A Model of Portfolio Delegation and Strategic Trading

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This article endogenizes information acquisition and portfolio delegation in a one-period strategic trading model. We find that, when the informed portfolio manager is relatively risk tolerant (averse), price informativeness increases (decreases) with the amount of noise trading. When noise trading is endogenized, the linear equilibrium in the traditional literature breaks down under a wide range of parameter values. In contrast, a linear equilibrium always exists in our model. In a conventional portfolio delegation model under a competitive partial equilibrium, the manager’s effort of acquiring information is independent of a linear incentive contract. In our strategic trading model, however, a higher-powered linear contract induces the manager to exert more effort for information acquisition. (JEL G14, G12, G11)

Institutional investors now dominate both equity ownership and trading activity. Gompers and Metrick (2001) report that, by December 1996, mutual funds, pension funds, and other financial intermediaries held discretionary control over more than half of the U.S. equity market. Jones and Lipson (2004) report that non-retail trading accounted for 96% of New York Stock Exchange trading volume in 2002. As pointed out by Bennett, Sias, and Starks (2003), “This institutionalization of equity holdings almost certainly means that, for most firms, the price-setting marginal investor is an institution.” It is thus of great importance to study the impact of institutional trading on stock prices.
and to integrate into one model both asset pricing and delegated portfolio management, as advocated by Allen (2001).

Although there is a voluminous literature on strategic informed trading, two fundamental issues remain unaddressed. First, this literature assumes that agents trade for their own accounts. Consequently, there still does not exist a strategic trading model that studies the impact of institutional trading on stock prices. Second, in an extension of the Kyle (1985) model, Spiegel and Subrahmanyam (1992) demonstrate that, when noise trading is endogenized, a linear equilibrium does not exist or the market breaks down under a wide range of parameter values. This result suggests that it is important to develop a robust strategic trading model in which an equilibrium always exists.

Admati and Pfleiderer (1997) endogenize information acquisition in the context of an agency problem between a portfolio manager and outside investors. They solve a competitive partial equilibrium model in which the portfolio manager is a price-taker. In addition to results on the use of benchmark portfolios in a manager’s compensation, Admati and Pfleiderer show that the manager’s effort is independent of the slope of a linear contract. This result challenges the traditional principal–agent literature, in which a portfolio manager is absent and in which a higher slope typically induces a higher level of effort from the agent.

In this article, we develop an integrated model of strategic trading and portfolio delegation. Specifically, we consider a linear equilibrium model in which asset prices, optimal contracts, and information acquisition are determined simultaneously. We illustrate that incentives do influence the manager’s effort and that a linear equilibrium always exists. We further show that more noise trading may lead to a more informative stock price due to information acquisition and optimal contracting. This result differs from those in the traditional market microstructure literature, where price informativeness is independent of or decreases with the amount of noise trading.

In our baseline model built upon Kyle (1985), there is an uninformed risk-neutral investor (the principal), a risk-averse informed portfolio manager (the

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2 For other competitive partial equilibrium models with information acquisition and portfolio delegation, see Stoughton (1993), Ding, Gervais, and Kyle (2008), and Garcia and Vanden (2008).


agent), competitive risk-neutral market makers, and noise traders. There are one risky stock and one risk-free bond available for trading. The uninformed investor entrusts her money to the informed manager. The manager has skill at acquiring private information about the stock’s liquidation value, and bases his trades on the acquired information. The manager’s trades affect the asset price as market makers take into account adverse selection in the determination of the asset price. At the end of one period, the asset’s liquidation value is realized and trading profits are determined. The manager is compensated according to a contract designed by the investor at the beginning of the period.

Moral hazard arises because acquiring information is costly to the manager, and the effort the manager spends acquiring information is unobservable to the investor. We require the contract to be a linear function of trading profits. The investor is a Stackelberg leader in the sense that, in the stages of the game after she announces the contract, other market participants take the contract as given and strategically trade with one another. Specifically, given the investor’s contract, the manager first chooses an effort level for information acquisition and then decides on the optimal portfolio allocation. The competitive risk-neutral market makers determine the equilibrium stock price, based on the total demand by the informed manager and noise traders. Consequently, as the Stackelberg leader, the investor takes the responses of other players into account when determining the optimal contract.

One of our main results states that, when the risk aversion ($R_a$) of the informed agent is relatively low (high), an increase in the variance of noise trading ($\sigma_u^2$) increases (decreases) the informativeness of stock prices ($Q$), measured by the precision of the asset’s liquidation value conditional on the equilibrium asset price. In the prior studies without information acquisition, $Q$ is independent of or decreases with $\sigma_u^2$ because an increase in the intensity of informed trading is exactly canceled out or dominated by an increase in noise trading. However, once information acquisition is possible, when noise trading becomes more volatile and the informed agent can thus better conceal his trading from market makers, a relatively less risk-averse agent first acquires more accurate information and then trades more aggressively, leading to a more informative price in equilibrium. Moreover, we show that, under portfolio delegation, $Q$ increases with $\sigma_u^2$ for a wider range of $R_a$ values than in the case without portfolio delegation. This is because portfolio delegation makes the agent effectively less risk averse, increasing the risk-taking capacity of the agent.

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5 We shall use principal (agent) and investor (manager or informed agent) interchangeably.

6 In the absence of risk aversion, portfolio delegation, and information acquisition, Kyle (1985) shows that $Q$ is independent of $\sigma_u^2$ because the risk-neutral agent scales up trading in such a way that $Q$ is unchanged. Subrahmanyam (1991) finds that, when the informed agent is risk averse, an increase in $\sigma_u^2$ decreases $Q$, because a risk-averse informed agent trades less aggressively than a risk-neutral one. Notice that, in Kyle and Subrahmanyam, the precision of private information is fixed, regardless of noise trading.
Different from the irrelevance result of Admati and Pfleiderer (1997), we show that higher incentives induce higher levels of effort, thus recovering the well-known result in the traditional principal–agent literature. In our model of strategic trading, market impact mitigates the manager’s incentive to “undo” changes in the linear contract, which occurs in the Admati–Pfleiderer model. The fact that, in reality, many of the contracts for portfolio managers, such as mutual fund, pension fund, or endowment fund managers, are linear, it is important to see that the incentive component of the contract comes out mattering.

We extend the baseline model by endogenizing noise trading. Following Spiegel and Subrahmanyam (1992), we assume that noise traders are risk-averse uninformed hedgers who hedge their endowment risk optimally. Hence, the optimal hedging demand of the hedgers creates endogenous noise trading. When noise trading is endogenized, equilibrium asset pricing, informed trading, and optimal contracting are affected by the trading behavior of the noise traders or uninformed hedgers. We demonstrate that the positive relationships between incentives and effort and between the informativeness of prices and the level of noise trading still hold in this case.

In our model, we show that an equilibrium always exists. By contrast, in the work by Spiegel and Subrahmanyam (1992), where both information acquisition and portfolio delegation are absent, no equilibrium exists under a wide range of parameter values, or the market breaks down. In the Spiegel–Subrahmanyam model, the uninformed risk-averse noise traders face an endowment risk. On the one hand, they would like to hedge this risk, but on the other hand, they would not like to lose to the informed trader. Hence, the trade-off is between the utility gain from hedging the endowment risk and the utility loss from losing to the informed trader. For example, if the risk aversion or the endowment risk of the noise traders is very low and if the quality of the informed trader’s private information is very high, then the noise traders do not take a position in the stock to avoid losing to the informed trader. As a result, the market breaks down.

The main intuition for the existence of an equilibrium in our model is as follows. When the informed trader can change effort to adjust the quality of private information, it is always in his best interest to lower the quality of information to avoid the market breakdown when the hedging demand from the hedgers is not strong enough. The informed trader’s effort choice is correctly anticipated ex ante by the hedgers and market makers. Therefore, an equilibrium always exists, no matter how little noise trading there may seem to be ex ante. This result highlights the important role of endogenous information acquisition.

Our article is closely related to those of Kyle (1985), Subrahmanyam (1991), and Spiegel and Subrahmanyam (1992). Kyle develops a multi-period model of strategic trading with a risk-neutral informed trader. Subrahmanyam extends the one-period version of the Kyle model by introducing a risk-averse informed
trader; Spiegel and Subrahmanyam endogenize noise trading. Portfoliodeligation is absent in all of these models.

Dow and Gorton (1997) construct an equilibrium model with strategic trading and portfolio delegation. The risk-neutral portfolio manager may or may not receive a valuable signal about the asset payoff. The signal is obtained without effort expenditure, but there is still an agency problem. When the manager does not receive a valuable signal, no trading is optimal for the manager and the principal, but the manager may still trade like a noise trader.

The rest of this article is organized as follows. Section 1 presents the baseline model with exogenous noise trading. Section 2 extends the baseline model by endogenizing noise trading. Section 3 considers a risk-averse principal and discusses the empirical implications of our model. Section 4 concludes the article. All proofs are given in the Appendix.

1. The Baseline Model

Following Kyle (1985), the vast majority of strategic trading models assume exogenous noise trading. For comparison, we first build the baseline model of portfolio delegation and strategic trading based on Kyle. We will extend it in the next section to allow for endogenous noise trading by following Spiegel and Subrahmanyam (1992). Consider a market with an informed trader, a number of noise traders, and competitive risk-neutral market makers. These traders buy and sell a single asset at a price \( \tilde{p} \) at time 0. At time 1, the liquidation value of the asset, \( \tilde{\nu} \sim N(\tilde{\nu}, \sigma^2) \), is announced, and the holders of the asset are paid. The asset price, determined by the competitive market makers who earn zero expected profit, is set to equal the expectation of the liquidation value. The demand of noise traders for the risky asset is denoted by \( \tilde{u} \sim N(0, \sigma^2_u) \).

Different from Kyle (1985), we assume that the informed trader can decide on the extent to which he is informed through an endogenous information acquisition process. In particular, upon input of a level of effort \( \rho \), the agent obtains a noisy signal about the asset value \( \tilde{\theta}(\rho) = \tilde{\nu} + \tilde{\epsilon} \), where \( \tilde{\epsilon} \sim N(0, \sigma^2_\epsilon) \) is uncorrelated with \( \tilde{\nu} \), and \( \sigma^2_\epsilon \) is inversely related to the agent’s effort \( \rho \), satisfying \( \sigma^2_\epsilon = \sigma^2/\rho \). The cost of exerting effort \( \rho \) is assumed to be \( C(\rho) = k\rho^2/2 \), where \( k \) is a positive constant. The informed trader thus bases his trade on the private information \( \tilde{\theta}(\rho) \), and his order, denoted by \( \tilde{x} \), is a function of \( \tilde{\theta}(\rho) \). The market makers observe only the total order flow \( \tilde{y} = \tilde{x} + \tilde{u} \) and set the price to be \( \tilde{p} = P(\tilde{x} + \tilde{u}) = \tilde{\nu} + \lambda(\tilde{x} + \tilde{u}) \).

We further assume that the informed trader sells his private information in the form of a fund in which a representative uninformed, risk-neutral-investor

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8 Throughout the article, a letter with the tilde symbol \( \tilde{ } \) (e.g., \( \tilde{u} \)) denotes a random variable, and the letter itself (e.g., \( u \)) denotes the realization of the random variable.
(principal) entrusts her money to the informed trader, who serves as the fund manager (agent). The principal designs an optimal linear sharing rule, denoted by \( S(\tilde{W}) = a + b\tilde{W} \), to induce the agent to exert effort both for information acquisition and for subsequent trading in the stock. Here, \( \tilde{W} \) denotes the agent’s trading profits, and \( a \) and \( b \) are constants. Notice that some mutual funds also use indexes as benchmarks in their compensation schemes and that mutual funds may not be allowed to take large short positions. These features may also break the Admati–Pfleiderer irrelevance result, because they make managers’ undoing incentives costly, very much like what this article does. For simplicity, we omit these features from our model.

Moral hazard arises due to the inability of the principal to observe effort. The agent has a negative exponential utility function:

\[
U_A(S(\tilde{W}), \rho) \equiv -\frac{1}{R_a} \exp \left[ -R_a (S(\tilde{W}) - C(\rho)) \right].
\]

where \( R_a \) is the informed agent’s risk-aversion coefficient. The agent’s reservation utility is denoted by \( \hat{U} \).

In summary, the timeline of the model is as follows.

1. In Stage 1, the principal assigns a linear contract \( S(\tilde{W}) = a + b\tilde{W} \) to the agent. The contract is publicly announced.
2. In Stage 2, the market makers believe that, under the contract \( S(\tilde{W}) \), the agent would exert effort \( \rho_m(b) \) that depends on \( b \).\(^9\) They are committed to this belief, which turns out to be correct in equilibrium (i.e., they have rational expectations).
3. In Stage 3, under the contract and taking into account the belief held by the market makers \( \rho_m(b) \), the informed agent exerts effort \( \rho = RH O(\rho_m(b), b) \) and obtains a signal \( \tilde{\theta}(\rho) \). Here, \( RH O(\rho_m(b), b) \) denotes the optimal effort policy, and \( \tilde{\theta}(\rho) = \tilde{v} + \tilde{\epsilon} \) is a noisy signal about the liquidation value whose precision increases with effort \( \rho \): \( \sigma^2\epsilon = \sigma^2/\rho \).
4. In Stage 4, the informed agent chooses the optimal trading strategy based on the realized signal \( \theta \) and submits his order \( x = X(\theta; \rho, \rho_m(b), b) \) to market makers.
5. In Stage 5, the risk-neutral competitive market makers determine the stock price \( p = P(y; \rho_m(b), b) \) based on the total order flow \( y \) and their belief about the informed agent’s effort \( \rho_m(b) \).
6. In Stage 6, the liquidation value \( \tilde{v} \) is realized. The principal and the informed agent are compensated.

We solve the model backward.

\(^9\) Note that the constant payment \( a \) in the contract does not affect the agent’s effort \( \rho \) or trade \( x \) due to the absence of wealth effect, thanks to the (CARA)–normal framework.
Step 1: In Stage 5, market makers set the stock price to earn zero expected profit. Given the total order flow \( y = x + u \) and the linear contract \( S(\tilde{W}) = a + b \tilde{W} \), they set the price based on their beliefs about the agent’s effort:

\[
P(y; \rho_m(b), b) = E \left[ \tilde{v} \mid y = X (\tilde{\theta}(\rho); \rho, \rho_m(b), b) + \tilde{u}, \rho = \rho_m(b) \right].
\]

Step 2: In Stage 4, the informed agent solves for the optimal trading strategy. After having exerted effort \( \rho \) and obtained signal \( \tilde{\theta}(\rho) = \theta \), the informed agent’s expected utility is given by

\[
U_A(x; \theta, \rho, \rho_m(b), a, b)
= E \left[ U_A(a + bx [\tilde{v} - P(x + \tilde{u}; \rho_m(b), b)], \rho) \mid \tilde{\theta}(\rho) = \theta \right].
\]

Note that the agent’s trading profits are given by \( \tilde{W} = x[\tilde{v} - P(x + \tilde{u}; \rho_m(b), b)] \). The informed agent’s optimal trading strategy maximizes his expected utility \( \bar{U}_A \), that is,

\[
X(\theta; \rho, \rho_m(b), b) = \arg\max_x \bar{U}_A(x; \theta, \rho, \rho_m(b), a, b).
\]

The constant in the contract \( a \) does not affect the agent’s trading strategy because of no wealth effect in the framework of constant absolute risk aversion (CARA)–normal. For the same reason, the agent’s optimal effort, determined in the next step below, does not depend on \( a \) either.

Step 3: To determine the agent’s optimal effort, we solve the game in Stage 3. The agent’s expected utility, before exerting effort \( \rho \) and obtaining signal \( \tilde{\theta}(\rho) \), is

\[
\bar{U}_A(\rho; \rho_m(b), a, b)
= E \left[ \bar{U}_A(X(\tilde{\theta}(\rho); \rho, \rho_m(b), b); \tilde{\theta}(\rho), \rho, \rho_m(b), a, b) \right].
\]

Thus, the agent’s optimal effort satisfies

\[
RHO(\rho_m(b), b) = \arg\max_{\rho} \bar{U}_A(\rho; \rho_m(b), a, b).
\]

Step 4: In Stage 2, market makers form rational expectations. That is, for a given contract, their belief \( \rho_m(b) \) coincides with the informed agent’s optimal effort choice:

\[
\rho_m(b) = RHO(\rho_m(b), b).
\]

Mathematically, \( \rho_m(b) \) is the solution to the above fixed point problem. Later, we prove the existence of such a solution in Proposition 2.
Step 5: In the final step, we solve for the optimal contract, which is designed by the principal in Stage 1. The principal is risk neutral, and her expected utility, denoted by $\overline{U}_p(a, b)$, is given by

$$\overline{U}_p(a, b)$$

$$\equiv E \left[ (1 - b) \tilde{W} - a \right]$$

$$= E \left[ -a + (1 - b) X (\tilde{\theta} (RHO (\rho_m (b), b)) ;
\times RHO (\rho_m (b), b), \rho_m (b), b)
\times (\tilde{\nu} - P \left( X (\tilde{\theta} (RHO (\rho_m (b), b)) ;
\times RHO (\rho_m (b), b), \rho_m (b), b) + \tilde{u}, b) \right) \right].$$

The optimal contract then maximizes $\overline{U}_p(a, b)$:

$$(a^*, b^*) = \arg \max_{(a, b)} \overline{U}_p(a, b)$$

subject to various constraints to be specified next.

We formally define the equilibrium as follows.

**Definition 1.** An equilibrium consists of an optimal contract $(a^*, b^*)$, an optimal effort choice $\rho^*(b) = RHO (\rho_m (b), b)$, an optimal trading strategy $x^*(\theta; \rho, b) = X (\theta; \rho, \rho_m (b), b)$, an optimal pricing function: $p^*(y; b) = P (y; \rho_m (b), b)$, and the rational prior belief $\rho^*(b) = \rho_m (b)$. The optimal contract $(a^*, b^*)$ maximizes the principal’s expected utility:

$$(a^*, b^*) = \arg \max_{(a, b)} \overline{U}_p(a, b),$$

subject to the following constraints:

$$\rho^*(b) = \arg \max_{\rho} \overline{U}_A (\rho; \rho_m (b), a, b),$$

$$x^*(\theta; \rho, b) = \arg \max_{x} \overline{U}_A (x; \theta, \rho, \rho_m (b), a, b),$$

$$\overline{U}_A (\rho^*(b); \rho_m (b), a, b) = \hat{U},$$

$$\rho_m (b) = \rho^*(b),$$

$$p^*(y; b) = E \left[ \tilde{v} \mid y = x^* (\tilde{\theta} (b) ; \rho^*(b), b) + \tilde{u} \right].$$

In Definition 1, Equation (1) determines the optimal contract, subject to the incentive compatibility constraints in Equations (2a) and (2b), the individual participation constraint in Equation (2c), the rational expectations constraint in Equation (2d), and the market efficiency constraint in Equation (2e).
Proposition 1. In Stage 5, given a linear contract \((a, b)\) and the belief \(\rho_m(b)\), market makers believe that the informed agent has exerted effort \(\rho = \rho_m(b)\) and his trading strategy is \(X(\theta; \rho_m(b), \rho_m(b), b) = \beta_m(\rho_m(b), b)(\theta - \bar{\theta}).\) Consequently, market makers set the pricing rule as \(P(y; \rho_m(b), b) = \bar{\theta} + \lambda_m(\rho_m(b), b)y\), where \(y\) is the total order flow and \(\lambda_m\) is given by

\[
\lambda_m = \frac{\beta_m}{\beta_m^2 m(1 + 1/\rho_m) + \sigma_u^2/\sigma^2}.
\]

Note that \(\lambda_m\) and \(\beta_m\) are both functions of \(\rho_m(b)\) and \(b\). For notational ease, we omit their arguments. We use the subscript \(m\) to indicate that these variables are determined under the belief \(\rho_m(b)\) of other market participants.

In Stage 4, the informed agent’s optimal trading strategy is shown as \(x^*(\theta; \rho, b) = X(\theta; \rho, \rho_m(b), b) = \beta^*(\rho, b)(\theta - \bar{\theta})\), with the trading intensity \(\beta\) given by

\[
\beta^*(\rho, b) = \frac{\rho}{2\lambda_m + Ra b[\sigma^2/(1 + \rho) + \lambda_m^2 \sigma_u^2]},
\]

where \(\rho\) is the agent’s effort chosen in Stage 3.

In Stage 3, the agent’s optimal effort \(\rho^*(b) = RH\Omega(\rho_m(b), b)\) satisfies a first-order condition:

\[
C_\rho(\rho^*(b)) = \frac{b \sigma^2}{2} Ra b \sigma^2 \beta^*(\rho^*(b), b) + 1 \frac{d \beta^*(\rho, b)}{d \rho} \bigg|_{\rho=\rho^*(b)},
\]

which can be simplified to the following cubic equation:

\[
\rho^*(b) (\rho^*(b) + 1) \left[ \left( 2 \lambda_m + Ra b \sigma_u^2 \lambda_m^2 \right) (\rho^*(b) + 1) + Ra b \sigma^2 \right] = \frac{b}{2k} \sigma^2.
\]

In Stage 2, market makers have rational expectations by correctly anticipating the agent’s effort choice and trading strategy. That is,

\[
\rho_m(b) = \rho^*(b), \quad \beta_m(\rho_m(b), b) = \beta^*(\rho^*(b), b).
\]

Note that the optimal responses are all functions of \(b\). Similarly, we denote \(\lambda_m(\rho^*(b), b)\) by \(\lambda^*(\rho^*(b), b)\).

In Stage 1, the optimal contract is determined through the following optimization:

\[
\max_b \left\{ (1 - b) \beta^*(\rho^*(b), b) \sigma^2 \right. \\
\left. \times \left[ 1 - \lambda^*(\rho^*(b), b) \beta^*(\rho^*(b), b) (1 + 1/\rho^*(b)) \right] - a^*(b) \right\},
\]

where \(a^*(b)\) is chosen to satisfy the participation constraint as follows:

\[
a^*(b) = -R_a^{-1} \log (-\hat{U} R_a) + \frac{1}{2} k \left[ \rho^*(b) \right]^2 - \frac{1}{2R_a} \log \left( R_a b \beta^*(\rho^*(b), b) \sigma^2 + 1 \right).
\]
In the proposition below, we prove the existence and uniqueness of the equi-
librium from Stage 2 on, following the announcement of contract \( S(\tilde{W}) = a + b \tilde{W} \). The existence of the overall equilibrium follows immediately from
the proposition because the optimal contract is determined by solving the prin-
cipal’s optimization problem over the compact interval of \([0, 1]\).

**Proposition 2.** Following the announcement of \( S(\tilde{W}) = a + b \tilde{W} \), the optimal response functions \( \rho^*, \beta^*, \) and \( \lambda^* \) exist and are unique.

The proof of Proposition 2, which is presented in the Appendix, suggests a
way to solve our model. First, holding \( b \) and \( \rho \) fixed, we can obtain the unique
solution \( \beta^* (\rho, b) \) by solving Equation (A3) in the Appendix, which is derived
from Equations (3) and (4). We can then determine \( \lambda^* (\rho, b) \) from Equation
(3). We next solve for the agent’s optimal effort \( \rho^* (b) \) from the first-order
condition in Equation (6), which admits a unique solution as proved in the
proposition. Finally, we substitute \( \rho^* (b), \beta^* (\rho^* (b), b), \) and \( \lambda^* (\rho^* (b), b) \)
into the principal’s objective function in Equation (8) and search for the opti-
mal \( b^* \) within the interval of \([0, 1]\).

1.1 The relationship between incentives and effort

Admati and Pfleiderer (1997) develop an innovative portfolio delegation model
under a competitive partial equilibrium. Among many important findings, they
obtain a striking result, that is, the manager’s effort of acquiring information
is independent of the incentive contract. This irrelevance result challenges the
traditional principal–agent literature in which a higher slope (\( b \)) in a linear
contract typically induces a higher level of effort (\( \rho \)) from the agent. This result
also highlights the differences between a traditional principal–agent model in
which the agent expends effort only and a portfolio delegation model in which
the manager first expends effort for information acquisition and then trades in
the stock based on the acquired information. In our portfolio delegation model,
which features both strategic trading and endogenous stock price, we recover
the traditional result that, all else being equal, a higher \( b \) induces a higher \( \rho \).

We first note that, in a competitive partial equilibrium, the portfolio man-
ger’s optimal position in the stock, after he has exerted effort \( \rho \), is given by

\[
X(\theta) = \frac{E_{\theta}(\tilde{v}) - P}{R_ab\text{Var}_{\theta}(\tilde{v})} = \frac{\rho (\theta - \tilde{v})}{R_ab^2} = \beta (\theta - \tilde{v}).
\]

Here, \( P \) denotes the stock price, which is equal to the unconditional mean
\( \tilde{v} \), and \( \beta \) is given by

\[
\beta = \frac{\rho}{R_ab^2}.
\]

The manager’s wealth is thus given by

\[
\tilde{W}_A = a + bX(\theta)(\tilde{v} - P) = a + b\beta (\theta - \tilde{v}) (\tilde{v} - P).
\]
Notice that $b\beta = \rho/R\sigma^2$ is independent of $b$, so the manager’s wealth $\tilde{W}_A$ does not depend on $b$. For example, if we double $b$, then the manager would reduce his position by half given a fixed $\rho$, resulting in the same amount of effective exposure $(b\beta)$ to the asset payoff. Consequently, the manager’s effort $\rho$ is independent of $b$.

In our strategic trading model, given a level of effort $\rho$, the manager’s optimal position $\beta$ in the stock is given in Equation (4). Only when $\lambda_m$ is zero will Equation (4) reduce to Equation (10). In general, because of the market impact cost associated with the manager’s trading, the manager cannot leverage up or down the position as much as in the price-taking case. Hence, the manager’s exposure to the risky asset payoff is higher, that is, $b\beta$ increases with $b$. As a result, a higher incentive slope $b$ leads to a higher level of $\rho$. We have performed numerous numerical calculations and confirmed that, in all of these calculations, a higher $b$ always leads to a higher $b\beta$, inducing a higher $\rho$ from the manager.

We present one of the calculations in Figure 1. The dashed lines in Figure 1 correspond to the competitive equilibrium of Admati and Pfleiderer (1997) in which the asset price is exogenously assumed (particularly $\lambda = 0$, as shown in Subplot A4) and $\rho$ is independent of $b$. To facilitate the comparison with our model, we fix $\rho$ to be 0.197 in Admati and Pfleiderer, which is the optimal effort level in our model without portfolio delegation (i.e., $b = 1$). From Subplot A3 of Figure 1, we can see that the optimal $\rho$ increases with $b$ in our model. This “relevance” result leads to different behavior of $\beta$ when $b$ tends to zero (Figure 1(A1)). By construction, the optimal $\beta$-values in both models coincide at $b = 1$. When $b$ converges to zero, $\beta$ approaches infinity per Admati and Pfleiderer due to zero price impact. In our model, however, when $b$ tends to zero, $\beta$ converges to zero as a result of deteriorating information quality (i.e., $\rho$ converges to zero, as shown in Figure 1(A3)), even though the price impact parameter $\lambda$ diminishes to zero (Figure 1(A4)). The “relevance” result and the resulting different equilibrium outcomes in our model suggest the importance of developing a strategic trading model in the context of delegated portfolio management.

1.2 Information acquisition and price informativeness

We next examine the impact of introducing information acquisition on the price informativeness. To distinguish its impact from that of portfolio delegation, we assume in this subsection that the agent trades for his own account. Portfolio delegation will be reintroduced in the next subsection. As did Kyle (1985), we define the price informativeness as the posterior precision of $\tilde{v}$ conditional on the equilibrium price:

$$Q = [Var (\tilde{v} | P)]^{-1} = \frac{1}{\sigma^2} + \frac{1}{\sigma^2 + \sigma^2/\beta^2} = \frac{1}{\sigma^2} + \frac{1}{\sigma^2/\rho + \sigma^2/\beta^2}. \quad (11)$$
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Figure 1
Comparison with Admati and Pfleiderer (1997)

Figure 1 gives a graphical illustration of the comparison between our model and Admati and Pfleiderer’s (1997). The dashed lines correspond to the competitive equilibrium of Admati and Pfleiderer (1997), where both the effort and the asset price are exogenously assumed and the manager is a price taker. The solid lines correspond to our benchmark model of portfolio delegation and strategic trading in the case of exogenous noise trading. Other parameters are $R_a = \sigma_u^2 = 2$, $k = \sigma^2 = 1$. The dash-dotted lines correspond to our general model of portfolio delegation and strategic trading in the case of endogenous noise trading. Other parameters are $R_a = \sigma_u^2 = 2$, $R_h = m = k = \sigma^2 = 1$.

Two effects determine the price informativeness $Q$. On the one hand, holding effort constant, an increase in noise trading or a decrease in informed trading decreases $Q$. In the Kyle model in which the informed agent is risk neutral, $Q$ is independent of the variance of noise trading $\sigma_u^2$. When the informed agent is risk averse, Subrahmanyam (1991) finds that $Q$ decreases with $\sigma_u^2$ because the risk-averse trader responds less aggressively to an increase in $\sigma_u^2$. On the other hand, an increase in effort $\rho$ not only has a direct positive effect on $Q$, since the private information is more accurate, but also indirectly enhances $Q$, since it enables the informed agent to trade more aggressively on better information (i.e., $\beta$ increases). We show that, when the agent is sufficiently risk tolerant, the second effect dominates the first one, hence, $Q$ increases with $\sigma_u^2$.

Figure 2 demonstrates the relation between $Q$ and $\sigma_u^2$ for three different values of the risk-aversion coefficient, $R_a = 0.1, 1, 2$. Let us focus on Subplots A4–A6 for now. We will study Subplots A1–A3 in the next subsection after we reintroduce portfolio delegation. In the absence of both information
Figure 2
Price informativeness ($Q$) vs. the variance of noise trading ($\sigma_u^2$)

Figure 2 plots the relation between the price informativeness ($Q$) and the variance of noise trading ($\sigma_u^2$). The solid lines correspond to the general case of information acquisition and portfolio delegation; the dashed lines correspond to the case of exogenous information and without portfolio delegation; the dash-dotted lines correspond to the case of endogenous information acquisition without portfolio delegation (i.e., $b = 1$). Other parameters are $\sigma^2 = k = 1$, $\sigma_u^2$ ranges from 0.1 to 5, and $R_a = 0.1, 1, 2$.

acquisition and portfolio delegation, depicted by the dashed lines in Subplots A4–A6, where effort is exogenously fixed at the level of 0.26 and $b = 1$, $Q$ always decreases with $\sigma_u^2$, which is consistent with the original result of Subrahmanyam (1991).\(^\text{10}\) When information acquisition is allowed, according to the dash-dotted lines in Subplots A4–A6, if $R_a$ is around 0.1, $Q$ increases monotonically with $\sigma_u^2$, whereas if $R_a$ is around 1, it decreases monotonically.

From unreported results, the relationship exhibits a hump shape when $R_a$ is between 0.1 and 1.

Absent information acquisition (i.e., $\rho$ is fixed at 0.26, depicted by the dashed line), when noise trading increases (e.g., $\sigma_u$ increases by 41% from $\sqrt{2}$ to 2), $Q$ decreases because the informed trader’s trading intensity $\beta$ does not increase as much (e.g., it increases only by 33% from 0.52 to 0.69). Once information

\(^{10}\) The exogenous effort of 0.26 is the optimal effort level under information acquisition and portfolio delegation in our baseline specification of parameters where $R_a = 1$, $\sigma^2 = k = 1$, and $\sigma_u^2 = 2$. 

acquisition is allowed, for the same increase of 41% in $\sigma_u$, the trader is now able to exert more effort to collect more accurate information. In particular, the level of his effort increases by 19% from 0.36 to 0.43, both of which are higher than the fixed level of 0.26 in the absence of information acquisition. With more accurate information, the agent trades more aggressively, increasing $\beta$ by 44% from 0.61 to 0.88. The significant increase in informed trading, along with the increase in effort, dominates the increase in noise trading, resulting in higher price informativeness. Therefore, the positive relation between price informativeness and noise trading for a small $R_a$ is attributable mainly to the dramatic increase in trading intensity that is fueled by better information acquired due to the increase in effort itself. This result highlights the importance of information acquisition.

From Subplots A5 and A6 in Figure 2, we can see that, if the informed trader is more risk averse (say, $R_a = 1$ or 2), $Q$ decreases monotonically with $\sigma_u^2$ even when information acquisition is allowed. This is because when the informed trader has an exponential utility function and all random variables are normally distributed, the marginal benefits of trading more aggressively and acquiring more accurate information decrease with $R_a$. When the informed agent is very risk averse, the increases in his effort for information acquisition and trading aggressiveness are dominated by the increase in noise trading, resulting in a less informative price in equilibrium.

We summarize the above results in Proposition 3, whose proof is given in the Appendix.

**Proposition 3.** If $R_a$ is sufficiently small (large), $Q$ increases (decreases) monotonically with $\sigma_u^2$.

### 1.3 Portfolio delegation, optimal contract, and price informativeness

In this subsection, we introduce portfolio delegation. We find that portfolio delegation allows $Q$ to increase with $\sigma_u^2$ for a wider range of $R_a$ values. For example, Subplot A5 of Figure 2 demonstrates that, without portfolio delegation, $Q$ decreases with $\sigma_u^2$ when $R_a = 1$. By contrast, once portfolio delegation is introduced, the same increase in noise trading can actually enhance the price informativeness, as shown in Subplot A2 of Figure 2.

The main intuition is that portfolio delegation makes the risk aversion of the fund a combination of the risk aversions of the investor and the manager, effectively reducing the risk aversion of the manager as long as the investor is less risk averse than the manager. For example, under the parameter specification used in Subplots A2 and A5 of Figure 2 when $R_a = 1$, in response to an increase in $\sigma_u^2$, the investor lowers the slope $b$ in the optimal contract (Figure 3 (A2)), which monotonically increases both $\rho$ and $\beta$ (Figure 3 (A5, A8)).

---

11 Because investors can diversify away idiosyncratic risk by investing in different funds, it is perhaps fine to assume that investors are less risk averse than managers. We thank a referee for the intuition.
Comparative statics with respect to the variance of noise trading ($\sigma_u^2$) for various degrees of risk aversion ($R_a$).

Figure 3 depicts the comparative statics results with respect to the variance of noise trading ($\sigma_u^2$) for various degrees of risk aversion ($R_a$). The solid lines correspond to the general case of information acquisition and portfolio delegation; the dashed lines correspond to the case of exogenous information and without portfolio delegation; the dash-dotted lines correspond to the case of endogenous information acquisition without portfolio delegation (i.e., $b = 1$). Other parameters are $\sigma^2 = k = 1$, $\sigma_u^2$ ranges from 0.1 to 5, and $R_a = 0.1, 1, 2$.

The positive effect on price informativeness by more aggressive trading upon better information dominates the negative one from more noise trading, resulting in the positive relation between $Q$ and $\sigma_u^2$. This explains Subplot A2 of Figure 2.

On the other hand, if the agent is very risk averse (e.g., $R_a = 2$), then the increases in $\rho$ and $\beta$ (Figure 3 (A6, A9)) become smaller. The reason is that, although a lower $b$ makes the manager effectively less risk averse, it also makes the manager less incentivized to exert effort $\rho$ because the manager receives a lower share of the profit. As an extreme example, $b = 0$ could make the manager essentially risk neutral but $\rho$ and $\beta$ would also be zero. In this case, the stock price would contain no private information. Hence, when the manager is very risk averse, say $R_a = 2$, his effective risk aversion, $R_ab$, can still remain high. As a result, the increase in noise trading eventually dominates the increase in the manager’s trading intensity, suggesting that $Q$ will eventually decrease with $\sigma_u^2$ after it reaches a certain level. This explains Subplot A3 of Figure 2.
2. The Extended Model with Endogenous Noise Trading

In this section, we extend the baseline model by endogenizing noise trading based on Spiegel and Subrahmanyam (1992). In the Spiegel–Subrahmanyam model, besides multiple risk-neutral informed traders and market makers, there are $m$ uninformed risk-averse hedgers who maximize their expected utilities to hedge their endowment risk. Each hedger $j$ has an endowment $\tilde{z}_j \sim N(0, \sigma^2_z)$ of the asset, and his order for the stock is a function of $\tilde{z}_j$, denoted by $\tilde{u}_j$. The sum of the hedgers’ orders is denoted by $\tilde{u} = \sum_{j=1}^{m} \tilde{u}_j$. The hedgers have negative exponential utility with a common risk-aversion coefficient $R_h$. Specifically, hedger $j$’s utility is given by

$$U_H(\tilde{V}_j; z_j) = -\frac{1}{R_h} \exp \left[-R_h \tilde{V}_j \right],$$

where, given the realization of his endowment $z_j$, $\tilde{V}_j (u_j; z_j)$ is his payoff given by

$$\tilde{V}_j (u_j; z_j) = \tilde{v} (u_j + z_j) - u_j \tilde{p}.$$

Spiegel and Subrahmanyam (1992) construct a linear equilibrium in which the optimal strategies for the uninformed hedgers are given by $\tilde{u}_j = \gamma \tilde{z}_j$.

We introduce portfolio delegation and study optimal contracting between a risk-neutral investor and a risk-averse informed agent, as in the previous section. The key difference is that the level of noise trading is now endogenously determined by the hedging demand of the hedgers. Therefore, when assigning a contract, the investor needs to consider the effects of the contract on the trading intensity of the hedgers and the informed agent, as well as on the pricing by the market makers.

The timeline of the model is similar as before, except that in Stage 2, following the announcement of a contract $S(\tilde{W}) = a + b \tilde{W}$, the hedgers share the same rational belief with the market makers that the agent would exert effort $\rho_m (b)$ that depends on $b$. And then in Stage 4, when the informed agent chooses the optimal trading strategy, simultaneously, uninformed hedger $j$ chooses his optimal trading strategy and submits order $\tilde{u}_j = U_j (z_j; \rho_m (b), b)$ to market makers, $j = 1, \cdots, m$. Following Spiegel and Subrahmanyam (1992), we assume that all hedgers are identical but that their initial endowments are independently distributed. Therefore, symmetric equilibrium trading strategies exist where they have identical equilibrium trading strategies: $U_j (\cdot; \rho_m (b), b) \equiv U (\cdot; \rho_m (b), b), \forall j$.

Specifically in Stage 4, hedger $j$’s payoff $V_j$ is given by

$$V_j = \tilde{v} (u_j + z_j) - u_j P = \tilde{v} (u_j + z_j) - u_j \left( \tilde{v} + \lambda_m \beta_m (\theta - \tilde{v}) + \lambda_m u_j + \lambda_m \sum_{k \neq j} \gamma z_k \right).$$
Conditional on $z_j$, $V_j$ is normally distributed with the following mean and variance:

$$E[V_j | z_j] = \bar{v} z_j - \lambda_m (u_j)^2,$$

$$\text{Var}[V_j | z_j] = ((1 - \lambda_m \beta_m) u_j + z_j)^2 \sigma^2 + (u_j \lambda_m \beta_m)^2 \sigma^2 \epsilon^2$$

$$+ (m - 1) \left( u_j \lambda_m \gamma \right)^2 \sigma^2 \epsilon^2.$$

Hedger $j$’s optimal trading strategy $u_j = U(z_j; \rho_m(b), b)$ maximizes his expected utility or equivalently maximizes the certainty equivalent $E[V_j | z_j] - 0.5 \mathbb{R}_h \text{Var}[V_j | z_j]$. The first-order condition is given by

$$2 \lambda u_j$$

$$= -\mathbb{R}_h \left\{ ((1 - \lambda_m \beta_m) u_j + z_j) (1 - \lambda_m \beta_m) \sigma^2$$

$$+ u_j \left[ (\lambda_m \beta_m)^2 \sigma^2 \epsilon + (m - 1) (\lambda_m \gamma)^2 \sigma^2 \epsilon \right] \right\}.$$

Therefore, we have

$$u_j^* = U(z_j; \rho_m(b), b)$$

$$= \frac{R_h (1 - \lambda_m \beta_m) \sigma^2 z_j}{2 \lambda_m + R_h [(1 - \lambda_m \beta_m)^2 \sigma^2 + (\lambda_m \beta_m)^2 \sigma^2 \epsilon + (m - 1) (\lambda_m \gamma)^2 \sigma^2 \epsilon]}$$

$$\equiv \gamma_m(\rho_m(b), b) z_j,$$

where

$$\gamma_m(\rho_m(b), b)$$

$$= \frac{R_h (1 - \lambda_m \beta_m) \sigma^2}{2 \lambda_m + R_h [(1 - \lambda_m \beta_m)^2 \sigma^2 + (\lambda_m \beta_m)^2 \sigma^2 \epsilon + (m - 1) (\lambda_m \gamma)^2 \sigma^2 \epsilon]}.$$ (12)

The informed agent’s trading strategy and the market makers’ pricing function are similar as before, except that $\sigma_u^2$ is now replaced by $m \gamma_m^2 \sigma_z^2$, that is,

$$\lambda_m(\rho_m(b), b) = \frac{\beta_m}{\rho_m^2 (1 + 1/\rho_m) + m \gamma_m^2 \sigma_z^2 / \sigma^2},$$ (13)

$$\beta(\rho; \rho_m(b), b) = \frac{\rho / (1 + \rho)}{2 \lambda_m + \mathbb{R}_b \sigma^2 / (1 + \rho) + \lambda_m^2 m \gamma_m^2 \sigma_z^2}.$$ (14)

The agent’s optimal effort choice problem and the principal’s optimization problem have the same functional forms as in Equations (5), (8), and (9).
The solution procedure is similar as before. For a given $b$, we start with an initial guess $\gamma^{(0)} = -1$. Treating the implied level of noise trading $\sigma_u^{(0)} = \sqrt{m \gamma^{(0)^2} \sigma_z^2}$ as exogenously given, we can then follow the methodology for the case of exogenous noise trading to solve for $\beta^{(0)}(b)$ and $\lambda^{(0)}(b)$, based on which we next obtain an updated value $\gamma^{(1)}$ from Equation (12). If $\gamma^{(1)}$ equals $\gamma^{(0)}$, then we are done; otherwise, repeat the previous steps until $\gamma^{(0)}, \gamma^{(1)}, \cdots$, converge. Upon convergence, we arrive at the optimal response functions $\beta^*(b), \lambda^*(b), \gamma^*(b)$ for a given $b$. Finally, we maximize the principal’s expected utility to solve for the optimal $b^*$ within the interval of $[0, 1]$.

2.1 A recap of Spiegel and Subrahmanyam (1992)

For convenience of comparison, we report the main results of Spiegel and Subrahmanyam when there is only one risk-neutral informed trader.

Proposition 4. If

$$R_h^2 m \sigma_z^2 \left( \sigma^2 + 2\sigma^2 \right)^2 > 4 \left( \sigma^2 + \sigma^2 \right),$$

then the unique linear equilibrium is given by

$$\lambda = \frac{R_h \sigma^2 \left[ (2m - 1)/m \sigma^2 + 4\sigma^2 \right]}{4 \sqrt{\sigma^2 + \sigma^2} \left[ R_h m^{1/2} \sigma_z \left( \sigma^2 + 2\sigma^2 \right) - 2 \sqrt{\sigma^2 + \sigma^2} \right]},$$

$$\beta = \frac{2 \left[ R_h m^{1/2} \sigma_z \left( \sigma^2 + 2\sigma^2 \right) - 2 \sqrt{\sigma^2 + \sigma^2} \right]}{R_h \sqrt{\sigma^2 + \sigma^2} \left[ (2m - 1)/m \sigma^2 + 4\sigma^2 \right]},$$

$$\gamma = -\frac{2 \left[ R_h m^{1/2} \sigma_z \left( \sigma^2 + 2\sigma^2 \right) - 2 \sqrt{\sigma^2 + \sigma^2} \right]}{R_h \sqrt{m \sigma_z} \left[ (2m - 1)/m \sigma^2 + 4\sigma^2 \right]}.$$  

Moreover, the stock price informativeness $Q$ is given by

$$Q = \left[ \text{Var} \left( \tilde{u} \mid P \right) \right]^{-1} = \frac{\beta^2 \left( \sigma^2 + \sigma^2 \right) + m \gamma^2 \sigma_z^2}{\sigma^2 \left[ \beta^2 \sigma_z^2 + m \gamma^2 \sigma_z^2 \right]} = \frac{1}{\sigma^2} + \frac{1}{\sigma^2 + 2\sigma_z^2}.$$  

Proof. See the proof of Proposition 1 in Spiegel and Subrahmanyam (1992). □

Note that, when a hedger is more risk averse (i.e., $R_h$ is larger) or his endowment is more volatile (i.e., $\sigma_z$ is larger), his hedging demand is higher (i.e., $|\gamma|$ and $\text{Var} \left( u_j \right)$ are both larger). In a limiting case in which there is
one hedger (i.e., \( m = 1 \)), who is infinitely risk averse (i.e., \( R_h = \infty \)), we have
\[ \gamma = -\frac{2\sigma_z^2 + 4\sigma_f^2}{\sigma_z^2 + 4\sigma_f^2}. \]
This case corresponds to the Kyle (1985) model with exogenous noise trading when we let \( \tilde{u} = \sum_{j=1}^{m} \tilde{u}_j \sim N(0, \sigma_u^2) \) with \( \sigma_u^2 = \gamma^2 \sigma_z^2. \)

Because the risk-averse hedgers are uninformed about the stock payoff, an increase in the uninformed hedging demand decreases \( Q \). On the other hand, when the uninformed hedging demand increases, the informed trader will increase his demand to take advantage of the uninformed trading, which increases \( Q \). When the informed trader is risk neutral, the two effects offset each other exactly, so that \( Q \) is independent of \( m, R_h, \) and \( \sigma_z \), as given in Equation (19).

### 2.2 Will the market break down?

In the Spiegel–Subrahmanyam model, the market breaks down when the condition in Equation (15) is violated. Specifically, this condition requires that \( R_h, \sigma_z^2, \) or \( m \) be large enough for an equilibrium to exist. In their model, the risk-averse noise traders face an endowment risk. On the one hand, they would like to hedge this risk, but on the other hand, they would not like to lose to the informed trader. Hence, their trade-off is between the utility gain from hedging the endowment risk and the utility loss from losing to the informed trader. For example, if the risk aversion or the endowment risk of the noise traders is very low and if the quality of the informed trader’s private information is very high, then the noise traders would not take any position in the stock to avoid losing to the informed trader. As a result, the market would break down.

We observe that the possibility of the market breakdown given by Spiegel and Subrahmanyam (1992) is due to the absence of information acquisition. We show that, once information acquisition is allowed, there always exists a linear equilibrium. The reason is that, even if there is not sufficient noise trading to support an equilibrium for a given \( \sigma_z^2 \), the informed agent is aware of this and will optimally lower his effort to become less informed, resulting in a higher \( \sigma_z^2 \). The agent’s effort choice is correctly anticipated ex ante by the hedgers and market makers. Therefore, an equilibrium always exists no matter how small \( R_h, \sigma_z^2, \) or \( m \) might be, as long as the hedgers are risk averse (\( R_h > 0 \)).

This result highlights the important role of information acquisition.

To illustrate this point, we conduct an asymptotic analysis regarding \( R_h \) in a special case in which \( R_d = 0 \) and \( m = 1 \). We show that, as long as \( R_h \) is strictly positive, no matter how close \( R_h \) is to zero, there always exists an equilibrium under information acquisition.

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12 Intuitively, the condition in Equation (15) can always be satisfied by increasing \( \sigma_z^2 \).
A Model of Portfolio Delegation and Strategic Trading

Figure 4
Comparative statics with respect to hedgers' risk aversion ($R_h$)
Figure 4 depicts the optimal response functions with respect to $R_h$ in the case of endogenous noise trading with a monopolistic risk-neutral informed trader ($R_a = 0$) with information acquisition only. Other parameters are $\sigma^2 = k = m = 1, \sigma^2_z = 5$.

Proposition 5. With endogenous information acquisition, no matter how small $R_h$ is, there always exists an equilibrium. The equilibrium solutions have the following asymptotic expressions if $R_a = 0$ and $m = 1$:

$$\rho^* \approx \sigma^2 \sigma^2_z R_h^2, \quad \beta^* \approx 2k\sigma^2 \sigma^4_z R_h^4, \quad \lambda^* \approx \left(4k\sigma^2_z\right)^{-1} R_h^{-2},$$

$$\gamma^* \approx -2k\sigma^2 \sigma^2_z R_h^3.$$

Figure 4 depicts the optimal response functions with respect to $R_h$. Unless otherwise specified, we use $\sigma^2 = 1, k = 1, \sigma^2_z = 5, \text{ and } m = 1$, similar to those used by Spiegel and Subrahmanyam (1992). This figure confirms the asymptotic expressions in the proposition above for small $R_h$ (e.g., $R_h < 0.01$). The optimal response functions depicted in Figure 4 have an intuitive interpretation. As $R_h$ becomes smaller, the hedger becomes less risk averse and thus has less motive to hedge, which implies less noise trading (i.e., smaller $|\gamma|$). In anticipation of this, the informed agent scales back his effort and trading intensity, and the market makers decrease $\lambda$.

We next prove the existence of an equilibrium in the general case. The sketch of the proof is as follows. First, from the existence of an equilibrium in the case of exogenous noise trading shown in Proposition 2, given a value $\gamma < 0$, if we...
define $\sigma_a^2 \equiv m \gamma^2 \sigma_z^2$, there always exists a set of solutions $\beta(\gamma)$, $\lambda(\gamma)$, and $\rho(\gamma)$ to Equations (3), (4), (6), and (7). Second, if we denote the right-hand side of Equation (12) as an operator $T(\gamma)$, that is,

\[
T(\gamma) \equiv -\frac{R_h(1 - \lambda(\gamma)\beta(\gamma))\sigma^2}{2\lambda(\gamma) + R_h[(1 - \lambda(\gamma)\beta(\gamma))^2\sigma^2 + (\lambda(\gamma)\beta(\gamma))^2\rho(\gamma) + (m - 1)(\lambda(\gamma)\gamma^2\sigma_z^2)]},
\]

then we prove that there exists a fixed point $\gamma^* < 0$, such that $T(\gamma^*) = \gamma^*$. Hence, the optimal responses are given by $\rho^* = \rho(\gamma^*)$, $\beta^* = \beta(\gamma^*)$, and $\lambda^* = \lambda(\gamma^*)$. To prove the existence of the fixed point, we demonstrate in the proof that there always exist $\gamma_a$ and $\gamma_b$, $\gamma_b < \gamma_a < 0$, such that $T(\gamma_a) < \gamma_a$ and $T(\gamma_b) > \gamma_b$.

In contrast, according to Spiegel and Subrahmanyam (1992), where $\rho$ is exogenously fixed, $\gamma_a$ that satisfies $T(\gamma_a) < \gamma_a$ does not always exist. Consequently, the condition in Equation (15) is needed to ensure the existence of $\gamma_a$.

**Proposition 6.** With endogenous information acquisition, there always exists an equilibrium.

### 2.3 Incentives, effort, and price informativeness

Notice that the informed manager’s trading intensity $\beta$, as expressed in Equation (14), takes the same form as in Equation (4). Due to the presence of market impact, the manager cannot leverage up or down as much as allowed by Admati and Pfleiderer (1997). As a result, $b\beta$ increases with $b$. Consequently, we find that higher incentives lead to higher effort, as in the case of exogenous noise trading.

Subplot A3 of Figure 1 presents one of the calculations. For a given $b$, the optimal $\rho$ in the current case is lower than that in the case of exogenous noise trading, whose results are depicted by the solid lines. The intuition is the following. When $b$ is close to zero, there is little private information or informed trading because $\rho$ and $\beta$ are both close to zero. In this case, the uncertainty about the asset’s liquidation value is very high and the informed trading is very low, allowing the uninformed hedgers to almost fully hedge their endowments, that is, $\gamma$ is close to $-1$. For any positive $b$, there is positive informed trading, and the hedgers do not fully hedge their endowments due to adverse selection.

---

13 Mathematically, when $b$ is small enough, we have

\[
\rho^*(b) \approx Ab^{2/3}, \quad \beta^*(b) \approx \left(\frac{2kA^2}{\sigma^2}\right)b^{1/3}, \quad \lambda^*(b) \approx \frac{\sigma^2 b^{1/3}}{4kA}, \quad \gamma^*(b) \approx -1 + \frac{b^{1/3}}{2kR_hA},
\]

where $A = \left(\frac{\sqrt{m\sigma_x^2}}{2k}\right)^{2/3}$. In the limiting case where $b = 0$, we obtain that $\gamma^* = -1$. In Figure 1, we choose $m = 1$ and $\sigma_x^2 = \sigma_y^2 = 2$ so that the noise levels in both cases of endogenous and exogenous noise trading are the same when $b = 0$. 

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resulting in less noise trading. As a result, the market is less liquid than in the case of exogenous noise trading (Figure 1 (A4)). The informed agent’s trading intensity $\beta$ is thus lower, and so is his exposure to the risky asset payoff, $b\beta$ (Figure 1 (A2, A3)). Consequently, the agent exerts lower effort than in the case of exogenous noise trading.

The optimal contract and price informativeness depend critically on the properties of the uninformed hedgers, such as the number of hedgers ($m$), their endowment risk ($\sigma^2_z$), and their risk-aversion coefficients ($R_h$). For example, we obtain that $b$ decreases as $m$ increases, similar to the negative relationship between $b$ and $\sigma_u^2$ in the case of exogenous noise trading as shown in Subplots A1–A3 of Figure 3. We have demonstrated that, when $m$ is fixed, $b\beta$ increases with $b$. When noise trading or $m$ goes up, increasing $b$ is not optimal because the induced higher level of effort not only increases the market impact cost but also reduces the uninformed hedgers’ incentive to trade. Consequently, when the number of uninformed hedgers goes up, lowering incentives is optimal for the principal.

In addition, one of the main results in the case of exogenous noise trading is that the price informativeness $Q$ generally increases (decreases) with the level of noise trading if $R_a$ is small (large) enough. We next demonstrate that this result still holds when noise trading is endogenized. Similar to Equation (11), we can express the price informativeness as

$$Q = [\text{Var} (\bar{v} | P)]^{-1} = \frac{\beta^2 (\sigma^2 + \sigma_z^2) + m\gamma^2 \sigma_z^2}{\sigma^2 \left[ \beta^2 \sigma_z^2 + m\gamma^2 \sigma_z^2 \right]} = \frac{1}{\sigma^2} + \frac{1}{\sigma_z^2 + m\gamma^2 \sigma_z^2 / \beta^2}.$$  

(20)

Figure 5 illustrates the relationship between $Q$ and $m$. In the work by Spiegel and Subrahmanyam (1992) where effort $\rho$ is exogenously fixed and the informed agent is risk neutral, $Q$, as given in Equation (19), is independent of $m$. Subplot A1 confirms this independence result. Subplots A2 and A3 show that, when the informed agent is risk averse, $Q$ decreases with $m$; the more risk averse the informed agent, the more quickly $Q$ decreases. This result makes sense because a risk-averse informed agent does not trade as aggressively as a risk-neutral one, and the less aggressive informed trading makes the price less informative.

The results for endogenous effort are reported in the second column of Figure 5. When the informed agent is risk neutral, $Q$ increases with $m$, as shown in Subplot B1. On the one hand, trading by more uninformed hedgers makes the price less informative. On the other hand, with more uninformed hedgers in the market, the informed agent can potentially profit more from trading against the uninformed hedgers. The informed agent thus exerts more effort in acquiring more accurate information. More aggressive trading by the informed agent with more accurate information leads to higher price
Figure 5

Price informativeness \( (Q) \) vs. the number of hedgers \( (m) \)

Figure 5 plots the relation between the price informativeness \( (Q) \) and the number of hedgers \( (m) \). The first, second, and third columns report the results for the cases of exogenous information, endogenous information alone, and both endogenous information and portfolio delegation, respectively. The first, second, and third rows report the results for \( R_a = 0, 0.1, \) and 2, respectively. Other parameters are \( R_h = 3, \sigma^2 = k = 1, \sigma^2_z = 5 \).

The second effect dominates the first one, so \( Q \) increases with \( m \).

When the informed agent’s risk aversion is low, say \( R_a = 0.1 \), \( Q \) increases with \( m \) initially when \( m \) is small. When \( m \) becomes large enough, the first effect dominates the second one, driving down \( Q \). When we further introduce portfolio delegation and keep \( R_a \) as low as 0.1, \( Q \) increases monotonically with \( m \), as opposed to increasing initially and decreasing later in the absence of portfolio delegation. As we discussed earlier, the optimal \( b \) decreases with \( m \). Hence, with optimal contracting, the effective risk aversion of the informed agent decreases as the number of uninformed hedgers increases. In equilibrium, the second effect dominates the first one, leading to higher price informativeness. When \( R_h \) becomes large, we cannot lower \( b \) too much because with a low \( b \), the agent’s effort for information will be low. As a result, the effective risk aversion of the informed agent, \( R_a b \), remains at a certain level, which limits the informed agent’s trading intensity or the amount of private information.

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14 Under different settings, Fishman and Hagerty (1992) and Leland (1992) discuss the impact of insider trading on the price informativeness.
being incorporated into the price. Consequently, when the informed agent is sufficiently risk averse, the effect of uninformed trading will dominate that of the informed trading, driving down the price informativeness, as shown in Subplot C2 of Figure 5.

3. Further Discussions

In this section, we extend the model to include a risk-averse principal as well as discuss the empirical implications of our model.

3.1 Risk-averse principal

When the principal is risk averse, the solution techniques are essentially the same as those for a risk-neutral principal. The expressions for the equilibrium $\lambda$, $\beta$, and $\rho$ take the same forms as in Proposition 1. Due to the risk aversion of the principal, his objective function contains an additional variance term, $1/2(1 - b)^2 \text{Var}[\tilde{W}]$. This term does not introduce much difficulty in our calculations.

For simplicity, we consider the case of exogenous noise traders and find that the results remain qualitatively the same. For example, Figure 6 demonstrates that, when the agent’s risk aversion is relatively small, price informativeness increases, the agent’s incentive and market liquidity decrease, and the agent’s effort and trading intensity increase with the amount of noise trading, as in the case of a risk-neutral principal. Due to the risk sharing between the principal and the manager, the optimal incentive slope $b$ increases with the principal’s risk aversion.

3.2 Empirical implications

Our model shows that an increase in noise trading may make the stock price more informative; and importantly, the presence of portfolio delegation makes this positive relationship more likely to hold. We identify two natural experiments in which there is an exogenous increase in the level of noise trading.

The first experiment is the addition of stocks to a stock index (e.g., S&P 500, Russell 1000). We investigate the price informativeness of a stock before and after its addition into the index. After a stock is added into an index, the passive index funds, whose trades are considered uninformed, will be required to purchase the stock. Hence, there will be an increase in uninformed trading in the stock. A traditional microstructure model would conclude that, due to increased uninformed trading, the price informativeness will decrease after a stock is added to the index. In our model, however, with more uninformed trading, investors, particularly active portfolio managers, will have more incentives to acquire private information. They will then trade more aggressively in the stock, which increases the price informativeness. Cross-sectionally, controlling for other factors, our model predicts that a stock with higher institutional
ownership should experience less decrease or even an increase in its price informativeness following its addition.

We propose the following panel regression to test our prediction:

\[ Q_{i,t} = a + b \times \text{Addition}_{i,t} + c \times (I O_{i,t} \times \text{Addition}_{i,t}) + \text{other terms}, \]

where \( Q_{i,t} \) is a measure of price informativeness for stock \( i \) at time \( t \), \( \text{Addition}_{i,t} \) is a dummy variable that equals zero (or one) before (at/after) the date when the stock is added to the index, and \( IO_{i,t} \) is stock \( i \)'s institutional ownership. \( IO_{i,t} \) measures the extent of portfolio delegation. According to our model, holding everything else equal, the same increase in noise trading results in less decrease in the price informativeness for stocks with higher institutional ownerships. That is, the coefficient for the interaction term \( c \) is predicted to be positive. Furthermore, a positive \( (b + c \times IO_{i,t}) \) is also consistent with our model, implying that the price informativeness may even increase with noise trading.

---

The second natural experiment is the passage of a country’s first-time enforcement of insider trading laws. Fernandes and Ferreira (2009) test the relation between a country’s first-time enforcement of insider trading laws and stock price informativeness, as measured by idiosyncratic stock return variation.\textsuperscript{16} They find that enforcement of insider trading laws improves price informativeness in developed countries, but it does not lead to significant improvement in emerging-market countries. They cast their findings as a puzzle for the traditional microstructure models. Their reasoning is that, with the enforcement of insider trading laws, more investors will have incentives to become informed, which should lead to higher price informativeness across markets, including emerging markets.

We argue that, in developed countries, there are more institutions that trade on behalf of individual investors. When there is less insider trading due to the enforcement of insider trading laws, there will be more liquidity trading. The reason is that the uninformed hedgers will be more likely to hedge their liquidity risk when they are less likely to lose to inside traders. There will also be more informed trading by institutions because they have more incentives to acquire private information when insider trading declines. Our model shows that, given the same increase in noise trading, portfolio delegation results in higher price informativeness because a portfolio manager trades more aggressively in the stock than an investor who trades for himself. Hence, our model can potentially provide an explanation for the empirical result of Fernandes and Ferreira (2009).

We can further test our model with an extension to the empirical setup of Fernandes and Ferreira (2009), who test

\[ Q_{i,t} = a + b \times \text{Enforcement}_{i,t} + c \times (IO_{i,t} \times \text{Enforcement}_{i,t}) + \text{other terms}, \]

where \( \text{Enforcement}_{i,t} \) is a dummy variable that takes the value of one in the year of country \( i \)’s first insider trading enforcement case and thereafter, and zero otherwise, and \( IO_{i,t} \) is country \( i \)’s average institutional ownership. The enforcement of the insider trading laws has a direct effect of reducing price informativeness, implying \( b < 0 \). The main prediction of our model is, however, on the interaction term \((IO_{i,t} \times \text{Enforcement}_{i,t})\), that is, \( c > 0 \). In other words, the larger extent of portfolio delegation in developed countries may explain why price informativeness increases after the law, but the opposite holds true for emerging countries. We argue that institutional ownership, a measure of the extent of portfolio delegation, is high (low) in developed (emerging) countries, so the overall effect of “Enforcement” on the price informativeness, measured by \( c \times IO_{i,t} + b \), may be positive (negative or zero) for developed

\textsuperscript{16}French and Roll (1986) and Roll (1988) argue that idiosyncratic stock return variation measures the rate of information incorporation into stock prices through trading. This measure can be estimated by \( 1 - R^2 \), where \( R^2 \) is from a regression of the firm’s return on the systematic returns.
(emerging) countries, which is consistent with the findings of Fernandes and Ferreira (2009).

Our model can also be used to explain certain empirical findings, which present challenges to the traditional microstructure models. For example, in the Kyle-type models, both the price informativeness $Q$ and the price impact $\lambda$ decrease with the level of noise trading. As a result, one would expect that the correlation between $Q$ and $\lambda$ is positive. Saar and Yu (2002) test this implication. They find that, “despite our expectation that more trading by investors who are informed about future cash flows would increase the informativeness of prices with respect to future earnings, the two permanent price impact measures seem to be either negatively correlated or not significantly correlated with the measures of price informativeness.” Based on this finding, they cast doubt on the role of the price impact measures in describing information asymmetry. Our result that $Q$ may increase with $\sigma^2_u$ suggests that the correlation between $Q$ and $\lambda$ may be negative, which provides a potential explanation for the empirical result of Saar and Yu (2002).

4. Conclusion

This article highlights the importance of developing an integrated model of portfolio delegation and strategic trading. An equilibrium always exists in our integrated model. We find that, when the risk aversion of the informed manager is small (large), the price informativeness increases (decreases) with the number, the risk aversion, and the endowment risk of the uninformed hedgers. We also find that, all else being equal, higher incentives lead to higher effort levels. These results differ significantly from those derived from pure strategic trading models or portfolio delegation models under competitive trading.

In conclusion, our model may be viewed as a first step toward the integration of portfolio delegation and strategic trading. For tractability, we consider only a one-period model. It would be of great interest to develop an integrated model in multiple periods.

Appendix

Proof of Proposition 1. In Stage 5, market makers determine the asset price under their prior about the agent’s effort and the conjecture about the agent’s trading strategies. In particular, they believe that the agent spent effort $\rho_m$ in Stage 3, and conjecture the agent’s trading strategy as $x = X(\theta; \rho = \rho_m(b), \rho_m(b), b) = \beta_m(\rho_m(b), b) (\theta - \pi)$. The competitive, risk-neutral market makers set the asset price to equal the expectation of its liquidation value conditioning on the total order flow under the belief $\rho_m(b)$. By Bayes’ rule, the posterior density function, $f(\tilde{v} | y)$, is normally distributed and its mean is given by

$$P(y; \rho_m(b), b) \equiv \tilde{v} + \lambda_m(\rho_m(b), b) y,$$

where $\lambda_m(\rho_m(b), b)$ is given in Equation (3).
In Stage 4, the informed agent’s maximization problem is

$$\max_x \left[ E_\theta(S) - \frac{1}{2} R_a V ar_\theta(S) \right] = \max_x \left[ a + b E_\theta(\bar{W}) - \frac{1}{2} R_a b^2 V ar_\theta(\bar{W}) \right],$$

where $\bar{W} = x(\bar{v} - 0.7) = x[\bar{v} - \lambda_m x - \lambda_m \bar{u}]$. Note that $E_\theta(\bar{W}) = x E_\theta(\bar{v} - \bar{v}) - \lambda_m x^2$, and $V ar_\theta(\bar{W}) = x^2 [V ar_\theta(\bar{v}) + \lambda_m^2 \sigma_u^2]$. Hence, the agent’s maximization problem becomes

$$\max_x \left[ a - b \lambda_m x^2 + b x E_\theta(\bar{v} - \bar{v}) - \frac{1}{2} R_a b^2 x^2 [V ar_\theta(\bar{v}) + \lambda_m^2 \sigma_u^2] \right].$$

Therefore, the agent’s optimal trading strategy is given by

$$X(\theta; \rho, \rho_m(b), a, b) = \frac{\sigma^2 / (\sigma^2 + \sigma_e^2)}{2 \lambda_m + R_a b [V ar_\theta(\bar{v}) + \lambda_m^2 \sigma_u^2]} (\theta - \bar{v}) \equiv \beta(\rho, b, (\theta - \bar{v})].$$

Hence, the expression for $\beta(\rho, b)$ in Equation (4) is derived. Under his optimal trading strategy, the agent’s expected utility, after exerting effort $\rho$ and obtaining signal $\hat{\theta} = \theta$, is given by

$$\overline{U}_A(\theta; \rho, \rho_m(b), a, b) = \frac{-R_a^{-1} \exp \left[ R_a C(\rho) - R_a a - \frac{R_a b \beta(\rho, b)}{2(1 + \rho)} (\theta - \bar{v})^2 \right]}{R_a \sqrt{b \sigma^2 \beta(\rho, b) + 1}}. \quad (A1)$$

Differentiating the right-hand side of the above equation with respect to $\rho$, we obtain the first-order condition as shown in Equation (5). The first (total) derivative of $\beta$ with respect to $\rho$ is derived below:

$$\frac{d\beta}{d\rho} = \beta(\rho, b) \rho^2 \frac{1}{2 \lambda_m + R_a b \left( \lambda_m^2 \sigma_u^2 + \sigma^2 \right)} > 0. \quad (A2)$$

Substituting Equation (A2) into Equation (5), we obtain the result in Equation (6).

In Stage 2, the market makers rationally anticipate the agent’s optimal effort in Stage 3. That is, $\rho_m$ is chosen such that $\rho^* = \rho_m$ in Stage 3. We can thus omit the subscript “m” hereafter. In Stage 1, we solve the principal’s optimization problem to determine the optimal contract. Specifically, $a$ is chosen to satisfy the agent’s participation constraint, and then $b$ is chosen to maximize the principal’s expected utility. To satisfy the individual participation constraint, we substitute the optimal $\rho^*$ and $\beta^*$ back into Equation (A1) and set the maximum expected utility to the agent’s reservation utility. We have

$$- \frac{1}{R_a \sqrt{R_a b \beta^* + 1}} \exp \left[ -R_a a + \frac{1}{2} R_a k \rho^* \right] = \hat{U},$$

which implies Equation (9). Given the expression for $a$ in Equation (9), we can show that the risk-neutral principal’s optimization problem is the one stated in Equation (8).

**Proof of Proposition 2.** Proving the existence and uniqueness of the equilibrium starting from Stage 2 is equivalent to proving that, for a given $b$, there exists a unique set of solutions $(\rho^*, \beta^*, \lambda^*)$ to the system of equations (3), (4), (6), and (7). The proof proceeds in two steps. In the first step, we prove that, holding $b$ and an arbitrary $\rho_m \geq 0$ fixed, under the belief $\rho = \rho_m$, the solutions to Equations (3) and (4), denoted by $\beta_m(\rho_m, b)$ and $\lambda_m(\rho_m, b)$, exist and are unique. In fact, if we substitute the expression of $\lambda$ from Equation 3 into Equation 4 and replace $\rho$ with $\rho_m$, then we obtain the following quintic equation for $\beta$:

$$(\rho + 1)^2 \sigma^0 R_a b \beta^5 + \rho(\rho + 1)^2 \sigma^4 \beta^4 + \rho(\rho + 1)(\rho + 2) \sigma^4 \sigma_u^2 R_a b \beta^3 + \rho^2 \sigma^2 \sigma_u^4 R_a b \beta - \rho^3 \sigma_u^4 = 0. \quad (A3)$$
If $b = 0$, then the solution is unique and positive, given by $\overline{\beta}_m \equiv \frac{\sigma_u}{\sigma} \sqrt{\frac{\rho_m}{\rho_m - \rho}} > 0$. If $\rho_m = 0$ and $b > 0$, then $\beta$ must be zero. If $b, \rho_m > 0$, we prove below that there always exists a unique positive solution in the interval $(0, \overline{\beta}_m)$. If we denote the left-hand side of Equation (A3) by $f (\beta)$, then we have

$$f' (\beta) = Ra_b \sigma^2 \left[ 5(\rho_m + 1)^2 \sigma^4 \beta^4 + 3 \rho_m (\rho_m + 1)(\rho_m + 2) \sigma^2 \sigma^2 \beta^2 + \rho_m^2 \sigma_u^4 \right] + 4 \rho_m (\rho_m + 1)^2 \sigma^4 \beta^3 > 0$$

and

$$f (0) = -\rho_m^3 \sigma_u^4 < 0$$

$$f (\overline{\beta}_m) = Ra_b \left[ (\rho_m + 1)^2 \sigma^6 \rho_m^5 + \rho_m (\rho_m + 1)(\rho_m + 2) \sigma^4 \sigma^2 \rho_m^3 + \rho_m^2 \sigma^2 \sigma_u^4 \right] > 0.$$ 

Thus, by continuity, there must exist a positive and unique solution within the interval $(0, \overline{\beta}_m)$.

To complete the proof, in the second step, we need to show that the second-order condition is satisfied, that is, the agent’s objective function is concave with respect to $\rho$ at the point of optimal solutions $\rho_m = \rho^*, \beta_m = \beta^*$, and $\lambda_m = \lambda^*$. The proof is the following. In Stage 3, before the agent exerts effort, his expected utility is given by

$$\overline{U}_A (\rho; \rho_m (b), a, b) = -\frac{\exp (Ra C (\rho) - Ra a)}{Ra \sqrt{Ra_b \sigma^2 \beta + 1}},$$

which implies

$$\frac{d\overline{U}_A (\rho; \rho_m (b), a, b)}{d\rho} = -\frac{\exp (Ra C (\rho) - Ra a)}{Ra \sqrt{Ra_b \sigma^2 \beta + 1}} \left[ -\frac{1}{2} \frac{Ra_b \sigma^2}{Ra_b \sigma^2 \beta + 1} \frac{d\beta}{d\rho} + Ra \frac{dC (\rho)}{d\rho} \right]$$

and

$$\frac{d^2\overline{U}_A (\rho; \rho_m (b), a, b)}{d\rho^2} = -\frac{\exp (Ra C (\rho) - Ra a)}{Ra \sqrt{Ra_b \sigma^2 \beta + 1}} \left[ -\frac{1}{2} \frac{Ra_b \sigma^2}{Ra_b \sigma^2 \beta + 1} \frac{d\beta}{d\rho} + Ra \frac{dC (\rho)}{d\rho} \right]^2$$

$$-\exp (Ra C (\rho) - Ra a) \left[ -\frac{1}{2} \frac{Ra_b \sigma^2}{Ra_b \sigma^2 \beta + 1} \frac{d\beta}{d\rho} + Ra \frac{dC (\rho)}{d\rho} \right] \frac{d^2 \beta}{d\rho^2} + \frac{Ra_b \sigma^2}{2 (Ra_b \sigma^2 \beta + 1)} \left( \frac{d\beta}{d\rho} \right)^2 + Ra k \right].$$

Denote $\chi \equiv 2\lambda^* + Ra_b \left( \lambda^* \sigma_u^2 + \sigma^2 \right)$. From Equation (4), we have

$$\frac{d\beta}{d\rho} \bigg|_{\rho = \rho^*} = \frac{\beta^*}{\rho^* \chi},$$

$$\frac{d^2 \beta}{d\rho^2} \bigg|_{\rho = \rho^*} = 2\beta^* (\rho^*)^{-2} \chi \frac{d\beta}{d\rho} \bigg|_{\rho = \rho^*} - 2\beta^* (\rho^*)^{-3} \chi.$$
Because at $\rho^*$ the first-order condition is satisfied (i.e., \( \frac{d\overline{U}_A(\rho; \rho_m(\cdot), a, b)}{d\rho} |_{\rho=\rho^*} = 0 \)), we have

\[
\frac{d\beta}{d\rho} |_{\rho=\rho^*} = \frac{2 \left( R_a b \sigma^2 \beta^* + 1 \right)}{R_a b \sigma^2} R_a k \rho^*,
\]

\[
\frac{d^2 \beta}{d\rho^2} |_{\rho=\rho^*} = \frac{4 \beta^* (R_a b \sigma^2 \beta^* + 1)}{R_a b \sigma^2} R_a k - 2 \beta^* (\rho^*)^{-3} \chi,
\]

and

\[
\frac{d^2 \overline{U}_A(\rho; \rho_m(\cdot), a, b)}{d\rho^2} |_{\rho=\rho^*} = -\frac{\exp(R_a V(\rho^*) - R_a a)}{\sqrt{R_a b \sigma^2 \beta^* + 1}} \left[ -\frac{2 \beta^* (R_a b \sigma^2 \beta^* + 1)}{\rho^*} - \frac{b \sigma^2 \beta^* (R_a b \sigma^2 \beta^* + 1)}{\rho^* \chi} + 2 R_a k \rho^* + 3 k \right].
\]

From the above equations, we obtain

\[
\frac{\beta^* \chi}{\rho^*} = \frac{2 \left( R_a b \sigma^2 \beta^* + 1 \right)}{R_a b \sigma^2} R_a k \rho^*.
\]

From Equations (4) and (5), we obtain

\[
\frac{\beta^* \chi}{\rho^*} = \frac{1}{1 + \rho^*} + \frac{R_a \sigma^2 \beta^*}{1 + \rho^*},
\]

\[
\sigma^2 = 2 k \rho^* (\rho^* + 1) / \beta^*.
\]

We thus arrive at

\[
\frac{d^2 \overline{U}_A(\rho; \rho_m(\cdot), a, b)}{d\rho^2} |_{\rho=\rho^*} = -\frac{\exp(R_a V(\rho^*) - R_a a)}{\sqrt{R_a b \sigma^2 \beta^* + 1}} \left[ -\frac{2 \beta^* \chi k + 2 R_a k \rho^* + 3 k}{\rho^* \chi} \right] = -\frac{\exp(R_a V(\rho^*) - R_a a)}{\sqrt{R_a b \sigma^2 \beta^* + 1}} \left[ -2 k \left( \frac{1}{1 + \rho^*} + \frac{R_a \sigma^2 \beta^*}{1 + \rho^*} \right) + 2 R_a k \rho^* + 3 k \right]
\]

\[
= -\frac{k \exp(R_a V(\rho^*) - R_a a)}{\sqrt{R_a b \sigma^2 \beta^* + 1}} \left[ 3 - \frac{2}{1 + \rho^*} \right] < 0.
\]

**Proof of Proposition 3.** When the agent is risk averse, Equation (4) gives

\[
\frac{d\beta}{d\rho} = \frac{2 \lambda + R_a \sigma^2 \beta^2 + R_a \sigma^2 \beta^2}{\left( 2 \lambda + R_a \sigma^2 \beta^2 \right) (1 + \rho) + R_a \sigma^2} = \frac{\beta + R_a \sigma^2 \beta^2}{\rho (1 + \rho)}.
\]

(A4)

Substituting Equation (A4) into the first-order condition in Equation (5), we have

\[
\beta = \frac{2 k}{\sigma^2 \rho^2 (1 + \rho)}.
\]

(A5)

When $R_a$ is very small, Equation (A3) is approximately given by

\[
\rho (\rho + 1)^2 \sigma^4 \beta^4 - \rho^3 \sigma^4 \beta^4 \approx 0.
\]
that is, \( \rho \approx \sqrt{1+4(\sigma_u/(2k))^2} - 1 \). It can be verified that \( \frac{d}{d\sigma_u} \frac{(\sigma_u/\rho)^2}{2} < 0 \). Together with the observations that \( \frac{d\rho}{d\sigma_u} > 0 \) and \( Q = \frac{1}{\sigma^*} + \frac{1}{\sigma^*/\rho + \sigma_u/\beta^2} \), we have

\[ \frac{dQ}{d\sigma_u} > 0 \text{ when } R_a \text{ is very small.} \]

When \( R_a \) is very large, Equation (A3) becomes the following:

\[
0 = 2\rho (\rho + 1) R_a k \left[ \rho^6 (\rho + 1)^6 \sigma^4 \left( \frac{2k}{\sigma^2} \right)^4 + \rho^3 (1 + \rho)^3 (\rho + 2) \sigma^2_\rho \left( \frac{2k}{\sigma^2} \right)^2 + \sigma^4_u \right]
\]

\[ + \rho^6 (\rho + 1)^6 \sigma^4 \left( \frac{2k}{\sigma^2} \right)^4 - \sigma^4_u \]

\[ \approx 2\rho (\rho + 1) R_a k \left[ \rho^6 (\rho + 1)^6 \sigma^4 \left( \frac{2k}{\sigma^2} \right)^4 + 2\rho^3 (1 + \rho)^3 \sigma^2_\rho \left( \frac{2k}{\sigma^2} \right)^2 + \sigma^4_u \right]
\]

\[ + \rho^6 (\rho + 1)^6 \sigma^4 \left( \frac{2k}{\sigma^2} \right)^4 - \sigma^4_u, \] (because \( \rho + 2 \) is close to 2).

Therefore, we have

\[
2\rho (\rho + 1) R_a k \left[ \rho^3 (\rho + 1)^3 \sigma^2_\rho \left( \frac{2k}{\sigma^2} \right)^2 + \sigma^4_u \right]^2 + \rho^6 (\rho + 1)^6 \sigma^4 \left( \frac{2k}{\sigma^2} \right)^4 - \sigma^4_u \approx 0.
\]

The above equation is approximately equal to \( 0 \approx 2\rho (\rho + 1) R_a k \sigma^4_u - \sigma^4_u \), because the first term in the square brackets is negligible compared with the second term \( \sigma^2_\rho \). Therefore, \( \rho (\rho + 1) \approx \frac{1}{2kR_h} \), which shows that the optimal \( \rho \) is independent of \( \sigma_u \) to the first-order approximation, and so is \( \beta \). Consequently, \( Q \) decreases with \( \sigma^2_\rho \) that is, \( \frac{dQ}{d\sigma_u} < 0 \), when \( R_a \) is very large. \( \blacksquare \)

**Proof of Proposition 5.** From Equations (6) and (12)–(14), and noting that \( \sigma^2_u = m\gamma^2 \sigma^2_z \), in the special case with endogenous information acquisition only and \( R_a = 0, m = 1 \), we have

\[
\beta^* = \frac{2k\rho^2 (1 + \rho^*)}{\sigma^2}, \quad \lambda^* = \frac{\sigma^2}{4k\rho^* (1 + \rho^*)^2}, \quad \gamma^* = -\frac{2R_h (\rho^* + 1) (\rho^* + 2)}{2(k\rho^*) + R_h (\rho^* + 1)(\rho^* + 4)},
\]

\[
\rho^* (\rho^* + 1) \left[ 2 + R_h k\rho^* (\rho^* + 1)(\rho^* + 4) \right] = \sigma^2 \sigma^2_\rho R_h^2 (\rho^* + 2)^2.
\]

From the last equation, if we let \( R_h \) go to zero, then \( \rho^* \) goes to zero as well. Therefore, it implies

\[
4\rho^* + o(\rho^*) = 4\sigma^2 \sigma^2_\rho R_h^2 + o(\rho^*)
\]

It follows that \( \rho^* \approx \sigma^2 \sigma^2_\rho R_h^2 \). Substituting this asymptotic expression into the first three equations regarding \( \lambda^*, \beta^*, \) and \( \gamma^* \), we can easily obtain the other three expressions in Proposition 5. \( \blacksquare \)

**Proof of Proposition 6.** We prove the existence of the equilibrium in two steps. First, start with the existence of a unique equilibrium in the case of exogenous noise trading proved in Proposition 2. Given a value \( \gamma < 0 \), if we define \( \sigma^2_\rho \equiv m\gamma^2 \sigma^2_z \), then there exists a set of solutions \( \beta(\gamma), \lambda(\gamma), \) and \( \rho(\gamma) \) to Equations (3), (4), (6), and (7). Second, we denote the operator \( T \) from Equation (12) as follows:

\[
T(\gamma) = -\frac{R_h (1 - \lambda(\gamma) \beta(\gamma)) \sigma^2_\rho}{2\lambda(\gamma) + R_h [1 - \lambda(\gamma) \beta(\gamma)]^2 \sigma^2_\rho + \lambda(\gamma) \beta(\gamma)^2 \sigma^2_\rho / \rho(\gamma) + (m - 1) \lambda(\gamma) \gamma^2 \sigma^2_z].
\]
we then prove that there exists a fixed point \( \gamma^* < 0 \), such that \( T(\gamma^*) = \gamma^* \), which completes the proof. In the following we prove the existence of such a fixed point.

First, because \( \lambda(\gamma) \beta(\gamma) = \beta^2 \left[ \beta^2 (1 + 1/\rho) + m \gamma^2 \sigma^2 / \sigma^2 \right]^{-1} < 1 \), \( T(\gamma) \) is always strictly negative. Furthermore, when \( \gamma \) goes to zero from below, we have

\[
\rho(\gamma) \approx \left( \frac{m^2 \sigma^2}{4k^2} \right)^{1/3} \gamma^{2/3}, \quad \beta(\gamma) \approx \left( \frac{m^2 \sigma^2}{2k}\right)^{1/3} \gamma^{4/3}, \quad \lambda(\gamma) \approx \left( \frac{\sigma^4}{16km \sigma^2} \right)^{1/3} \gamma^{-2/3},
\]

which implies

\[
T(\gamma) \approx -\frac{R_b \sigma^2}{2} \left( \frac{\sigma^4}{16km \sigma^2} \right)^{-1/3} \gamma^{2/3}.
\]

As a result, there exists a \( \gamma_a < 0 \) that is close enough to zero that \( T(\gamma_a) < \gamma_a \).\(^{17}\)

Next, we show that there exists a \( \gamma_b < \gamma_a \) that is negative enough that \( T(\gamma_b) > \gamma_b \). Then, by the intermediate value theorem and the continuity of \( T(\gamma) \), there must exist a value \( \gamma^* \in (\gamma_b, \gamma_a) \), so that \( T(\gamma^*) = \gamma^* \). To prove the former statement, we investigate the asymptotic behavior of \( T(\gamma) \) when \( \gamma \) goes to \(-\infty\). We first consider the case where \( R_a = 0 \). Let \( \sigma_u^2 \equiv m \gamma^2 \sigma^2 \) for a given \( \gamma \), then when the agent is risk neutral, from Equations (3) and (4) in Proposition 1, we have

\[
\beta = \frac{\sigma_u}{\sigma} \sqrt{\frac{\rho}{1 + \rho}}, \quad \lambda = \frac{1}{2} \frac{\sigma}{\sigma_u} \sqrt{\frac{\rho}{1 + \rho}}.
\]

Furthermore, from Equation (6), we can easily show that

\[
\rho = \frac{1 + (4\sigma \sigma_u / k)^{2/3}}{2} - 1.
\]

Therefore, when \( \gamma \) is sufficiently small (i.e., \( \sigma_u^2 \) sufficiently large),

\[
\rho(\gamma) \approx \left( \frac{\sigma_u \sigma}{2k} \right)^{1/3} = \left( \frac{m \sigma^2}{2k} \right)^{1/3} \gamma^{2/3}, \quad \beta(\gamma) \approx -\frac{m^{1/2} \sigma}{\sigma} \gamma, \quad \lambda(\gamma) \approx -\frac{\sigma}{2m^{1/2} \sigma^2} \gamma^{-1}.
\]

It follows that \( T(\gamma) \) converges to \(-\frac{2m}{(2m - 1)} \approx 2\) when \( \gamma \) goes to \(-\infty\). Therefore, there must exist a sufficiently small \( \gamma_b \) such that \( T(\gamma_b) > \gamma_b \).

When the agent is risk averse, from Equation (6), as \( |\gamma| \) or \( \sigma_u^2 \) goes to infinity, it must be true that \( \rho \) converges to a finite number \( \rho_{\infty} \equiv \frac{1}{2} \left( \sqrt{1 + 2/(R_b k)} - 1 \right) \), and \( \sigma_u^2 \) converges to zero, implying that \( \lambda \) converges to zero as well. It is easy to see from Equation (4) that \( \beta \) converges to \( \beta_{\infty} \equiv \rho_{\infty} / \left( R_b \sigma^2 \right) \). As a result, \( T(\gamma) \) converges to \(-1\). Thus again, in this case, there must exist a sufficiently small \( \gamma_b \) such that \( T(\gamma_b) > \gamma_b \).

\^[17] By contrast, in Spiegel and Subrahmanym (1992) where \( \rho \) is exogenously fixed, a \( \gamma_a \) that satisfies \( T(\gamma_a) < \gamma_a \) does not always exist. In fact, the condition in Equation (15) is needed to ensure the existence of \( \gamma_a \). We can prove it by contradiction. For simplicity, we assume \( R_a = 0 \) as in Spiegel and Subrahmanym. Suppose that the condition in Equation (15) does not hold, which is equivalent to \( R_b \left( 1 - \beta \lambda \right) \sigma^2 \leq -2\gamma \lambda \), where \( \beta = \frac{\sigma_u}{\sigma} \sqrt{\frac{\rho}{1 + \rho}}, \lambda = \frac{1}{2} \frac{\sigma}{\sigma_u} \sqrt{\frac{\rho}{1 + \rho}} \). Moreover, since \( \gamma < 0 \), it follows that \( R_b \left( 1 - \beta \lambda \right) \sigma^2 \leq -2\gamma \lambda < -\gamma \left( 2\lambda + R_b \left( 1 - \beta \lambda \right)^2 \sigma^2 / \rho + \left( m - 1 \right) \left( \lambda \sigma^2 / \sigma^2 \right) \right) \). That is, \( T(\gamma) \geq \gamma \) for all \( \gamma < 0 \), implying that no equilibrium exists. On the other hand, when the condition holds (i.e., \( R_b (1 - \beta \lambda) \sigma^2 > -2\gamma \lambda \)), if we let \( \gamma \) go to zero from below, then there must exist a \( \gamma_a \) so that \(-2\gamma \lambda \approx -\gamma \left( 2\lambda + R_b \left( 1 - \beta \lambda \right)^2 \sigma^2 / \rho + \left( m - 1 \right) \left( \lambda \sigma^2 / \sigma^2 \right) \right) \), implying \( T(\gamma_a) < \gamma_a \). This result, together with the existence of \( \gamma_b \) that satisfies \( T(\gamma_b) > \gamma_b \), guarantees the existence of an equilibrium.
References


A Model of Portfolio Delegation and Strategic Trading


