right\}$$
$$= R_p f(t)\frac{1}{N}\mathrm{cov}\left\{\left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right]b_Q dB_t, \sum_{i=1}^{N}\left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right]b_Q dB_t\right\},$$

where matrix H is defined as in Equation (16).

Notice that $E^i(t)$ represents the expected excess dollar return at time t on firm i, adjusted for the manager's expected compensation. It can then be shown that

$$E^i(t) = \left(\frac{\mathbf{1}_i H b_Q b_Q^T H^T \mu_M^T}{\mu_M H b_Q b_Q^T H^T \mu_M^T}\right) E^M(t) = \frac{\mathrm{cov}\left(dP_{it}^{adj}, dP_{Mt}^{adj}\right)}{\mathrm{var}\left(dP_{Mt}^{adj}\right)} E^M(t),$$

where $dP_{it}^{adj} = dP_{it} - 1/f(t)dS_{it}$ and $dP_{Mt}^{adj} = \sum_{i=1}^{N}[dP_{it} - (1/f(t))dS_{it}]$. This is a CAPM-type linear relation in terms of the expected dollar returns. Notice that the derivation of this relation does not require the number of assets, N, to go to infinity.

We next show that the expected excess dollar return on stock i is independent of the firm-specific risk σ_{ii} and A_{it}^* and that the equilibrium stock price decreases with σ_{ii} and increases with A_{it}^*. Recall that the expected excess dollar return can be expressed as

$$E^i(t) = R_p f(t)\frac{1}{N}\mathrm{cov}\left\{\left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right]b_Q dB_t, \sum_{i=1}^{N}\left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right]b_Q dB_t\right\}.$$

Straightforward calculation yields

$$b_Q dB_t = \begin{pmatrix} \lambda_{11}\sigma_{1c} & \lambda_{1i}\sigma_{1i} & 0 & \cdots & 0 & 0 \\ \lambda_{22}\sigma_{2c} & 0 & \lambda_{22}\sigma_{2i} & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \lambda_{NN}\sigma_{Nc} & 0 & 0 & \cdots & 0 & \lambda_{NN}\sigma_{Ni} \end{pmatrix}\begin{pmatrix} dB_{ct} \\ dB_{1t} \\ dB_{2t} \\ \cdot \\ \cdot \\ \cdot \\ dB_{Nt} \end{pmatrix} = \begin{pmatrix} \lambda_{11}(\sigma_{1c}dB_{ct} + \sigma_{1t}) \\ \lambda_{22}(\sigma_{2c}dB_{ct} + \sigma_{2t}) \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{NN}(\sigma_{Nc}dB_{ct} + \sigma_{Nt}) \end{pmatrix}$$

$$\begin{aligned} \mathbf{1}_i b_Q dB_t &= \lambda_{ii}(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}), \\ \bar{h}_i dB_t &= \bar{h}_{i1}\lambda_{11}(\sigma_{1c}dB_{ct} + \sigma_{1i}dB_{1t}) + \bar{h}_{i2}\lambda_{22}(\sigma_{2c}dB_{ct} + \sigma_{2i}dB_{2t}) + \cdots \\ &\quad + k_i A_i(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}) + \cdots + \bar{h}_{iN}\lambda_{NN}(\sigma_{Nc}dB_{ct} + \sigma_{Ni}dB_{Nt}), \end{aligned}$$

where $\bar{h}_i = (\bar{h}_{i1}\bar{h}_{i2}\cdots k_i A_i/\lambda_{ii}\cdots \bar{h}_{iN})$ and $\bar{h}_{ij} = (1/N-1)\left[\left(R_p/R_a + R_p\right)f(t) - k_i A_i/\lambda_{jj}\right)$ $\left(\sigma_{ic}/\sigma_{jc}\right)]$. We then have

$$\begin{aligned} \left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right]b_Q dB_t &= \lambda_{ii}(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}) - \frac{1}{f(t)}\bar{h}_{i1}(\lambda_{11}\sigma_{1c}dB_{ct} + \sigma_{1i}dB_{1t}) \\ &\quad - \frac{1}{f(t)}\bar{h}_{i2}(\lambda_{22}\sigma_{2c}dB_{ct} + \sigma_{2i}dB_{2t}) - \cdots - \frac{k_i A_i}{f(t)}(\sigma_{ic}dB_{ct} + \sigma_{1i}dB_{it}) - \cdots \\ &\quad - \frac{1}{f(t)}\bar{h}_{iN}(\lambda_{NN}\sigma_{Nc}dB_{ct} + \sigma_{Ni}dB_{Nt}) \\ &= \lambda_{ii}(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}) \\ &\quad - \left[\frac{R_p}{R_a + R_p} - \frac{1}{f(t)}\frac{k_i A_i}{\lambda_{11}}\frac{\sigma_{ic}}{\sigma_{1c}}\right]\lambda_{11}\frac{1}{N-1}(\sigma_{1c}dB_{ct} + \sigma_{1i}dB_{1t}) \\ &\quad - \left[\frac{R_p}{R_a + R_p} - \frac{1}{f(t)}\frac{k_i A_i}{\lambda_{22}}\frac{\sigma_{ic}}{\sigma_{2c}}\right]\lambda_{22}\frac{1}{N-1}(\sigma_{2c}dB_{ct} + \sigma_{2i}dB_{2t})\cdots \\ &\quad - k_i A_i(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}) - \cdots \\ &\quad - \left[\frac{R_p}{R_a + R_p} - \frac{1}{f(t)}\frac{k_i A_i}{\lambda_{NN}}\frac{\sigma_{ic}}{\sigma_{Nc}}\right]\lambda_{NN}\frac{1}{N-1}(\sigma_{Nc}dB_{ct} + \sigma_{Ni}dB_{Nt}) \\ &= \lambda_{ii}(\sigma_{ic}dB_{ct} + \sigma_{ii}dB_{it}) - \frac{R_p}{R_a + R_p}\frac{1}{N-1}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc}dB_{ct} - \frac{1}{f(t)}k_i A_i\sigma_{1i}dB_{it} \\ &= \left(\lambda_{ii}\sigma_{ic} - \frac{R_p}{R_a + R_p}\frac{1}{N-1}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc}\right)dB_{ct} + \frac{R_a k_i \lambda_{ii}\sigma_{ii}^3}{1 + R_a k_i \sigma_{ii}^2}dB_{it}, \end{aligned}$$

and

$$\sum_{i=1}^{N}\left[\mathbf{1}_i - \frac{1}{f(t)}\bar{h}_i\right] b_Q dB_t = \frac{R_p}{R_a + R_p}\sum_{i=1}^{N}\lambda_{ii}\sigma_{ic} dB_{ct} + \sum_{i=1}^{N}\frac{R_a k_i \lambda_{ii}\sigma_{ii}^3}{1 + R_a k_i \sigma_{ii}^2} dB_{it}.$$

Consequently, we arrive at the expected excess dollar return:

$$E^i(t) = R_p f(t)\frac{1}{N}\frac{R_a}{R_a + R_p}\left(\lambda_{ii}\sigma_{ic} - \frac{R_p}{R_a + R_p}\frac{1}{N-1}\sum_{j\neq i}^{N}\lambda_{jj}\sigma_{jc}\right)\sum_{i=1}^{N}\lambda_{ii}\sigma_{ic}.$$

Finally, we examine the impact of σ_{ii} and A_{it}^* on the equilibrium asset price. Because $\lambda_{ii}(t)$ is independent of σ_{ii} and A_{it}^*, we only need to show that $\lambda_{i0}(t)$ is an increasing function of A_{it}^*. Recall that the expected excess return $E^i(t)$ is defined as

$$E^i(t) = a_{Q_t}^i - \frac{1}{f(t)}\left[\frac{1}{2}k_i(t)A_{it}^2 + \frac{R_a}{2}\bar{h}_i b_{Q_t} b_{Q_t}^T \bar{h}_i^T\right],$$

where $a_{Q_t}^i = \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \lambda_{ii}(t)A_{it}$. A calculation yields

$$\begin{aligned}
\bar{h}_i b_{Q_t} b_{Q_t}^T \bar{h}_i^T &= \bar{h}_{i1}\left[\sum_{j\neq 1}^{N}\bar{h}_{ij}\lambda_{11}\lambda_{jj}\sigma_{1c}\sigma_{jc} + \bar{h}_{i1}\lambda_{11}^2\left(\sigma_{1c}^2 + \sigma_{1i}^2\right) + k_i A_i \lambda_{11}\sigma_{1c}\sigma_{ic}\right] \\
&\quad + \bar{h}_{i2}\left[\sum_{j\neq 2}^{N}\bar{h}_{ij}\lambda_{22}\lambda_{jj}\sigma_{2c}\sigma_{jc} + \bar{h}_{i2}\lambda_{22}^2\left(\sigma_{2c}^2 + \sigma_{2i}^2\right) + k_i A_i \lambda_{22}\sigma_{2c}\sigma_{ic}\right] + \cdots \\
&\quad + \frac{k_i A_i}{\lambda_{ii}}\left[\sum_{j\neq i}^{N}\bar{h}_{ij}\lambda_{ii}\lambda_{jj}\sigma_{ic}\sigma_{jc} + k_i A_i \lambda_{ii}\left(\sigma_{ic}^2 + \sigma_{ii}^2\right)\right] + \cdots \\
&= \bar{h}_{i1}\left(\lambda_{11}\sigma_{1c}\sum_{j\neq i}^{N}\bar{h}_{ij}\lambda_{jj}\sigma_{jc} + \bar{h}_{i1}\lambda_{11}^2\sigma_{1i}^2 + k_i A_i \lambda_{11}\sigma_{1c}\sigma_{ic}\right) \\
&\quad + \bar{h}_{i2}\left(\lambda_{22}\sigma_{2c}\sum_{j\neq i}^{N}\bar{h}_{ij}\lambda_{jj}\sigma_{jc} + \bar{h}_{i2}\lambda_{22}^2\sigma_{2i}^2 + k_i A_i \lambda_{11}\sigma_{1c}\sigma_{ic}\right) + \cdots \\
&\quad + k_i A_i\left[\lambda_{ii}\sigma_{ic}\sum_{j\neq i}^{N}\bar{h}_{ij}\lambda_{jj}\sigma_{jc} + k_i A_i\left(\sigma_{ic}^2 + \sigma_{ii}^2\right)\right] + \cdots \\
&= \frac{1}{N-1}\frac{R_p}{R_a + R_p}f(t)\sum_{l\neq i}^{N}\lambda_{ll}\sigma_{ll}\sum_{j\neq i}^{N}\bar{h}_{ij}\lambda_{jj}\sigma_{jc} + \sum_{j\neq i}^{N}\bar{h}_{ij}^2\lambda_{jj}^2\sigma_{ji}^2 + (k_i A_i)^2\sigma_{ii}^2 \\
&\quad \Rightarrow [k_i(t)A_{it}]^2\sigma_{ii}^2,
\end{aligned}$$

where we have ignored all the terms independent of σ_{ii}. The expression for $E^i(t)$ then yields

$$\begin{aligned}
E^i(t) &= \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \lambda_{ii}(t)A_{it} - \frac{1}{f(t)}\left[\frac{1}{2}k_i(t)A_{it}^2 + \frac{R_a}{2}\bar{h}_i b_{Q_t} b_{Q_t}^T \bar{h}_i^T\right] \\
&= \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \left[\lambda_{ii}(t) - \frac{1}{f(t)}\frac{1}{2}k_i(t)\left(1 + R_a k_i \sigma_{ii}^2\right)A_{it}\right]A_{it} \qquad (31) \\
&= \dot{\lambda}_{i0}(t) - r\lambda_{i0}(t) + \frac{1}{2}\lambda_{ii}A_{it},
\end{aligned}$$

where the boundary condition is given by $\lambda_{i0}(T) = 0$. Ignoring the terms independent of σ_{ii} [e.g., $E^i(t)$] and solving ordinary differential equation (31), we get

$$\lambda_{i0}(t) = \frac{1}{2}\int_t^T e^{r(t-u)}\lambda_{ii}(u)A_{iu}du,$$

which is clearly an increasing function of A_{it}. Q.E.D.

Proof of Theorem 2. Given the linear contract form defined in Equation (26), the manager's maximization problem is given by

$$\max_{A_i} E\left[-\frac{1}{R_a}e^{-R_a\left(S_i-\frac{1}{2}k_iA_i^2\right)}\right].$$

It can be shown that the manager's FOC yields

$$h_{ii} = k_iA_i. \tag{32}$$

The FOC is both necessary and sufficient, because the manager's utility function is increasing and concave. In addition, the manager's PC leads to an expression for g_i:

$$g_i = \varepsilon_{i0} - \sum_{j=1}^N h_{ij}A_j + \frac{1}{2}k_iA_i^2 + \frac{1}{2}R_a\sum_{j=1}^N h_{ij}^2\sigma_{ji}^2 + \frac{1}{2}R_a\left(\sum_{j=1}^N h_{ij}\sigma_{jc}\right)^2. \tag{33}$$

The investor's objective is to maximize her expected utility over the terminal consumption, $C_1 \equiv W_1 - \sum_{i=1}^N S_i(\mu)$, with respect to $\{A_i, \mu_i, h_{ij}(i \neq j)\}$:

$$\max E\left\{-\frac{1}{R_p}\mathbf{e}^{-R_p\left[W_1-\sum_{i=1}^N S_i(\mu)\right]}\right\}. \tag{34}$$

The investor's terminal consumption C_1 is given by

$$\begin{aligned} C_1 &= W_0(1+r) + \sum_{i=1}^N \mu_i[D_i - S_i - P_{i0}(1+r)] \\ &= W_0(1+r) - \sum_{i=1}^N \mu_i\left[-g_i + A_i - \sum_{j=1}^N h_{ij}A_j + (\sigma_{ii}-h_{ii})\varepsilon_i - \sum_{j\neq i}^N h_{ij}\varepsilon_i \right. \\ &\quad \left. + \left(\sigma_{ic} - \sum_{j=1}^N h_{ij}\sigma_{jc}\right)\varepsilon_c - P_{i0}(1+r)\right]. \end{aligned}$$

Evaluating the investor's expected utility function (34) and taking the derivative with respect to her control variables, we obtain that the FOC with respect to A_i :

$$-k_iA_i - R_ak_i^2A_i\sigma_{ii} + 1 - R_ak_i\sigma_{ic}\sum_{j=1}^N h_{ij}\sigma_{jc} + R_pk_i\sigma_{ic}\frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N h_{ij}\sigma_{jc} + R_pk_i\sigma_{ic}\frac{1}{N}\sum_{i=1}^N \sigma_{ic} = 0;$$

the FOC with respect to h_{ij} $(i \neq j)$:

$$-R_a\sigma_{jc}\sum_{j=1}^N h_{ij}\sigma_{jc} + R_p\sigma_{jc}\frac{1}{N}\sum_{j=1}^N \sigma_{jc} - R_p\sigma_{jc}\frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N h_{ij}\sigma_{jc} = 0;$$

and the FOC with respect to μ_i:

$$\begin{aligned} P_{i0} &= \frac{1}{1+r}\left(F + \frac{1}{2}A_i\right), \\ F &= \frac{1}{2}R_a\left(\frac{1}{N}\frac{R_p}{R_a+R_p}\sum_{j=1}^N \sigma_{jc}\right)^2 - R_p\left(\sigma_{ic} - \frac{1}{N}\sum_{j=1}^N h_{ij}\sigma_{jc}\right)\sum_{i=1}^N\left(\sigma_{ic} - \sum_{j=1}^N h_{ij}\sigma_{jc}\right); \end{aligned}$$

where N approaches infinity.[42] Here, the equilibrium price is clearly an increasing function of the manager's PPS A_i.

It can be verified that $A_i^* = (1/k_i)(1/1 + R_a k_i \sigma_{ii}^2)$ and $h_{ij}(i \neq j) = 1/N\left[\left(R_p/R_a + R_p\right) - \left(k_i A_i^* \sigma_{ic}/\sigma_{jc}\right)\right]$ satisfy the FOCs. To determine the interest rate, we impose the condition that the risk-free bond is in zero-net supply or that the identical investors hold no bond positions in equilibrium. Equivalently, investors invest all their wealth in the risky assets, that is,

$$W_0 = \sum_{i=1}^{N} \mu_i P_{i_0} = \frac{1}{N}\left(\frac{1}{1+r}\right)\sum_{i=1}^{N}\left(F + \frac{1}{2}A_i^*\right) = \left(\frac{1}{1+r}\right)\left(F + \frac{1}{2N}\sum_{i=1}^{N} A_i^*\right)$$

or

$$r = \frac{1}{W_0}\left(F + \frac{1}{2N}\sum_{i=1}^{N} A_i^*\right) - 1.$$

We now derive the CAPM-type linear equation, adjusted for managers' compensation. The investor's FOC with respect to her demand for the risky assets yields

$$E\{U'(C_1)[D_i - S_i - P_{i0}(1+r)]\} = 0,$$

where "prime" denotes the derivative of the investor's utility U with respect to her terminal consumption C_1 and where $U(C_1)$ denotes the investor's utility function which is concave and twice differentiable. Notice that, within the linear contract space, investor's net wealth (after managers' compensation) is normally distributed and that $\Delta P^{adj} \equiv [D_i - S_i - P_{i0}(1+r)]$ is the excess dollar return adjusted for managerial compensation. Using Stein's lemma,[43] we obtain

$$E[U'(C_1)]E\left(\Delta P_i^{adj}\right) = -E[U''(C_1)]\mathrm{cov}\left(C_1, \Delta P_i^{adj}\right).$$

We further obtain that

$$E\left(\Delta P_i^{adj}\right) = \beta^{adj} E\left(\Delta P_M^{adj}\right),$$

where $\beta^{adj} = \mathrm{cov}\left(C_1, \Delta P_i^{adj}\right)/\mathrm{cov}\left(C_1, \Delta P_M^{adj}\right)$ and $\Delta P_M^{adj} = \sum_{i=1}^{N} \Delta P_i^{adj}$, which is the excess return for the market portfolio adjusted for managers' compensation. When the risk-free bond is in zero-net supply, C_1 represents the value of the market portfolio adjusted for managers' compensation because investors' entire wealth is invested in the risky assets. We thus have

$$\mathrm{cov}\left(C_1, \Delta P_i^{adj}\right) = \mathrm{cov}\left[(C_1 - W_0), \Delta P_i^{adj}\right] = \mathrm{cov}\left(\Delta P_M^{adj}, \Delta P_i^{adj}\right),$$

which leads to

$$\beta^{adj} = \frac{\mathrm{cov}\left(\Delta P_M^{adj}, \Delta P_i^{adj}\right)}{\mathrm{var}\left(\Delta P_M^{adj}\right)}.$$

As shown in the continuous-time case, β^{adj} is independent of idiosyncratic risk. Q.E.D.

[42] Mathematically, we omit all individual terms that involve $1/N$ in the derivation.

[43] See, for example, Rubinstein (1976) for a derivation of this lemma.

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