What Do Nominal Rigidities and Monetary Policy Tell Us about the Real Yield Curve?*

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Abstract

We study term and inflation risk premia in real and nominal bonds, respectively, in an equilibrium model calibrated to United States data. Nominal wage and price rigidities, and an interest-rate monetary policy rule characterize our model economy. Wage rigidities induce positive term and inflation risk premia for permanent productivity shocks: they generate high marginal utility, expected consumption growth, inflation, and bond yields, simultaneously. Policy and inflation-target shocks increase real and nominal yield variability, respectively. Real-nominal bond return correlations are increased by the rigidities. Stronger policy responses to output and inflation reduce real term premia and increase inflation risk premia.

Keywords: Term structure of interest rates, general equilibrium, nominal rigidities, monetary policy.

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1 Introduction

What are the economic drivers of real interest rates of short and long maturities? Are long-term real bonds risky instruments? Are real rates volatile? The importance of these and related questions for finance, macroeconomics, and policymaking drastically contrasts with our current understanding of the real yield curve. For finance, this curve provides us with true risk-free rates for all maturities. Real rates are then a rich source to learn about the dynamics of real discount factors and price future cashflows without the need of inflation projections. For macroeconomics, real rates allow us to better understand how agents substitute consumption and transfer risks over time. For policymaking, we are interested in learning about the government ability to affect real rates through bond issuance or monetary policy, for instance. Unfortunately, short- and long-term real rates are mostly unobserved. It is only recently that developed markets for government inflation-indexed bonds have surfaced, offering hints on real yield dynamics.[1] These markets, however, are not exempt from liquidity and institutional issues that complicate their study.[2] In this context, economic theory can provide us with additional guidelines to understand real rates and their link to economic conditions.

Significant theoretical progress has been made in understanding the link between macroeconomic dynamics and monetary policy. This progress precisely relies on making monetary policy work by indirectly affecting real rates of all maturities through nominal price and wage rigidities.[3] The literature, however, has mostly focused on linking economic dynamics to the short-term real rate. This motivates us to explore the implications of nominal rigidities and monetary policy on long-term real rates. Specifically, in this paper we focus on understanding the effects of these two ingredients on the real term and inflation risk premia, the volatility of real yields, and the

[1] See Garcia and van Rixtel (2007) for recent history on inflation-linked bond markets across the world.

[2] See D'Amico, Kim and Wei (2008) and Pflueger and Viceira (2012) for evidence on liquidity premia in inflation-indexed government bonds in United States.

[3] See Woodford (2003) for the standard theoretical framework. Bils and Klenow (2004) and Nakamura and Steinsson (2008) report estimates for the median duration of prices in United States of 4 to 11 months. Kahn (1997) and Barattieri, Basu and Gottschalk (2010) report estimates for the median duration of wages in the United States of 12 to 17 months.

diversification benefits of real bonds.

Real term premia are expected return compensations for buying long-term over short-term real bonds. Inflation risk premia are expected return compensations for buying nominal over real bonds. These premia capture the cost for the government of issuing long-term vs. short-term debt, and nominal vs. real bonds, respectively. Identifying their properties is fundamental to obtain, for instance, inflation expectations from yield curve information. Our analysis provides several related results. First, nominal rigidities affect the sign and size of bond risk premia. In particular, wage rigidities generate positive real term and inflation risk premia. That is, upward sloping average real and nominal yield curves. This is mainly a compensation for the macroeconomic risk induced by permanent productivity shocks. Quantitatively, wage rigidities have more significant effects than price rigidities. Second, the volatility of short-term real rates is significantly increased by monetary policy shocks, and the volatility of real and nominal long-term rates depends on the persistence of transitory productivity and inflation-target shocks, respectively. Third, a stronger monetary policy reaction to inflation and/or output decrease real term premia and increase inflation risk premia. This is the result of monetary policy affecting the joint dynamics of consumption, inflation, and real rates. Finally, the correlations between nominal and real bond returns increase in the presence of nominal rigidities.

We begin in section 2 with an empirical analysis of inflation-linked bonds (TIPS) in the United States. Although TIPS are not exactly comparable to real bonds, the analysis allows us to ask what observed properties of TIPS can be explained by our economic model.[4] We confirm previous findings in the literature and provide new evidence. There is an average upward sloping curve for TIPS, with slopes that are lower than for nominal bonds of comparable maturities. TIPS yields have been as volatile as nominal bond yields for similar maturities. However, TIPS yield variability is not completely captured by nominal bond yield variability, suggesting the existence

[4]TIPS are not directly comparable to real bonds since their nominal value is indexed to lagged inflation and embed a protection option against deflation. Also, several studies have documented liquidity issues in the TIPS market that affect their implied yields.

of specific factors affecting the TIPS market. Finally, there is a significant statistical link between TIPS yields, consumption and inflation: correlations between TIPS yields with inflation and consumption growth tend to be negative and positive, respectively, but unstable across subsamples.

We present our model in section 3. It extends the standard New Keynesian framework to capture bond pricing dynamics. Following Li and Palomino (2012), the model contains four important ingredients. First, a representative household with Epstein and Zin (1989) recursive preferences over consumption and labor. As shown by Tallarini (2000), it allows us to keep a low level of elasticity of intertemporal substitution to match macroeconomic dynamics, and a high degree of risk aversion to match high compensations for risk in financial assets. Second, Calvo (1983) rigidities on nominal wages and prices. The representative household sets their wages from supplying labor, but at each point of time faces the probability of not being able to adjust these wages optimally. Firms set their product prices, but at each point of time face the probability of not being able to adjust these prices optimally. Third, monetary policy is captured by a Taylor (1993) interest rate rule. As a result of nominal rigidities, monetary policy has effects on real economic activity and, then, on real bond yields. Fourth, the economy is affected by three types of shocks: permanent and transitory productivity shocks, policy shocks, and inflation-target shocks. In equilibrium, real and nominal yields and expected bond returns are driven by these shocks, and preference, production, and policy parameters.

We describe the model implications in section 4. The model is calibrated to capture key United States macroeconomic and nominal yield properties. A novel aspect of the calibration is that it matches the contribution of the model shocks to the variability of macroeconomic variables found in the data.[5] The implied average real and nominal yield curves are upward sloping as a result of positive real term and inflation risk premia. The volatility of real yields is lower than that for nominal yields, and decreases faster across maturities.

Wage and price rigidities significantly affect the compensation for permanent productivity

[5]A similar approach is used by Li and Palomino (2012) to study stock returns.

shocks in bond yields. Wage rigidities induce a procyclical and mean reverting labor demand, and a countercyclical inflation with respect to permanent productivity shocks. Bad news for productivity growth is bad news for consumption growth and labor demand, and generates inflation. Labor demand decreases since wages (marginal costs) are higher than if they were perfectly flexible. However, expected future labor demand increases since wages will adjust downwards, and induces positive expected consumption growth. Simultaneously, as a result of higher wages, producers increase their product prices to restore their markup, generating inflation. The positive effect on expected consumption growth increases interest rates and decreases real bond returns. Real term premia are positive since real bond returns are low when marginal utility is high. The increase in prices reduces the real return of nominal bonds. Therefore, inflation risk premia are positive since returns on nominal bonds are low when marginal utility is high. Similarly, price rigidities keep prices too high after a negative productivity shock, wages stay high to keep labor markups high, and consumption is more negatively affected than under flexible prices. However, as prices decline over time, expected consumption growth increases and have a positive effect on real rates. Therefore, price rigidities have a positive effect on real term and inflation risk premia. In the calibration, the combination of wage and price rigidities generate positive real term and inflation risk premia of 132 and 96 bps., respectively, in five-year bonds.

The volatility of real and nominal bond yields is also affected by the rigidities. Monetary policy shocks to the nominal short-term rate induce significant volatility in the real short-term rate. This effect is absent in an economy with perfectly flexible prices and wages. Very persistent transitory productivity and inflation-target shocks have long-lasting effects on consumption growth and inflation, respectively. This translates into volatility for long-term yields that can be high or low depending on the rigidities.

The systematic response to economic conditions in the policy rule affects the magnitudes of real and inflation risk premia. A stronger response to inflation or output induce a smaller (stronger) negative covariance between consumption growth and real rates (inflation). These changes are reflected in lower real term premia and higher inflation risk premia.

We gain some insights into the diversification benefits of real bonds by computing the correlations between real returns of real and nominal bonds. Nominal rigidities increase these correlations, meaning that real bonds are more important for diversification in the absence of wage and price rigidities. Rigidities link real economic activity to inflation and, then, increases the link between real and nominal bond performance. Policies that reduce the volatility of inflation make real bonds behave more like real bonds, resulting in more correlated returns.

Related literature

Our paper joins the literature that analyzes the term structure from a macroeconomic perspective. Harvey (1988) finds an empirical link between real rates and consumption growth. Ang, Bekaert and Wei (2008) study the real yield curve with a regime-switching model. They find that the curve can be flat or upward sloping depending on the regime, and suggest monetary policy as a potential regime driver. Recently, the inflation risk premia in nominal bonds have been widely study. Empirical and theoretical studies such as Buraschi and Jiltsov (2005), Hördahl (2008), and Christensen, Lopez and Rudebusch (2010), among others, report very different properties for these premia. While most studies agree on the existence of positive inflation risk premia in United States treasury bonds, they disagree on how large and volatile these premia are.[6] We use an equilibrium macroeconomic model that links risk premia in real bonds and inflation risk premia to determinants of consumption growth and inflation. We find that the size and sign of real term and inflation risk premia are significantly affected by nominal rigidities and monetary policy.

Our macroeconomic model is based on the New Keynesian framework with recursive preferences. Recursive preferences in endowment economies have been used for the analysis of the nominal term structure by Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2012). They find that these preferences can capture important properties of yields such as positive and

[6] Hördahl (2008) and D'Amico, Kim and Wei (2008) provide summaries of the different estimates of inflation risk premia in the literature. The average inflation risk premium for 10-year bonds ranges from -16 bps. in Grishchenko and Huang (2012) to 140 bps. in Buraschi and Jiltsov (2005).

time-varying inflation risk premia. We incorporate these preferences to a New Keynesian model in the spirit of Woodford (2003) and Christiano, Eichenbaum and Evans (2005). Rudebusch and Swanson (2012) present a similar model to study the nominal yield curve and do not study the implications on the real yield curve. While Rudebusch and Swanson (2012) rely on price rigidities and transitory shocks, our results mainly depend on wage rigidities and permanent productivity shocks. The difference between permanent and transitory shocks for the term structure was previously analyzed by Campbell (1986) and Labadie (1994). They show that positively autocorrelated trend-stationary and difference-stationary consumption shocks generate positive and negative yield slopes, respectively. Kaltenbrunner and Lochstoer (2010) report similar results under recursive preferences. Empirically, it is difficult to disentangle permanent and transitory shocks as shown by Christiano and Eichenbaum (1990). However, Alvarez and Jermann (2005) show that permanent shocks to marginal utility are important to capture differences in expected returns of financial assets.

Finally, Campbell and Shiller (1996), Campbell, Shiller and Viceira (2009), and Bekaert and Wang (2010) analyze the potential diversification benefits of TIPS. We study the correlations between real and nominal bond returns for different specifications of the model to understand how the diversification benefits of real bonds depend on nominal rigidities and monetary policy.

2 Empirical Evidence

This section presents descriptive statistics of United States government TIPS and nominal bonds, and their link to consumption growth and inflation. We use quarterly data from January 1999 to September 2008.[7] This data period is motivated by two reasons. First, TIPS data are only available since 1999. Second, the period September - December 2008 coincides with the collapse of Lehman Brothers and a switch to unconventional monetary policy given the zero interest-rate

[7] Results using comparable monthly data are very similar. We present results for quarterly data to be consistent with the model calibration.

bound. We stop in September 2008, since we want to focus on understanding the effects of a monetary authority setting the level of a short-term rate (conventional monetary policy) on bond yields. We report statistics for the whole sample and for the subsample January 2004 to September 2008, since the literature emphasizes differences in TIPS liquidity across time.

The consumption growth and inflation series were constructed using quarterly data from the Bureau of Economic Analysis, following the methodology in Piazzesi and Schneider (2007). These series capture only consumption of non-durables and services and its related inflation. These data are consistent with the variables of the economic model below. The data on zero-coupon nominal bond and TIPS yields are constructed following the procedure in Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2008), respectively. The data are obtained from the Federal Reserve website. The short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. The three-month real rate was estimated using the methodology described in Pflueger and Viceira (2011). Specifically, the computation is based on the regression

$$i_t - \pi_{t+1} = \text{constant} + \beta_i i_t + \beta_r (i_{t-1} - \pi_t) + \varepsilon_t, \tag{1}$$

where i_t is the three-month nominal rate and π_t is the three-month inflation rate. The real rate is then computed as

$$r_t = i_t - \mathbb{E}_t[\pi_{t+1}] = \text{constant} + \beta_i i_t + \beta_r (i_{t-1} - \pi_t), \tag{2}$$

under the assumption that the inflation risk premium in three-month nominal bonds is negligible.[8]

Panels A and B of table 1 report yield and excess bond return statistics, respectively. Excess bond returns are computed as the difference of realized nominal returns on TIPS and nominal bonds with the 3-month T-Bill rate. The term structure of both yields and excess returns are

[8] The coefficients in the regression are computed using data for $1982-2008$ to be consistent with the calibration in the model. Estimated rates using data for $1999 - 2008$ are very similar.

upward sloping for both TIPS and nominal bonds.[9] The term structure of breakeven (BE) inflation rates (difference between the nominal and TIPS yields for bonds with the same maturity) is upward sloping for maturities longer than 2 years. These findings suggest that long-term TIPS and nominal bonds are riskier than short-term bonds, and that the inflation risk premia are positive. The table also shows that the standard deviation of TIPS and nominal bond yields are similar for the same maturities, raising the question on which factors can generate similar variability for the two instruments. Sharpe Ratios are higher for nominal bonds than for TIPS. Correlations of yields and excess returns between nominal bonds and TIPS tend to decrease with maturity, suggesting the possibility of diversification benefits of including TIPS in a nominal bond portfolio.

Table 2 presents the principal component analysis for TIPS, nominal bond, and BE inflation rates. Panel A shows that two principal components capture most of the variability of TIPS yields and BE rates. Three principal components explain the variability of nominal bonds, as extensively reported in the literature. However, when the analysis is performed for TIPS and nominal bonds jointly, the first four principal components can only capture 97% of the variability. These findings suggest the existence of factors that are specific to TIPS and nominal bond yields. To confirm this intuition, panel B reports the highest R^2's of regressions of TIPS and nominal bond yields on each other principal components. While nominal bond yields explain less that 78% of the variability of TIPS yields, TIPS yields explain less than 67% of the variability of nominal bond yields, for the whole sample. These numbers increase to 91% and 88%, respectively, for the 2004-2008 subsample. The difference may signal an improved liquidity in the TIPS market relative to the nominal bond market over time.

Finally, in order to motivate the economic model below, table 3 links bond yields and macroeconomic variables through correlations and regression results. The correlations of TIPS and nominal bond yields are significantly positive with consumption growth, and significantly negative with inflation for the sample period. These relations hold for the 2004-2008 subsample except for the

[9] Excess returns on TIPS were negative for the period 2004-2008. This may be related to liquidity concerns about TIPS during 2007-2008.

correlation between consumption growth and TIPS yields which become negative. These correlations suggest a link between macroeconomic information and bond yields. However, the table also shows R^2's of regressions of yield changes on contemporaneous and lagged values of consumption growth and inflation. They show a limited ability of consumption growth and inflation to explain movements in TIPS and nominal bond yields, respectively. This ability is greater for the 2004-2008 subsample. In summary, the empirical evidence suggests a complex link between TIPS, nominal bonds, and macroeconomic variables. We use the model in section 3 to learn about this link.

3 Economic Model

We model a production economy with a representative household, a production sector for differentiated goods, and monetary policy. The representative household derives utility from the consumption of a basket of goods and disutility from supplying labor to the production sector. The labor market and the production sector are characterized by monopolistic competition and nominal wage and price rigidities, respectively. Monetary policy is modeled as an interest-rate policy rule that reacts to economic conditions. When nominal prices and/or wages are not adjusted optimally, price and wage inflation generates distortions that affect production decisions. Different monetary policy rules have different implications not only on inflation but also on real activity. As a result, real and nominal bond yields for all maturities are affected by both nominal rigidities and monetary policy. All markets are complete and default-free real and nominal bonds are in zero net supply. The model can be seen as an extension of the standard New-Keynesian framework (see Woodford (2003), for instance) to capture bond pricing dynamics. Based on Li and Palomino (2012), it incorporates recursive preferences for households and is solved using a second-order perturbation. Recursive preferences disentangle risk aversion from the elasticity of intertemporal substitution of consumption. It allows us to match observed macroeconomic dynamics, while increasing risk aversion to capture large expected excess returns. A second-order perturbation helps us capture non-zero bond expected excess returns.

3.1 Household

The representative agent in this economy chooses consumption C_t and labor supply N_t^s to maximize the Epstein and Zin (1989) recursive utility function

$$V_t = (1-\beta)U(C_t, N_t^s)^{1-\psi} + \beta \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1-\psi}{1-\gamma}}, \tag{3}$$

where $\beta > 0$ is the subjective discount factor, $\psi^{-1} > 0$ captures the elasticity of intertemporal substitution of consumption, and $\gamma > 0$ determines the coefficient of relative risk aversion. The recursive utility formulation allows us to relax the strong assumption of $\gamma = \psi$ implied by constant relative risk aversion. The intra-temporal utility is

$$U(C_t, N_t^s) = \left(\frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} \right)^{\frac{1}{1-\psi}}, \tag{4}$$

where $\omega^{-1} > 0$ captures the Frisch elasticity of labor supply, and the process κ_t is chosen to ensure balanced growth. The consumption good is a basket of differentiated goods produced by a continuum of firms. Specifically, the consumption basket is

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $\theta > 1$ is the elasticity of substitution across differentiated goods, and $C_t(j)$ is the consumption of the differentiated good j. Labor supply is the aggregate of a continuum of different labor types supplied to the production sector, such that

$$N_t^s = \int_0^1 N_t^s(k) dk, \tag{6}$$

where $N_t^s(k)$ is the supply of labor type k.

The representative consumer is subject to the intertemporal budget constraint

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty} M^{\$}_{t,t+s} P_{t+s} C_{t+s}\right] \leq \mathbb{E}_t\left[\sum_{s=0}^{\infty} M^{\$}_{t,t+s} P_{t+s}\left(LI_{t+s} + D_{t+s}\right)\right], \tag{7}$$

where $M^{\$}_{t,t+s}$ is the nominal discount factor for cashflows at time $t+s$, P_t is the nominal price of a unit of the basket of goods, LI_t is the real labor income from supplying labor to the production sector, and D_t captures aggregate cashflows from additional claims on production such as dividends.

Appendix A shows that the household's optimality conditions imply that the one-period real and nominal discount factors are

$$M_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left(\frac{V_{t+1}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{\psi-\gamma}, \quad \text{and} \quad M^{\$}_{t,t+1} = M_{t,t+1}\left(\frac{P_{t+1}}{P_t}\right)^{-1}, \tag{8}$$

respectively. The nominal discount factor allows us to price the one-period nominal bond as

$$B_t^{\$} = e^{-i_t} = \mathbb{E}_t\left[M^{\$}_{t,t+1}\right], \tag{9}$$

where the one-period (continuously compounded) nominal interest rate i_t is the instrument of monetary policy.

Wage Setting

Following Schmitt-Grohe and Uribe (2007) and Li and Palomino (2012), we model an imperfectly competitive labor market where the representative household monopolistically provides a continuum of labor types indexed by $k \in [0,1]$.[10] The supply of labor type k satisfies the demand

[10] This approach is different from the standard heterogeneous households approach to model wage rigidities in Erceg, Henderson and Levin (2000), where each household supplies a differentiated type of labor. In the presence of recursive preferences, this approach introduces heterogeneity in the marginal rate of substitution of consumption across households since it depends on labor. We avoid this difficulty and obtain a unique marginal

equation

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} N_t^d\,, \tag{10}$$

where N_t^d is the aggregate labor demand of the production sector, $W_t(k)$ is the wage for labor type k, and W_t is the aggregate wage index given by

$$W_t = \left[\int_0^1 W_t^{1-\theta_w}(k)\,dk\right]^{\frac{1}{1-\theta_w}}\,. \tag{11}$$

The labor demand equation (10) is obtained from the production sector problem presented in the section below. The household chooses wages $W_t(k)$ for all labor types k under Calvo (1983) staggered wage setting. Specifically, at each time t the household is only able to adjust wages optimally for a fraction $1-\alpha_w$ of labor types. The remaining fraction α_w of labor types adjust their previous period wages by the wage indexation factor $\Lambda_{w,t-1,t}$. (The specific functional form of the indexation factor is presented in the calibration section.) The optimal wage maximizes (3), subject to demand functions (10) for all labor types k, and the budget constraint (7). Notice that real labor income is given by

$$LI_t = \int_0^1 \frac{W_t(k)}{P_t} N_t^s(k)dk\,. \tag{12}$$

Since the demand curve and the cost of labor supply are identical across different labor types, the household chooses the same wage W_t^* for all labor types subject to an optimal wage change at time t. Appendix A shows that the optimal wage satisfies

$$\frac{W_t^*}{P_t} \quad = \quad \mu_w \kappa_t \left(N_t^s\right)^{\omega} C_t^{\psi} \frac{G_{w,t}}{H_{w,t}}\,, \tag{13}$$

rate of substitution by modeling a representative agent who provides all different types of labor.

where $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$. The recursive equations describing $G_{w,t}$ and $H_{w,t}$ are presented in the appendix. Equation (13) can be interpreted as follows: In the absence of wage rigidities ($\alpha_\omega = 0$), the marginal rate of substitution between labor and consumption is $\kappa_t \left(N_t^s\right)^\omega C_t^\psi$, and the optimal wage is this rate adjusted by the optimal markup μ_w. Wage rigidities generate the time-varying markup $\mu_w \frac{G_{w,t}}{H_{w,t}}$, since the wage of some labor types is not adjusted optimally.

3.2 Production Sector

The production of differentiated goods is characterized by monopolistic competition and price rigidities in a continuum of firms. Firms set the price of their differentiated goods in a Calvo (1983) staggered price setting: At each time t, with probability α, a firm sets the good price as the previous period price adjusted by the price indexation factor $\Lambda_{p,t-1,t}$. (The specific functional form of the indexation factor is presented in the calibration section.) With probability $1-\alpha$, the firm sets the product price to maximize the present value of profits. The maximization problem for firm j can be written as

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^{\$} \left[\Lambda_{p,t,t+s} P_t(j) Y_{t+s|t}(j) - W_{t+s|t}(j) N_{t+s|t}^d(j) \right] \right\}, \tag{14}$$

subject to the production function

$$Y_{t+s|t}(j) = A_{t+s} N_{t+s|t}^d(j), \tag{15}$$

and the demand function

$$Y_{t+s|t}(j) = \left(\frac{P_t(j)\Lambda_{p,t,t+s}}{P_{t+s}} \right)^{-\theta} Y_{t+s}. \tag{16}$$

The output $Y_{t+s|t}(j)$ is the production of firm j at time $t+s$ given that the last optimal price change was a time t. The wage $W_{t+s|t}(j)$ and the labor demand $N_{t+s|t}^d(j)$ have a similar interpretation.

The production problem takes into account the probability of not being able to adjust the price optimally in the future, and the corresponding indexation $\Lambda_{p,t,t+s}$.

The production function depends on labor productivity A_t and labor. We assume that labor productivity contains permanent and transitory components. Specifically, $A_t = A_t^p Z_t$, where the permanent and transitory components follow processes

$$\Delta \log A_{t+1}^p = \phi_a \Delta \log A_t^p + \sigma_a \varepsilon_{a,t+1}, \tag{17}$$

and

$$\log Z_{t+1} = \phi_z \log Z_t + \sigma_z \varepsilon_{z,t+1}, \tag{18}$$

respectively, with Δ as the difference operator, and innovations $\varepsilon_{a,t}$ and $\varepsilon_{z,t} \sim \text{IID}\mathcal{N}(0,1)$. Labor demand is a composite of a continuum of differentiated labor types indexed by $k \in [0,1]$ via the aggregator

$$N_t^d(j) = \left[\int_0^1 N_t^d(j,k)^{\frac{\theta_w - 1}{\theta_w}} \, dj\right]^{\frac{\theta_w}{\theta_w - 1}}, \tag{19}$$

where $\theta_w > 1$ is the elasticity of substitution across differentiated labor types.

All firms that set prices optimally are identical and set the same optimal price P_t^*. Appendix B shows that the optimal price satisfies

$$\left(\frac{P_t^*}{P_t}\right) H_t = \frac{\mu}{A_t} \frac{W_t}{P_t} G_t, \tag{20}$$

where $\mu = \frac{\theta}{\theta - 1}$. The recursive equations for H_t and G_t are presented in the appendix. Equation (20) can be interpreted as follows: In the absence of price rigidities, the product price is the markup-adjusted marginal cost of production, with optimal markup μ. Price rigidities generate the time-varying markup $\mu\frac{G_t}{H_t}$, since some firms do not adjust their prices optimally.

3.3 Monetary Policy

We model a monetary authority that sets the level of the one-period nominal interest rate. Monetary policy is described by the policy rule

$$i_t = \rho i_{t-1} + (1-\rho)\left[\bar{\imath} + \imath_\pi(\pi_t - \pi^\star_{t-1}) + \imath_x x_t\right] + u_t. \tag{21}$$

The one-period nominal interest rate, i_t, has an interest-rate smoothing component captured by ρ, and is set responding to aggregate inflation, the output gap, and a policy shock u_t. The ouput gap x_t is defined as the log deviation of total output, Y_t, from the output in an economy under flexible prices and wages, Y_t^f. That is, $X_t \equiv \frac{Y_t}{Y_t^f}$, and $x_t \equiv y_t - y_t^f$. The coefficients $\imath_x$, and $\imath_\pi$ capture the response of the monetary authority to the output gap and inflation deviations from a target, respectively. The process $\pi_t^\star$ denotes the time-varying inflation target. The inflation target is time-varying as in Ireland (2007) and Rudebusch and Swanson (2012). Its process is

$$\pi^\star_t = (1-\phi_{\pi^\star})g_\pi + \phi_{\pi^\star}\pi^\star_{t-1} + \sigma_{\pi^\star}\varepsilon_{\pi^\star,t}, \tag{22}$$

where $\varepsilon_{\pi^\star,t} \sim \text{IID}\mathcal{N}(0,1)$. The policy shocks u_t follow the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{23}$$

where $\varepsilon_{u,t} \sim \text{IID}\mathcal{N}(0,1)$.

3.4 The Term Structure of Interest Rates

The price of a real and nominal bonds with maturity at $t+n$ can be written as

$$B_t^{(n)} = \exp\left(-nr_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad B_t^{\$,(n)} = \exp\left(-ni_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}^{\$}], \tag{24}$$

respectively, where $r_t^{(n)}$ and $i_t^{(n)}$ are the associated real and nominal bond yields, and $M_{t,t+n}$ and $M_{t,t+n}^{\$}$ are the real and nominal discount factors for payoffs at time $t+n$.

Real Term Premia

Real term premia capture the compensation for macroeconomic risk in long-term real bonds. Specifically, we define the term premium of an n-period real bond as

$$rTP_t^{(n)} \quad \equiv \quad \log \frac{\mathbb{E}_t\left[B_{t+1}^{(n-1)}\right]}{B_t^{(n)}} - r_t. \tag{25}$$

That is, the real term premium is the one-period expected excess return of a real bond with respect to the real risk-free rate. Under the assumption of joint normality for the log-pricing kernel and bond yields, appendix E shows that the premium is

$$rTP_t^{(n)} = \text{cov}_t\left(\log M_{t,t+1}, (n-1)r_{t+1}^{(n-1)}\right). \tag{26}$$

It captures the correlation between the marginal utility of consumption and the bond one-period return. A positive correlation between the future marginal utility and the bond yield implies low bond real returns during periods of high marginal utility and, therefore, positive expected excess bond returns. The appendix also shows that the unconditional spread can be written as the average

$$\mathbb{E}\left[r_t^{(n)} - r_t\right] = \text{J.I.}_r^{(n)} + \frac{1}{n-1}\sum_{s=0}^{n-2}\left(1 - \frac{s+1}{n}\right)\mathbb{E}\left[rTP_{t+s}^{(n-s)}\right], \tag{27}$$

where J.I. denotes Jensen's inequality terms. Therefore, the average spread can be seen as a weighted average of the one-period expected excess returns during the life of the bond. We analyze the model determinants of this spread and the real risk premium in section 4.

Inflation Risk Premia

We define the n-period inflation risk premium $\pi RP_t^{(n)}$ as the expected excess real return for investing in an n-period nominal bond over an n-period real bond for n-periods. Appendix E shows, under normality assumptions for bond yields, the log-pricing kernel, and inflation, that this premium is

$$\pi TP_t^{(n)} \equiv \log \frac{\mathbb{E}_t\left[\exp(-\pi_{t,t+n})\right]}{B_t^{\$,(n)}} - \log \frac{1}{B_t^{(n)}} = \mathrm{cov}_t\left(m_{t,t+n}, \pi_{t,t+n}\right), \tag{28}$$

where

$$m_{t,t+n} = \sum_{s=0}^{n-1} \log M_{t+s,t+s+1}, \quad \text{and} \quad \pi_{t,t+n} = \sum_{s=1}^{n} \pi_{t+s}, \tag{29}$$

are the log-real pricing kernel that discounts cashflows at $t+n$ to t, and inflation during the life of the bond, respectively. The inflation risk premium then is an expected return compensation in nominal bonds for the correlation between the marginal utility of consumption and inflation. If this correlation is positive, the expected returns of nominal bonds are higher than for real bonds: during periods of high marginal utility, high inflation has a negative impact on nominal bond returns. It can be shown, that the average spread between nominal and real rates is

$$\mathbb{E}\left[i_t^{(n)} - r_t^{(n)}\right] = \mathbb{E}[\pi_t] + \text{J.I.}_\pi^{(n)} + \frac{1}{n}\mathbb{E}\left[\pi TP_t^{(n)}\right]. \tag{30}$$

We analyze the model determinants of this spread and the inflation risk premium in section 4.

3.5 Equilibrium

Equilibrium in this economy requires product, labor, and financial market clearing. Equilibrium conditions in the product market is characterized by $C_t = Y_t$. Labor market clearing requires that supply and demand of labor type k employed by firm j are equal, $N_t^s(j,k) = N_t^d(j,k)$. As shown in appendix C, it implies equilibrium in the aggregate labor market given by $N_t^s = N_t^d F_{w,t}$ and $N_t^d = \frac{Y_t}{A_t} F_t$,. The distortions $F_{w,t}$ and F_t measure wage and price dispersion caused by wage and price rigidities, respectively, and are defined in the appendix. Equilibrium in the financial market implies that the nominal interest rate from household maximization in equation (9) is equal to the interest rate set by the monetary authority. That is,

$$-\log\left[M_{t,t+1}^{\$}\right] = \rho\, i_{t-1} + (1-\rho)\left(\bar{\imath} + \imath_\pi(\pi_t - \pi_{t-1}^\star) + \imath_x x_t\right) + u_t\,. \tag{31}$$

Also, equilibrium in the bond market implies the absence of arbitrage opportunities. This condition can be written recursively as

$$B_t^{(n)} = \mathbb{E}_t\left[M_{t,t+1} B_{t+1}^{(n-1)}\right], \quad \text{and} \quad B_t^{\$,(n)} = \mathbb{E}_t\left[M_{t,t+1}^{\$} B_{t+1}^{\$,(n-1)}\right], \tag{32}$$

for real and nominal bonds, respectively.

Appendix D provides a summary of the system of equations describing the equilibrium of the model. We solve the model numerically, applying a second-order approximation of the optimality conditions. A second-order approximation is required to capture expected excess returns on financial claims.[11]

[11] A first-order approximation of the equilibrium conditions implies expected returns for all financial assets equal to the risk-free rate. We use the Dynare package available from www.dynare.org to solve the model.

4 Model Implications

We explore the real bond pricing implications of the model based on a calibration that captures important properties of macroeconomic variables and the nominal yield curve. We compare the baseline calibration with different model specifications to understand the effects of each shocks, nominal rigidity, and policy parameter. We focus on the study of real term and inflation risk premia, the volatility of real and nominal rates, and the correlation of real and nominal bond returns.

4.1 Calibration

We calibrate the model to match key quarterly statistics of consumption, inflation, and the nominal yield curve. The data are described in section 2. The sample period 1982:Q1 to 2008:Q3 is chosen for two reasons. First, we want to focus on a period of stable monetary policy that can be described by an interest-rate policy rule. This period corresponds to the Paul Volcker and Alan Greenspan eras and the early years of Ben Bernanke's tenure at the the Federal Reserve. Clarida, Galí and Gertler (2000) provide empirical evidence of a change in monetary policy after 1979. We exclude the monetary experiment period $1979 - 1981$, since during this period the short-term rate was replaced by monetary aggregates as the policy instrument. We do not include data after the third quarter of 2008 because the ability to conduct policy using the Federal Funds rate was limited by the zero bound after December 2008. Second, while our main focus is to capture and understand properties of the real term structure, a calibration of the model-implied real curve to the short sample of TIPS data does not seem appropriate. Instead, we explore the implications of this calibration for the real yield curve in section 4, and make comparisons of it with the nominal yield curve to obtain additional insights.

Table 4 presents the parameter values used in the baseline calibration. We assume perfect indexation of prices and wages to the inflation target, such that $\log \Lambda_{p,t,t+1} = \pi^{\star}_t$, and $\log \Lambda_{w,t,t+1} =$

$g_a + \pi_t^\star$. Notice that wage indexation implies no deviations from real wages on average. The rigidity parameters α and α_w imply price and wage durations of $-1/\log(\alpha) \approx 2.4$ quarters and $-1/\log(\alpha_w) \approx 4$ quarters, respectively, consistent with the empirical evidence. The elasticity parameters θ and θ_w imply price and wage markups of 20% and 30%, respectively. The parameter value for ω implies a Frisch labor elasticity of $1/\omega \approx 2.86$, consistent with values used in the macro literature to capture labor and wage dynamics. The policy responses to inflation $\imath_\pi$ and the output gap $\imath_x$ are standard in the literature.

The remaining parameters were chosen to match different moments in four steps. First, we choose parameter values for the policy and productivity shock processes based on the variance decomposition reported by Altig, Christiano, Eichenbaum and Linde (2011). Table 5 reports the data and model-implied variance decompositions in terms of volatilities. Specifically, we target the volatility of the three-month T-Bill rate, the inflation rate, and de-trended consumption. This approach takes into account the fact that policy and productivity shocks do not capture all the variability in these macroeconomic variables. The variance decomposition for policy shocks in the data is matched by choosing parameter values for ϕ_u, σ_u, and ψ. This decomposition is very sensitive to the elasticity of intertemporal substitution of consumption, which is set at $1/\psi = 0.20$. The variance decomposition of productivity shocks depends on the dynamics of permanent and transitory shocks. While Altig, Christiano, Eichenbaum and Linde (2011) model only has permanent productivity shocks, we find that our model calibration with only permanent shocks implies a counterfactually high consumption growth volatility. For this reason, we choose a combination of ϕ_a, σ_a, ϕ_z, σ_z, and ρ that simultaneously allows us to capture the variance decomposition of productivity shocks for the selected values and the volatility of consumption growth.[12] Notice that the transitory productivity shocks have persistence and volatility parameters that are very similar to those found in the real business cycle literature. Although the calibration captures the variance of the interest rate and consumption explained by productivity shocks, the

[12] Unfortunately, Altig, Christiano, Eichenbaum and Linde (2011) do not report the variance decomposition for consumption growth.

table shows that the model-implied inflation volatility explained by these shocks is lower than in the data. This decomposition is very sensitive to the smoothing parameter ρ. The chosen value $\rho = 0.725$ is similar to those reported in Clarida, Galí and Gertler (2000).

Second, we choose parameters $\phi_{\pi^\star}$ and $\sigma_{\pi^\star}$ for the inflation target to match the total volatility of inflation, and a high volatility of the 5-year nominal bond yield relative to the short-term rate, as reported in table 6. These values are in line with those used by Rudebusch and Swanson (2012). We want to capture the total volatility of inflation in the model since one of our purposes is to understand the driving forces of the inflation risk premium.[13] We obtain a high ratio of long-term to short-term yield volatility by setting an extremely persistent inflation target ($\phi_{\pi^\star} = 0.9995$). As in standard affine term structure models, this value is not enough to make long-term yields more volatile than short-term yields. However, the volatility of the long-term yield is significant. Lower values for $\phi_{\pi^\star}$ considerably reduce this volatility. Table 6 also shows that the calibration captures the negative correlation between consumption growth and inflation. This correlation will be important for explaining the inflation risk premium. On the other hand, the calibration implies a slightly negative autocorrelation of consumption growth, while the autocorrelation in the data is positive. However, it is well known that this parameter is poorly measured due to factors such as the time-aggregation upward bias reported by Working (1960).[14] The results of empirical studies are mixed for this autocorrelation. While Campbell and Mankiw (1989) and Cochrane (1994), for instance, find that U.S. consumption growth is almost unforecastable, Kandel and Stambaugh (1991) and Mehra and Prescott (1985) imply a small persistent predictability component in this variable. The small negative autocorrelation implied by our model will be important to capture positive real term premia.

[13]It is a strong assumption to claim that inflation target shocks completely capture the volatility of inflation that is not captured by policy and policy shocks. However, in the absence of additional shocks in the model, this assumption allows us to simultaneously assign an appropriate degree of variability to policy and productivity shocks, while capturing the high volatility of inflation.

[14]Our data for monthly consumption growth imply an autocorrelation of -0.26. Suppose that the true process for monthly consumption growth is AR(1) with this autocorrelation. In an experiment with 10,000 simulations and 321 monthly data points in each simulation, the 95% confidence interval for the autocorrelation of consumption growth in time-aggregated quarterly data is (-0.04, 0.33).

Third, we choose a high value for the risk aversion parameter γ to obtain the slope of the curve between 3-month and 5-year maturities.[15] In the presence of recursive preferences on consumption and labor, Swanson (2012) shows that the average coefficient of risk aversion is given by

$$\frac{\psi}{1+\frac{\psi}{\omega\mu}}+\frac{\gamma-\psi}{1-\frac{1-\psi}{1+\omega}}\approx 250. \tag{33}$$

This value is high according to experimental evidence, which highlights the difficulties of production economy models to capture asset risk premiums. However, the value is significantly smaller that the one required by Tallarini (2000), for instance, to match the equity premium. This improvement is the result of incorporating permanent productivity shocks. On the other hand, the value contrasts with a $\gamma = 5$ in Kaltenbrunner and Lochstoer (2010) who are able to capture the equity premium in a production economy with recursive preferences. The difference is driven by their high volatility of productivity shocks and the fact that they do not match inflation data.

Finally, we choose g_a to match the average consumption growth, and β and g_π to simultaneously match the average inflation and three-month T-bill rates.[16] We define $\bar{\imath} = -\log\beta + \psi g_a + g_\pi$, which is the nominal rate when the inflation rate is at the target and there is no output gap.

4.2 The Real and Nominal Term Structures of Interest Rates

Column (1) of table 7 reports model-implied statistics for the real and nominal 5-year bonds in the baseline calibration. The real and nominal yield curves are upward sloping and the slope of the real yield curve is lower than the slope of the nominal yield curve. This is qualitative similar to the data statistics in table 1. In the model, upward sloping curves are the result of

[15] The unconditional expected excess quarterly return and Sharpe Ratio of 5-year bonds in the model are 34 bps. and 0.57, respectively. These numbers contrast with their data counterparts of 63 bps. and 0.25 , respectively, reflecting the model limitation to capture the high volatility of excess returns.

[16] We solve the model using a second-order approximation around the non-stochastic steady state. The high value for γ generates large precautionary savings terms that create distortions in the mean of inflation and the interest rate. We offset these distortions by choosing a large value for g_π, which reduces its interpretation as a long-term inflation target. However, this approach does not generate significant distortions in expected excess bond returns and the slope of the yield curve.

positive real term and inflation risk premia. The volatility of the short-term real rate is lower than the volatility of the short-term nominal rate, and both curves exhibit downward sloping term structures of volatility, as in the data. However, the decay in volatility across maturities of real yields is significantly more pronounced than for nominal yields. Therefore, the calibration does not capture the empirical evidence on similar volatility for the yields of the two instruments with the same maturity.

Tables 7 and 8 allow us to identify the model elements delivering the main results. Columns (2) to (4) of table 7 present statistics for three alternative model specifications: no rigidities, only wage rigidities, and only price rigidities. The alternative models share the same parameter values as in the baseline calibration, except for the rigidity parameters. Similarly, columns (2) to (5) of table 8 present statistics for four alternative models with the same parameter values as in the baseline case but when only one specific shock is turned on in each specification. Table 9 allows us to understand the yield curve effects of changes in monetary policy rule parameters.[17]

We summarize the main findings here and explain them below. First, wage rigidities generate upward sloping real and nominal yield curves. This is a result of positive real term and inflation risk premia for permanent productivity shocks. Price rigidities increase the slope of the curves relative to an economy with flexible prices and wages, but the slopes, and the term and inflation risk premia are still negative. Second, transitory productivity, policy, and inflation-target shocks have a minimal contribution to risk premia relative to the contribution of permanent productivity shocks. The first three shocks have transitory effects in the marginal utility of consumption and, then, investors do not require a significant compensation for these risks in financial assets. However, these shocks affect the volatility of short- and long-term bonds. Policy shocks significantly increase the volatility of the real risk-free rate under nominal rigidities. Persistent transitory productivity shocks and inflation-target shocks increase the volatility of real and nominal bond yields, respectively. Third, stronger reactions to both inflation and output in the policy rule decrease term

[17]For the alternative specifications in tables 7, 8, and 9, we also adjust β and g_π such that the average inflation and short-term interest rates are the same as in the baseline calibration.

premia and increase inflation risk premia. These changes in the policy affect the joint dynamics of consumption, inflation, and interest rates. Fourth, nominal rigidities increase the correlation between real and nominal bond returns. These rigidities then tend to reduce the diversification benefits of real bonds.

The effect of nominal rigidities and permanent productivity shocks on bond risk premia

We focus on the analysis of risk premia for permanent productivity shocks since the quantitative contribution of the other shocks to term and inflation risk premia is very small.[18] Column (3) of table 7 shows that wage rigidities generate upward sloping average real and nominal yield curves. This is the result of a positive compensation for permanent productivity shocks in real term and inflation risk premia. Wage rigidities change the joint dynamics of consumption, labor, inflation, and yields relative to an economy with perfectly flexible wages. To understand why, we can focus on an economy with expected utility preferences ($\gamma = \psi$) and only permanent productivity shocks. Appendix E explains the general recursive preference case.

Under expected utility, the log-pricing kernel and the one-period interest rate are

$$\log M_{t,t+1} = \text{const}_m - \psi \Delta c_{t+1}, \quad \text{and} \quad r_t = \text{const}_r + \psi \mathbb{E}_t[\Delta c_{t+1}], \tag{34}$$

respectively, where "const" denotes constant terms. From equation (26), the real term premium of a two-period bond is

$$rTP_t^{(2)} = \text{cov}_t(\log M_{t,t+1}, r_{t+1}) = -\psi^2 \text{cov}_t(\Delta c_{t+1}, \mathbb{E}_{t+1}[\Delta c_{t+2}]), \tag{35}$$

and, from equation (28), the inflation risk premium of a one-period bond is

$$\pi TP_t^{(1)} = \text{cov}_t(\log M_{t,t+1}, \pi_{t+1}) = -\psi \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}). \tag{36}$$

[18]There are interesting qualitative implications of nominal rigidities on the compensation for transitory productivity, policy, and inflation-target shocks. These implications can be understood from the risk premium equations and figures 2, 3, and 4, respectively.

Under flexible wages, consumption growth is positively autocorrelated and positively correlated with inflation, generating negative real term and inflation risk premia. Figure 1 shows that a negative permanent productivity shock reduces prices and real wages so that they reflect an optimal wage markup over the marginal rate of substitution of consumption and labor, and an optimal price markup relative to labor productivity. Equilibrium labor does not change and, then, consumption growth inherits the positive persistence of the AR(1) process for productivity growth. Since expected consumption growth and the real rate are simultaneously low, real bond returns are high during periods of high marginal utility. Therefore, real bonds are a consumption hedge and involve a negative premium. Since inflation is negative, real returns on nominal bonds are high, therefore a consumption hedge, and the inflation risk premium is negative. On the contrary, under wage rigidities, labor supply/demand becomes mean-reverting and affects the dynamics of consumption growth. After a negative permanent productivity shock, some wages do not adjust downwards due to the rigidity. This reduces labor demand, and consumption declines by more than in a flexible-wage economy. As wages gradually adjust down over time, expected future labor demand increases. If the mean-reverting labor effect dominates the positive persistence of the permanent shock, expected future consumption growth and the real risk-free rate increase, leading to a positive real term premium. Simultaneously, the negative shock increases prices so that producers charge a constant markup over the higher real marginal costs. The positive inflation reduces the real return on nominal bonds during times of high marginal utility, inducing a positive inflation risk premium.

A similar mechanism explains the effect of price rigidities on real term and inflation risk premia. Figure 1 shows that, after a negative permanent productivity shock, product prices drop by less than in a flexible-price economy, and then consumption, labor demand, and wages are more negatively affected, initially. However, as prices decline over time, future labor demand, wages and, therefore, expected future consumption growth increase, partially offsetting the negative movement in the real risk-free rate. Therefore, bond returns are not as low as in a flexible-price economy during periods of high marginal utility, and the slope of the real term structure is less negative. Also, the figure and table 7 show that inflation and consumption growth are positively correlated, generating a negative inflation risk premium.

Monetary policy shocks and the volatility of the short-term real interest rate

The small effect of policy shocks on expected bond excess returns radically contrasts with the significant effect of these shocks on the volatility of the real risk-free rate. Column (4) in table 8 shows statistics for an economy with only policy shocks. The table shows that policy shocks have the most significant contribution to the volatility of the real risk-free rate r_t. This is a result of price and wage rigidities. The impulse responses in figure 3 show that the risk-free rate does not react to policy shocks in the absence of rigidities. In a flexible price and wage economy, monetary policy does not have any real effects. Changes in the policy rule are completely reflected in inflation changes and have no effect on real activity. On the other hand, the figure shows that nominal rigidities induce a positive response in the real risk-free rate to positive policy shocks. A policy change in the nominal risk-free rate translates not only into changes in inflation but also into changes in the real risk-free rate. Since prices and/or wages do not adjust optimally to changes in the nominal rate, output and consumption are affected. A positive policy shock has a transitory negative effect on output, that is followed by positive output growth. As a result, the real risk-free rate has to increase to be consistent with the higher expected consumption growth. The effect of this shock is also reflected in a higher volatility for the entire real yield curve. However, the most significant effect is on the short-term rate.

Transitory productivity shocks, inflation-target shocks, and the volatility of long-term yields

Columns (3) and (5) in table 8 reveal the minor contribution of transitory productivity and inflation-target shocks to the slope and risk premia in real and nominal bonds. However, the table also shows that these shocks have a significant contribution to the volatility of real and nominal long-term bonds, respectively. While the ratio of the volatility of the 5-year real bond yield with respect to the short-term real rate is 0.16 in the baseline calibration, it is 0.59 in the presence of (only) transitory shocks Z_t. This is the result of the high persistence of these shocks, similar to the one found in the real business cycle literature. In our model, these shocks have to be persistent to match the contribution of productivity shocks to the volatility of the short-term nominal rate and consumption. The high persistence in the shock is reflected in persistent effects on macroeconomic variables, and therefore on long-term real bonds. Similarly, the high persistence in inflation-target shocks generates volatility in long-term nominal interest rates similar to the volatility of the one-period nominal rate. These shocks have a long-lasting effect on inflation that affects the level of long-term nominal rates. Notice that these shocks have a reduced effect on real yield volatility as a result of nominal rigidities. Therefore, changes in the properties of inflation-target shocks may explain differences between real and nominal yield volatility over time.

The interest-rate policy rule, and real term and inflation risk premia

We use our model to understand how changes in the reaction coefficients of the policy rule affect some properties of the real yield curve and the inflation risk premia. We analyze four policy parameters: $\imath_\pi$, $\imath_x$, ρ, and $\phi_{\pi^\star}$. Table 9 reports model statistics under different values for these parameters. For each parametrization, we adjust β and g_π to keep the same average nominal interest rate and inflation.

Column (2) in the table shows the effect of a change in the response to inflation $\imath_\pi$ from 1.5 to 1.7. The stronger response reduces the volatility of inflation, the output gap, and real rates, and increases in absolute value the negative correlation between consumption growth and inflation. The larger stabilization effect of the policy reduces the covariance of consumption growth and real bond yields (returns), lowering real term premia. On the other hand, the inflation risk premia increase: inflation is less volatile, but a unit of inflation has stronger negative effects on consumption growth.

Column (3) in table 9 shows the effect of increasing the response to the output gap $\imath_x$ from 0.125 to 0.25. The volatility of the output gap and real rates declines at the expense of higher inflation volatility. More stable consumption growth and real rates translate into lower real term premia. The higher inflation volatility is reflected in larger inflation risk premia.

Column (4) in table 9 reports the effect of a change in the interest-rate smoothing coefficient ρ from 0.725 to 0.80.[19] A higher ρ decreases the volatility of the output gap and inflation. Nominal rate volatility declines as a result of the larger weight on the lagged interest rate. Real rate volatility increases as a result of a lower relative response of the nominal rate to current economic conditions. This translates into opposite effects on real term premia that completely offset each other. The inflation risk premia increase since a higher real rate volatility generates a higher volatility in consumption growth and then a stronger negative covariance between consumption growth and inflation.

Finally, column (5) in table 9 shows the effect of a reduction in the autocorrelation coefficient of the inflation target $\phi_{\pi^\star}$ from 0.9995 to 0.90. The lower persistence reduces the volatility of the output gap, inflation, and nominal yields. Simultaneously, the negative correlation between consumption growth and inflation becomes stronger. As a result, the real term and inflation risk premia do not change significantly.

The diversification benefits of real bonds

An interesting question to ask is whether real bonds improve investors' opportunities to share risks and smooth consumption over time. In our complete-market model, real bonds are redundant assets since their returns can be replicated by portfolios of nominal bonds. However, we can gain some insights into the potential diversification benefits of real bonds by analyzing correlations between returns of real and nominal bonds with the same maturity. A constrained investor with only access to a nominal bond faces an incomplete market since four sources of risk affect the marginal utility of consumption and inflation. Therefore, the diversification benefits of adding a real bond to her portfolio are captured by this correlation. In our baseline calibration, table 7 shows that the correlations are 0.95 and 0.86 for one- and five-year bonds, respectively. The

[19] For this exercise, we adjusted $\imath_\pi$ and $\imath_x$ so that $(1-\rho)\imath_\pi$ and $(1-\rho)\imath_x$ remain at their baseline values.

correlations are lower in the absence of nominal rigidities, and higher under wage rigidities only. The intuition is that nominal rigidities generate a strong link between inflation and real economic performance, reducing the differences between holding real or nominal bonds. In the absence of rigidities, inflation does not have effects on economic activity and the link between real and nominal bond returns is weaker. Table 8 presents the correlations generated by the individual model shocks. Permanent and productivity shocks have similar effects on the correlations of real and nominal bonds. These shocks have more pronounced effects on short-term bond correlations than on longer term bonds. Policy shocks have very similar effects on real and nominal bonds. Inflation-target shocks, on the other hand, generate negative correlations between real and nominal bonds. Interestingly, table 9 shows that changes in the policy rule parameters do not have significant quantitative effects on these correlations.

5 Conclusion

This paper provides a theoretical analysis of the link between nominal price rigidities, monetary policy, and long-term real bond yields using an equilibrium model. The model calibrated to the United States economy implies an average upward sloping real curve, volatile long-term rates, and positive inflation risk premia. These properties are consistent with those observed in inflation-indexed and nominal bonds in recent years. There are four main findings. First, nominal rigidities increase term premia in real bonds and inflation risk premia in nominal bonds, as a compensation for permanent productivity shocks. Quantitatively, wage rigidities have larger effects than price rigidities. The main mechanism is explained by the procyclical mean-reverting labor demand induced by wage rigidities: It simultaneously generates high marginal utility of consumption, high inflation, and low returns on real and nominal bonds. Second, the model transitory shocks do not have significant effects on risk premia, but important effects on yield volatility. Monetary policy shocks considerably increase the volatility of short-term real bond yields. Persistent productivity transitory and inflation-target shocks increase the volatility of long-term real and nominal yields, respectively. Third, the stabilization effects of a stronger response to inflation or output in the interest-rate policy rule decrease real term premia and increase inflation risk premia. Fourth, the existence of nominal rigidities increase the correlation between real and nominal bonds, and may reduce the diversification benefits of inflation-indexed bonds.

The analysis can be extended in several dimensions. First, an empirical study of the model testable implications across countries. For instance, the model predicts lower real yield curve slopes in economies with more flexible wages. This is consistent with the average inverted real yield curve in United Kingdom, and the findings in Smith (2000) and Dickens et al. (2007) of less rigid wages in United Kingdom than in United States. Second, the model abstracts from capital accumulation. This component can be introduced to understand the link between the marginal product of capital and long-term real rates. Third, the model implies low volatility in nominal bond risk premia, inconsistent with empirical findings. Introducing sources of time-varying volatility in marginal utility can help us understand the link between time variation in real and nominal bond risk premia. Finally, the framework can be used to learn about the effects of optimal monetary policy on real rates and their economic content.

References

Altig, D., L. Christiano, M. Eichenbaum and J. Linde. 2011. "Firm-Specific Capital, Nominal Rigidities and the Business Cyle." *Review of Economic Dynamics* 14(2):225–247.

Alvarez, Fernando and Urban J. Jermann. 2005. "Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth." *Econometrica* 73(6):1977–2016.

Ang, Andrew, Geert Bekaert and Min Wei. 2008. "The Term Structure of Real Rates and Expected Inflation." *Journal of Finance* 63(2):797–849.

Bansal, Ravi and Ivan Shaliastovich. 2012. A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets. Nber working papers National Bureau of Economic Research, Inc.

Barattieri, Alessandro, Susanto Basu and Peter Gottschalk. 2010. Some Evidence on the Importance of Sticky Wages. NBER Working Papers 16130 National Bureau of Economic Research, Inc.

Bekaert, Geert and Xiaozheng Wang. 2010. "Inflation risk and the inflation risk premium." *Economic Policy* 25:755–806.

Bils, Mark and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112(5):947–985.

Buraschi, Andrea and Alexei Jiltsov. 2005. "Inflation risk premia and the expectations hypothesis." *Journal of Financial Economics* 75(2):429–490.

Calvo, Guillermo. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12:383–398.

Campbell, John Y. 1986. "Bond and Stock Returns in a Simple Exchange Model." *The Quarterly Journal of Economics* 101(4):785–803.

Campbell, John Y. and N. Gregory Mankiw. 1989. Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual 1989, Volume 4*. NBER Chapters National Bureau of Economic Research, Inc pp. 185–246.

Campbell, John Y. and Robert J. Shiller. 1996. "A Scorecard for Indexed Government Debt." *National Bureau of Economic Research, Inc* pp. 155–208.

Campbell, John Y., Robert J. Shiller and Luis M. Viceira. 2009. "Understanding Inflation-Indexed Bond Markets." *Brookings Papers on Economic Activity* 40(1 (Spring)):79–138.

Christensen, Jens H. E., Jose A. Lopez and Glenn D. Rudebusch. 2010. "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields." *Journal of Money, Credit and Banking* 42(s1):143–178.

Christiano, Lawrence J. and Martin Eichenbaum. 1990. “Unit roots in real GNP: Do we know, and do we care?” *Carnegie-Rochester Conference Series on Public Policy* 32(1):7–61.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans. 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy* 113(1):1–45.

Clarida, Richard, Jordi Galí and Mark Gertler. 2000. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *The Quarterly Journal of Economics* pp. 147–180.

Cochrane, John H. 1994. “Permanent and Transitory Components of GNP and Stock Prices.” *The Quarterly Journal of Economics* 109(1):241–65.

D’Amico, Stefania, Don H. Kim and Min Wei. 2008. Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices. Technical report.

Dickens, William T., Lorenz Goette, Erica L. Groshen, Steinar Holden, Julian Messina, Mark E. Schweitzer, Jarkko Turunen and Melanie E. Ward. 2007. “How Wages Change: Micro Evidence from the International Wage Flexibility Project.” *Journal of Economic Perspectives* 21(2):195–214.

Epstein, Larry G and Stanley E Zin. 1989. “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework.” *Econometrica* 57(4):937–69.

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin. 2000. “Optimal monetary policy with staggered wage and price contracts.” *Journal of Monetary Economics* 46(2):281–313.

Garcia, Juan Angel and Adrian van Rixtel. 2007. Inflation-linked bonds from a Central Bank perspective. Occasional Paper Series 62 European Central Bank.

Grishchenko, Olesya V. and Jingzhi Huang. 2012. Inflation risk premium: evidence from the TIPS market. Finance and Economics Discussion Series 2012-06 Board of Governors of the Federal Reserve System (U.S.).

Gurkaynak, Refet S., Brian Sack and Jonathan H. Wright. 2006. The U.S. Treasury yield curve: 1961 to the present. Technical report.

Gurkaynak, Refet S., Brian Sack and Jonathan H. Wright. 2008. The TIPS yield curve and inflation compensation. Technical report.

Harvey, Campbell R. 1988. “The real term structure and consumption growth.” *Journal of Financial Economics* 22(2):305–333.

Hördahl, Peter. 2008. “The inflation risk premium in the term structure of interest rates.” *BIS Quarterly Review* .

Ireland, Peter N. 2007. “Changes in the Federal Reserve’s Inflation Target: Causes and Consequences.” *Journal of Money, Credit and Banking* 39(8):1851–1882.

Kahn, Shulamit. 1997. “Evidence of Nominal Wage Stickiness from Microdata.” *American Economic Review* 87(5):993–1008.

Kaltenbrunner, Georg and Lars A. Lochstoer. 2010. "Long-Run Risk through Consumption Smoothing." *Review of Financial Studies* 23(8):3190–3224.

Kandel, Shmuel and Robert F. Stambaugh. 1991. "Asset returns and intertemporal preferences." *Journal of Monetary Economics* 27(1):39–71.

Labadie, Pamela. 1994. "The term structure of interest rates over the business cycle." *Journal of Economic Dynamics and Control* 18(3-4):671–697.

Li, Erica X. N. and Francisco Palomino. 2012. "Nominal Rigidities, Asset Returns and Monetary Policy." *Working Paper, University of Michigan* .

Mehra, Rajnish and Edward C. Prescott. 1985. "The equity premium: A puzzle." *Journal of Monetary Economics* 15(2):145–161.

Nakamura, Emi and Jn Steinsson. 2008. "Five Facts about Prices: A Reevaluation of Menu Cost Models." *The Quarterly Journal of Economics* 123(4):1415–1464.

Pflueger, Carolin E. and Luis M. Viceira. 2011. Inflation-Indexed Bonds and the Expectations Hypothesis. NBER Working Papers 16903 National Bureau of Economic Research, Inc.

Pflueger, Carolin E. and Luis M. Viceira. 2012. An Empirical Decomposition of Risk and Liquidity in Nominal and Inflation-Indexed Government Bonds. NBER Working Papers 16892 National Bureau of Economic Research, Inc.

Piazzesi, Monika and Martin Schneider. 2007. "Equilibrium Yield Curves." pp. 389–442. NBER Macroeconomics Annual 2006, MIT Press.

Rudebusch, Glenn D. and Eric T. Swanson. 2012. "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks." *American Economic Journal: Macroeconomics* 4(1):105–43.

Schmitt-Grohe, Stephanie and Martin Uribe. 2007. "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model." *Monetary Policy under Inflation Targeting* 11(5):125–186.

Smith, Jennifer C. 2000. "Nominal Wage Rigidity in the United Kingdom." *Economic Journal* 110(462):C176–95.

Swanson, Eric T. 2012. "Risk Aversion and the Labor Margin in Dynamic Equilibrium Models." *American Economic Review* .

Tallarini, Thomas D. 2000. "Risk-sensitive real business cycles." *Journal of Monetary Economics* 45(3):507–532.

Taylor, John B. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39(1):195–214.

Woodford, Michael. 2003. *Interest and Prices*. New Jersey: Princeton University Press.

Working, Holbrook. 1960. "Note on the Correlation of First Differences of Averages in a Random Chain." *Econometrica* 28(4):916–918.

A Household's Utility Maximization under Wage Rigidities (based on Li and Palomino (2012))

The household's problem is

$$\max_{\{C_t, N_t^s, W_t^*\}} \quad V_t = U_t + \beta Q_t^{\frac{1-\psi}{1-\gamma}}$$

where

$$U_t = \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega}, \quad \text{and} \quad Q_t = \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right],$$

subject to the budget constraint

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} C_{t+\tau} \right] \leq \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} \right) \right],$$

where LI_t and D_t are aggregate labor income and firm profits, respectively. The Lagrangian associated with this problem is

$$\mathcal{L} \quad = \quad \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} + \beta Q_t^{\frac{1-\psi}{1-\gamma}} + \lambda \, \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} - C_{t+\tau} \right) \right].$$

It can be shown that utility maximization implies $\lambda = \frac{C_t^{-\psi}}{P_t}$, and

$$\begin{aligned} M_{t,t+1}^{\$} &= \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \frac{P_t}{P_{t+1}} = \beta \frac{\frac{\partial Q_t}{\partial C_{t+1}} \frac{\partial V_t}{\partial Q_t}}{C_t^{-\psi}} \frac{P_t}{P_{t+1}} \\ &= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{V_{t+1}^{1/(1-\psi)}}{Q_t^{1/(1-\gamma)}} \right)^{\psi-\gamma} \frac{P_t}{P_{t+1}}. \end{aligned}$$

The τ-period nominal pricing kernel is

$$M_{t,t+\tau}^{\$} = \prod_{s=1}^{\tau} M_{t,t+s}^{\$}.$$

The household cannot change wages for α_w fraction of labor types. For the remaining $1-\alpha_w$ fraction of labor types k, the household chooses wages $W_t^*(k)$ to maximize V_t. We assume that the wage choice for one labor type has negligible effects on the aggregate wage index and the aggregate labor demand. To see the impact of $W_t^*(k)$ on the household's utility, we rewrite the labor supply at $t+\tau$ as

$$N_{t+\tau}^s = \int_0^1 N_{t+\tau}^s(k)\, dk = N_{t+\tau}^d \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{-\theta_w} dk,$$

and the aggregate labor income at $t+\tau$ as

$$LI_{t+\tau} = \int_0^1 \frac{W_{t+\tau}}{P_{t+\tau}}(k) N_{t+\tau}^s(k)\, dk = \frac{N_{t+\tau}^d W_{t+\tau}}{P_{t+\tau}} \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{1-\theta_w} dk.$$

For the wage of type k labor at $t+\tau$, there are $\tau+2$ possible values:

$$W_{t+\tau}(k) = \begin{cases} W^*_{t+\tau-s}(k), & \text{with prob} = (1-\alpha_w)\alpha_w^s \text{ for } s = 0, 1, \cdots, \tau \\ W_{t-1}\Lambda_{w,t-1,t+\tau}, & \text{with prob} = \alpha_w^{\tau+1}. \end{cases}$$

We obtain derivatives

$$\begin{aligned} \frac{\partial N^s_{t+\tau}}{W^*_t(k)} &= N^d_{t+\tau}(1-\alpha_w)\alpha_w^\tau \left(\frac{-\theta_w}{W^*_t(k)}\right)\left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}}\right)^{-\theta_w}, \\ \frac{\partial LI_{t+\tau}}{\partial W^*_t(k)} &= \frac{N^d_{t+\tau}}{P_{t+\tau}}(1-\alpha_w)\alpha_w^\tau(1-\theta_w)\left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}}\right)^{-\theta_w}. \end{aligned}$$

The first-order condition of the Lagrangian with respect to $W^*_t(k)$ is given by

$$\frac{\partial \mathcal{L}}{\partial W^*_t(k)} = \frac{\partial V_t}{\partial W^*_t(k)} + \lambda\,\mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}P_{t+\tau}\frac{\partial LI_{t+\tau}}{\partial W^*_t(k)}\right] = 0\,,$$

where

$$\frac{\partial V_t}{\partial W^*_t(k)} = -\mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\frac{P_{t+\tau}}{P_t}\left(\frac{C_{t+\tau}}{C_t}\right)^{\psi}\kappa_{t+\tau}(N^s_{t+\tau})^{\omega}\frac{\partial N^s_{t+\tau}}{\partial W^*_t(k)}\right].$$

Rearranging terms, we get

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\Lambda_{w,t,t+\tau}\alpha_w^\tau W^{\theta_w}_{t+\tau}N^d_{t+\tau}\frac{W^*_t(k)}{P_t}C_t^{-\psi}\right] = \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\Lambda_{w,t,t+\tau}\alpha_w^\tau\left(\frac{P_{t+\tau}}{P_t}\right)W^{\theta_w}_{t+\tau}N^d_{t+\tau}\mu_w\kappa_{t+\tau}(N^s_{t+\tau})^{\omega}\left(\frac{C_{t+\tau}}{C_t}\right)^{\psi}\right].$$

Since all labor types face the same demand curve, we have $W^*_t(k) = W^*_t$ for all k. We can write the left-hand side of the equation as

$$LHS = C_t^{-\psi}W_t^{\theta_w}N^d_t H_{w,t}\frac{W^*_t}{P_t},$$

where

$$H_{w,t} = 1 + \alpha_w\mathbb{E}_t\left[M^{\$}_{t,t+1}\Lambda_{w,t,t+1}\left(\frac{N^d_{t+1}}{N^d_t}\right)\left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w}H_{w,t+1}\right].$$

Similarly, the right-hand side of the first-order condition can be written as

$$RHS = \mu_w W_t^{\theta_w}N^d_t(N^s_t)^{\omega}G_{w,t} = \mu_w W_t^{\theta_w}N^d_t\kappa_t\left(N^s_t\right)^{\omega}G_{w,t}$$

where

$$G_{w,t} = 1 + \alpha_w\,\mathbb{E}_t\left[M^{\$}_{t,t+1}\Lambda_{w,t,t+1}\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\psi}\left(\frac{N^d_{t+1}}{N^d_t}\right)\left(\frac{\kappa_{t+1}}{\kappa_t}\right)\left(\frac{N^s_{t+1}}{N^s_t}\right)^{\omega}\left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w}G_{w,t+1}\right].$$

The optimal real wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\frac{W^*_t}{P_t} = \mu_{w,t}C_t^{\psi}\kappa_t\left(N^s_t\right)^{\omega} \quad \text{and} \quad \mu_{w,t} = \mu_w\frac{G_{w,t}}{H_{w,t}}\,.$$

B Profit Maximization under Price Rigidities (based on Li and Palomino (2012))

Consider the Dixit-Stiglitz aggregate (5) as a production function, and a competitive "producer" of a differentiated good facing the problem

$$\max_{\{C_t(j)\}} P_t C_t - \int_0^1 P_t(j) C_t(j) dj$$

subject to (5). Solving the problem, we find the demand function

$$P_t(j) = P_t \left(\frac{C_t(j)}{C_t} \right)^{-1/\theta} \tag{37}$$

The zero-profit condition implies

$$P_t C_t = \int_0^1 P_t(j) C_t(j) dj = \int_0^1 P_t C_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta} dj.$$

Solving for P_t, it follows that

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{38}$$

which can be written as the demand function for each differentiated good

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} C_t\,. \tag{39}$$

Therefore, when prices are flexible, prices of all differentiated goods are the same.

The profit maximization problem is

$$\max_{\{P_t(j)\}} \quad \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau} \alpha^{\tau} \left[\Lambda_{p,t,t+\tau} P_t(j) Y_{t+\tau|t}(j) - W_{t+\tau|t}(j) N^d_{t+\tau|t}(j) \right] \right]$$

subject to

$$Y_{t+\tau|t}(j) = Y_{t+\tau} \left(\frac{P_t(j)\Lambda_{p,t,t+\tau}}{P_{t+\tau}} \right)^{-\theta}, \quad \text{and} \quad Y_{t+\tau|t}(j) = A_t N^d_{t+\tau|t}(j)\,.$$

The first-order condition of this problem with respect to $P_t(j)$ is

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau} \alpha^{\tau} Y_{t+\tau|t}(j) \Lambda_{p,t,t+\tau} P^*_t(j) \right] = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau} \alpha^{\tau}_I Y_{t+\tau|t}(j) \mu \frac{W_{t+\tau|t}(j)}{A_{t+\tau}} \right].$$

The left-hand side (LHS) of the equation can be written recursively as

$$LHS = P_t^* \left(\frac{P_t^*}{P_t}\right)^{-\theta} Y_t H_t,$$

where

$$H_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+\tau}^{1-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} H_{t+1} \right].$$

Similarly, the right-hand side (RHS) of the equation can be written as

$$RHS = \frac{\mu}{A_t} Y_t \left(\frac{P_t^*}{P_{I,t}}\right)^{-\theta} \frac{W_t}{P_t} P_t G_t,$$

where

$$G_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+\tau}^{-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} \left(\frac{W_{t+1}}{W_t}\right) \left(\frac{A_t}{A_{t+1}}\right) G_{t+1} \right].$$

The optimal price is hence given by

$$\left(\frac{P_t^*}{P_t}\right) H_t = \frac{\mu}{A_t} \frac{W_t}{P_t} G_t \, .$$

Here, $P_t^*(j) = P_t^*$ because all firms changing prices face the same demand curve and hence the same optimization problem. Based on the definition of markup, the optimal time-varying product markup is given by

$$\mu_t = \mu \frac{G_t}{H_t} \quad \text{and} \quad P_t^* = \mu_t \frac{W_t}{A_t} \, .$$

Price inflation is given by

$$1 = (1-\alpha) \left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha \, \Lambda_{p,t-1,t}^{1-\theta} \left(\frac{P_{t-1}}{P_t}\right)^{(1-\theta)} .$$

C Labor Market Clearing Conditions (based on Li and Palomino (2012))

The total supply of type k labor is given by

$$N_t^s(k) = \int_0^1 N_t^s(j,k)\,dj = \int_0^1 N_t^d(j,k)\,dj = \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} \int_0^1 N_t^d(j)\,dj\,.$$

From the production function $Y_t(j) = A_t N_t^d(j)$, we obtain

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} \int_0^1 \frac{Y_t(j)}{A_t}\,dj = \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} dj\,.$$

where the second equality follows from the product demand function $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$. Defining the price dispersion aggregator and the wage dispersion aggregator by

$$F_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} dj\,, \quad \text{and} \quad F_{w,t} \equiv \int_0^1 \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} dk\,,$$

respectively, it follows that aggregate labor supply is $N_t^s = \frac{Y_t F_t F_{w,t}}{A_t}$. From the resource constraint $N_t^d = \int_0^1 N_t^d(j)dj$, it can be shown that $N_t^d = N_t^s / F_{w,t} = \frac{Y_t F_t}{A_t}$. Note that the wage dispersion $F_{w,t}$ is bounded below by one.

$$F_{w,t} \quad = \quad \int_0^1 \left[\left(\frac{W_t(k)}{W_t}\right)^{1-\theta_w}\right]^{\frac{-\theta_w}{1-\theta_w}} dk \geq \left[\int_0^1 \left(\frac{W_t(k)}{W_t}\right)^{1-\theta_w} dk\right]^{\frac{-\theta_w}{1-\theta_w}} = 1^{-\theta_w} = 1\,,$$

where the second equality is due to Jensen's inequality for $\frac{-\theta_w}{1-\theta_w} > 1$. Similarly, we can show that F_t is bounded below by one.

D Equilibrium Conditions

This appendix provides a summary of the equilibrium equations for the model. These conditions need to be expressed in terms of de-trended variables. In order to obtain balanced growth, we make $\kappa_t = (A_t^p)^{1-\psi}$. This condition ensures that Y_t, W_t, and W_t^* are growing at the same rate. Therefore, the equations can be written in terms of $\hat{Y}_t = \frac{Y_t}{A_t^p}$, $\hat{W}_t = \frac{W_t}{A_t^p}$, and $\hat{W}_t^* = \frac{W_t^*}{A_t^p}$.

Wage Setting

$$
\begin{aligned}
\frac{W_t^*}{P_t} &= \mu_w \kappa_t \left(N_t^s\right)^{\omega} C_t^{\psi} \frac{G_{w,t}}{H_{w,t}} . \\
H_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} H_{w,t+1} \right], \\
\text{and} \quad G_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{\psi} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \left(\frac{N_{t+1}^s}{N_t^s} \right)^{\omega} \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} G_{w,t+1} \right].
\end{aligned}
$$

Price Dispersion

$$
F_t = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta} dj = (1-\alpha) \left(\frac{P_t^*}{P_t} \right)^{-\theta} + \alpha \Lambda_{p,t-1,t}^{-\theta} \left(\frac{P_{I,t-1}}{P_{I,t}} \right)^{-\theta} F_{I,t-1}.
$$

Wage Dispersion

$$
F_{w,t} = \int_0^1 \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} dk = (1-\alpha_w) \left(\frac{W_t^*}{W_t} \right)^{-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{-\theta_w} \left(\frac{W_{t-1}}{W_t} \right)^{-\theta_w} F_{w,t-1} .
$$

Wage Aggregator

$$
\left(\frac{W_t}{P_t} \right)^{1-\theta_w} = \int_0^1 \left(\frac{W_t(k)}{P_t} \right)^{1-\theta_w} dk = (1-\alpha_w) \left(\frac{W_t^*}{P_t} \right)^{1-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{1-\theta_w} \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta_w} \left(\frac{W_{t-1}}{P_{t-1}} \right)^{1-\theta_w},
$$

Price Setting

$$
\begin{aligned}
\left(\frac{P_t^*}{P_t} \right) H_t &= \frac{\mu}{A_t} \frac{W_t}{P_t} G_t , \\
H_t &= 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{1-\theta} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_{t+1}}{P_t} \right)^{-\theta} H_{t+1} \right], \\
\text{and} \quad G_t &= 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{-\theta} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta} \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{A_t}{A_{t+1}} \right) G_{t+1} \right].
\end{aligned}
$$

Price Aggregator

$$
1 = (1-\alpha) \left(\frac{P_t^*}{P_t} \right)^{1-\theta} + \alpha \Lambda_{p,t-1,t}^{1-\theta} \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta} .
$$

Aggregate Labor Supply and Demand

$$N_t^s = F_{w,t} N_t^d, \qquad N_t^d = \frac{Y_t}{A_t} F_t.$$

Markup

$$\mu_t = \frac{Y_t}{LI_t} = \frac{A_t}{F_t} \left(\frac{W_t}{P_t} \right)^{-1}.$$

Pricing Kernel

$$\begin{aligned}
M_{t,t+1} &= \left[\beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left(\frac{1}{R_{YL,t+1}} \right)^{1-\frac{1-\gamma}{1-\psi}}, \\
R_{YL,t+1} &= (1-\nu_t) R_{C,t+1} + \nu_t R_{LI^*,t+1}, \\
R_{Y,t+1} &= \frac{C_{t+1} + S_{Y,t+1}}{S_{Y,t}}, \qquad R_{LI^*,t+1} = \frac{LI^*_{t+1} + S_{LI^*,t+1}}{S_{LI^*,t}}, \\
\nu_t &= \frac{\bar{\nu} S_{LI^*,t}}{\bar{\nu} S_{LI^*,t} - S_{Y,t}}.
\end{aligned}$$

Real and Nominal Bond Yields

$$\exp\left(-n r_t^{(n)}\right) = \mathbb{E}_t \left[M_{t,t+1} \exp\left(-(n-1) r_{t+1}^{(n-1)}\right) \right], \quad \exp\left(-n i_t^{(n)}\right) = \mathbb{E}_t \left[M_{t,t+1}^{\$} \exp\left(-(n-1) i_{t+1}^{(n-1)}\right) \right].$$

Indexation

$$\begin{aligned}
\log \Lambda_{p,t,t+1} &= \pi_t^\star, \\
\log \Lambda_{w,t,t+1} &= g_a + \pi_t^\star.
\end{aligned}$$

Policy Rule

$$i_t = \rho i_{t-1} + (1-\rho) \left[\bar{\imath} + \imath_\pi (\pi_t - \pi_{t-1}^\star) + \imath_x x_t \right] + u_t.$$

E Real Term and Inflation Risk Premia

Real term premium

Consider the no arbitrage equation for the n-period real bond:

$$B_t^{(n)} = e^{-nr_t^{(n)}} = \mathbb{E}_t\left[M_{t,t+1}B_{t+1}^{(n-1)}\right] = \mathbb{E}_t\left[e^{m_{t,t+1}-(n-1)r_{t+1}^{(n-1)}}\right],$$

where $m_{t,t+1} \equiv \log M_{t,t+1}$. Assuming normality for the log-pricing kernel and bond yields, it follows that

$$e^{-nr_t^{(n)}} = \mathbb{E}_t\left[e^{m_{t,t+1}}\right]\mathbb{E}_t\left[e^{-(n-1)r_{t+1}^{(n-1)}}\right]e^{-\text{cov}_t(m_{t,t+1},(n-1)r_{t+1}^{(n-1)})}.$$

From the definition of real term premium, equation (26) follows.

Inflation risk premium

Similarly, the n-period nominal bond yield satisfies the equation

$$B_t^{\$,(n)} = e^{-ni_t^{(n)}} = \mathbb{E}_t\left[e^{m_{t,t+n}-\pi_{t,t+n}}\right] = \mathbb{E}_t\left[e^{m_{t,t+n}}\right]\mathbb{E}_t\left[e^{-\pi_{t+n}}\right]e^{-\text{cov}_t(m_{t,t+n},\pi_{t+n})}.$$

From the definition of inflation risk premium, equation (28) follows.

Recursive preferences and risk premia

Li and Palomino (2012) show that the real pricing kernel in equation (8) can be written as

$$\log M_{t,t+1} = \varphi(\log\beta - \psi\Delta c_t) - (1-\varphi)\log R_{YL,t+1},$$

where $\varphi = \frac{1-\gamma}{1-\psi}$. Assuming a log-normal pricing kernel, the real risk-free rate is

$$r_t = -\frac{1}{2}\text{var}_t(\log M_{t,t+1}) + \varphi\psi\mathbb{E}_t[\Delta c_{t+1}] + (1-\varphi)\mathbb{E}_t[\log R_{YL,t+1}].$$

From equation (26), the two-period real term premium is

$$\begin{aligned}\text{cov}_t(\log M_{t,t+1}, r_{t+1}) &= -(\theta\psi)^2\text{cov}_t(\Delta c_{t+1}, \mathbb{E}_{t+1}[\Delta c_{t+2}]) - \theta\psi(1-\theta)\text{cov}_t(\Delta c_{t+1}, \mathbb{E}_{t+1}[\log R_{YL,t+2}]) \\ &- \theta\psi(1-\theta)\text{cov}_t(\mathbb{E}_{t+1}[\Delta c_{t+2}], \log R_{YL,t+1}) - (1-\theta)^2\text{cov}_t(\log R_{YL,t+1}, \mathbb{E}_{t+1}[\log R_{YL,t+2}]).\end{aligned}$$

Four terms determine the premium. The first term is the risk premium under expected utility (analyzed in the main text). The remaining terms are the result of recursive preferences ($\varphi \neq 1$). Our calibration implies ($\varphi > 1$). Therefore, a positive or negative contribution of these terms to the risk premium depend on the joint process of consumption growth and the portfolio return $R_{YL,t+1}$.

Similarly, from equation (28), the one-period inflation risk premium is

$$\text{cov}_t(\log M_{t,t+1}, \pi_{t+1}) = -\theta\psi\text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) - (1-\theta)\text{cov}_t(\log R_{YL,t+1}, \pi_{t+1}).$$

The first term is the premium under expected utility. For $\varphi > 1$, the second term has a positive contribution to the premium if the return portfolio $R_{YL,t+1}$ is positively correlated with inflation.

F Tables and Figures

Table 1: **Descriptive Quarterly Statistics of U.S. Government TIPS and Nominal Bond Yields and Excess Returns**

"BE" refers to the difference between nominal bond and TIPS yields (breakeven rate). Statistics are quarterly (non-annualized). Yields are annual rates. Excess returns on TIPS are computed as $\log P_{tip,t+1} - \log P_{tip,t} + \pi_{t+1} - i_t$. Excess returns on nominal bonds are computed as $\log P_{nom,t+1} - \log P_{nom,t} - i_t$. Correlations are computed between a nominal bond and a TIPS with same maturity. Information for 2-year TIPS is only available since 2004.

	1999:Q1 - 2008:Q3			2004:Q1 - 2008:Q3		
	TIPS	Nominal	BE	TIPS	Nominal	BE
Panel A: Bond Yields						
Average						
3 months	0.43	3.08	2.65	0.23	2.98	2.75
2 years	N.A.	3.70	N.A.	1.27	3.57	2.30
5 years	2.27	4.24	1.97	1.62	3.93	2.31
10 years	2.64	4.92	2.28	2.02	4.52	2.50
20 years	2.79	5.38	2.59	2.19	4.92	2.73
Standard Deviations						
3 months	1.47	1.72		1.34	1.48	
2 years	N.A.	1.54	N.A.	1.10	1.15	0.68
5 years	1.14	1.10	0.49	0.68	0.72	0.39
10 years	0.88	0.76	0.31	0.39	0.36	0.21
20 years	0.75	0.60	0.31	0.25	0.31	0.28
Correlations						
3 months	0.97			0.97		
2 years	N.A.			0.82		
5 years	0.90			0.85		
10 years	0.94			0.84		
20 years	0.92			0.50		
Panel B: Bond Excess Returns						
Average						
2 years	N.A.	0.94	N.A.	-0.04	0.78	0.82
5 years	0.68	1.22	0.54	-0.07	0.86	0.93
10 years	0.92	1.46	0.54	-0.05	0.98	1.03
20 years	1.17	1.96	0.79	-0.25	1.76	2.01
Sharpe Ratios						
2 years	N.A.	0.85	N.A.	-0.03	0.65	0.53
5 years	0.29	0.45	0.26	-0.03	0.31	0.40
10 years	0.27	0.34	0.20	-0.01	0.25	0.37
20 years	0.25	0.30	0.18	-0.05	0.28	0.45
Correlations						
2 years	N.A.			0.36		
5 years	0.67			0.62		
10 years	0.77			0.73		
20 years	0.73			0.70		

Table 2: **Principal Component Analysis for U.S. Government TIPS and Nominal Bond Yield Changes**

"All" refers to results from principal component analysis when both nominal bond and TIPS yields are included. "TIPS" refers to results from principal component analysis when only TIPS yields are included. "Nominal" refers to results from principal component analysis when only nominal bond yields are included. "BE" refers to results from principal component analysis when only breakeven rates are included. "Total" in the "Principal Component Explanatory Power" section is the variability explained by the first four principal components (pc's). The entries in the "Maximum Explanatory Power" correspond to the R^2's of the instrument specified in the row on all the principal components specified in the column. Two values separated by "/" in a single entry correspond to R^2's for regressions with and without 2,3, and 4 year TIPS yields and breakeven rates in the calculation of principal components.

	1999:Q1 - 2008:Q3				2004:Q1 - 2008:Q3			
	All	TIPS	Nominal	BE	All	TIPS	Nominal	BE
Panel A: Principal Component Explanatory Power								
1st pc	74.82	94.91	77.99	89.55	67.43	90.24	73.28	86.92
2nd pc	11.35	4.64	14.37	9.28	14.67	8.48	17.08	10.85
3rd pc	7.35	0.37	4.99	1.01	11.29	1.06	6.27	1.57
4th pc	3.73	0.07	1.25	0.15	2.98	0.18	1.83	0.53
Total	97.25	99.99	98.60	99.99	96.36	99.96	98.46	99.87
Panel B: Maximum Explanatory Power (R^2)								
TIPS	100	100	78	31	100	100	91	74/59
Nominal	100	67	100	76	100	88/83	100	70/55
BE	100	19	70	100	100	77/70	87	100

Table 3: **Descriptive Quarterly Statistics of U.S. Government TIPS, Nominal Bond Yields, Consumption Growth, and Inflation**

"AR" refers to first-order autocorrelation. "BE" refers to the difference between nominal bond and TIPS yields (breakeven rate). Statistics are quarterly (non-annualized). Yields are annualized. The regressions of yield changes are on the the contemporaneous value and 3 quarterly lags of the macroeconomic variable (consumption growth or inflation). Information for 2-year TIPS is only available since 2004.

	1999:Q1 - 2008:Q3			2004:Q1 - 2008:Q3		
	Mean	Std. Dev.	AR	Mean	Std. Dev.	AR
Macroeconomic Variables						
Δc	0.36	0.37	0.61	0.24	0.38	0.71
π	0.78	0.32	0.18	0.93	0.31	-0.01
corr$(\Delta c, \pi)$	-0.32			-0.50		

	TIPS	Nominal	BE	TIPS	Nominal	BE
Correlations of Yields and Consumption Growth						
3 months	0.46	0.46		0.34	0.33	
2 years	N.A.	0.51	N.A.	-0.09	0.36	0.75
5 years	0.42	0.54	0.22	-0.04	0.36	0.75
10 years	0.42	0.51	0.05	-0.14	0.20	0.59
20 years	0.39	0.51	0.05	-0.46	0.31	0.61
Correlations of Yields and Inflation						
3 months	-0.24	-0.04		-0.50	-0.33	
2 years	N.A.	-0.08	N.A.	-0.30	-0.36	-0.12
5 years	-0.34	-0.18	0.39	-0.35	-0.38	-0.10
10 years	-0.37	-0.27	0.40	-0.33	-0.34	0.01
20 years	-0.36	-0.37	0.14	-0.09	-0.25	-0.19
Regressions of Yield Changes on Consumption Growth (R^2)						
3 months	0.28	0.44		0.42	0.46	
2 years	N.A.	0.22	N.A.	0.30	0.29	0.50
5 years	0.01	0.14	0.32	0.25	0.21	0.49
10 years	0.01	0.11	0.31	0.24	0.23	0.44
20 years	0.03	0.13	0.22	0.22	0.18	0.43
Regressions of Yield Changes on Inflation (R^2)						
3 months	0.36	0.04		0.53	0.11	
2 years	N.A.	0.03	N.A.	N.A.	0.02	0.30
5 years	0.18	0.06	0.19	0.21	0.01	0.24
10 years	0.15	0.07	0.18	0.20	0.02	0.19
20 years	0.19	0.05	0.20	0.23	0.01	0.19

Table 4: **Baseline Calibration Parameter Values**

Parameter	Description	Value
Panel A: Preferences		
β	Subjective discount factor	0.98675
ψ	Inverse of elasticity of intertemporal substitution	5
γ	Risk aversion parameter	1000
ω	Inverse of Frisch labor elasticity	0.35
Panel B: Rigidities and Monopolistic Competition		
α	Price rigidity parameter	0.66
θ	Elasticity of substitution of differentiated goods	6
$\tilde{\alpha}$	Wage rigidity parameter	0.78
θ_w	Elasticity of substitution of labor types	4.47
Panel C: Interest Rate Rule		
ρ	Interest-rate smoothing coefficient in policy rule	0.725
$\bar{\imath}$	Constant in the policy rule	0.1136
$\imath_\pi$	Response to inflation in the policy rule	1.5
$\imath_x$	Response to output gap in the policy rule	0.125
Panel D: Policy and Productivity Shocks		
ϕ_u	Autocorrelation of policy shock	0.145
$\sigma_u \times 10^2$	Conditional vol. of policy shock	0.1955
ϕ_a	Autocorrelation of permanent productivity shock	0.231
$\sigma_a \times 10^2$	Conditional vol. of permanent productivity shock	0.249
ϕ_z	Autocorrelation of transitory productivity shock	0.9719
$\sigma_z \times 10^2$	Conditional vol. of transitory productivity shock	0.133
Panel E: Growth Rates and Inflation Target		
$g_a \times 10^2$	Unconditional Mean of Productivity Growth	0.4695
$g_{\pi^\star} \times 10^2$	Unconditional Mean of Inflation Target	8.47
$\phi_{\pi^\star}$	Autocorrelation of Inflation Target	0.9995
$\sigma_{\pi^\star} \times 10^4$	Conditional vol. of Inflation Target	0.261

Table 5: **Data and Model-Implied Volatility Decomposition**
The table reports quarterly standard deviations (in percentage) explained by policy and productivity shocks. The values for inflation and the nominal rate are annual rates. The data cover the period 1982:Q1 to 2008:Q3 and are described in section 2. The variance decomposition for the data is obtained from Altig, Christiano, Eichenbaum and Linde (2011). The variable $\hat{c}_t$ is the Hodrick-Prescott filtered series of consumption. The model parameter values are reported in table 4.

	Total Volatility	Volatility Explained by Shock			
		Data		Model	
	Data	Policy	Productivity	Policy	Productivity
i_t	2.592	0.968	0.368	0.968	0.368
π_t	1.360	0.304	0.472	0.304	0.352
$\hat{c}_t$	0.760	0.170	0.215	0.170	0.216
Δc_t	0.373	-	-	0.143	0.340

Table 6: **Data and Model-Implied Statistics**
The table reports quarterly data and model-implied statistics of selected variables. The values for inflation and the nominal rate are annual rates. Means and standard deviations are reported in percentage terms. The quarterly data cover the period 1982:Q1 to 2008:Q3 and are described in section 2. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $\mathrm{AR}(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. The variable $\hat{c}_t$ is the Hodrick-Prescott filtered series of consumption. The model parameter values are reported in table 4.

	Data	Model
$\mathbb{E}[\Delta c]$	0.469	0.469
$\mathbb{E}[\pi]$	3.263	3.258
$\sigma(\hat{c})$	0.763	0.399
$\sigma(\Delta c)$	0.373	0.369
$\sigma(\pi)$	1.360	1.364
$\mathrm{corr}(\Delta c, \pi)$	-0.141	-0.109
$\mathrm{AR}(\Delta c)$	0.400	-0.013
$\mathbb{E}[i]$	5.043	5.043
$\mathbb{E}[i^{(20)} - i]$	1.400	1.364
$\sigma(i)$	2.592	1.648
$\sigma\left(i^{(20)}\right)/\sigma(i)$	1.044	0.805

Table 7: **Model-Implied Summary Statistics for Different Rigidity Specifications**
The table reports quarterly model-implied statistics of selected variables. The values for inflation and the nominal rate are annual rates. Means and standard deviations are reported in percentage terms. The baseline parameter values are presented in table 4. "Baseline" indicates an economy with both price and wage rigidities and all four exogenous shocks. "No Rig." indicates no price and wage rigidities ($\alpha = \alpha_w = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities (α_w=0). The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and AR$(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi RP^{(20)}$ are the 5-year real and inflation risk premia, respectively.

	(1) Baseline	(2) No Rig.	(3) Only WR	(4) Only PR
$\mathbb{E}[\Delta c]$	0.47	0.47	0.47	0.47
$\mathbb{E}[\pi]$	3.26	3.26	3.28	3.30
$\sigma(x)$	0.35	0.00	0.32	0.12
$\sigma(\Delta c)$	0.37	0.26	0.40	0.33
$\sigma(\pi)$	1.36	2.79	1.61	1.31
corr$(\Delta c, \pi)$	-0.11	0.38	-0.41	0.26
AR(Δc)	-0.01	0.23	-0.04	-0.01
$\mathbb{E}[r]$	1.19	5.84	-0.61	2.47
$\mathbb{E}[r^{(20)} - r]$	0.64	-4.46	2.59	-0.90
$\mathbb{E}[i]$	5.04	5.05	5.03	5.04
$\mathbb{E}[i^{(20)} - i]$	1.36	-1.98	2.50	-0.66
$\sigma(r)$	1.22	1.18	1.22	1.03
$\sigma(r^{(20)})/\sigma(r)$	0.16	0.08	0.15	0.12
$\sigma(i)$	1.65	1.15	1.58	1.17
$\sigma(i^{(20)})/\sigma(i)$	0.79	0.87	0.80	0.85
$\sigma(r)/\sigma(i)$	0.74	1.03	0.77	0.88
$\mathbb{E}[rTP^{(20)}]$	1.32	-1.57	2.27	-0.48
$\mathbb{E}[\pi RP^{(20)}]$	0.96	-4.74	3.07	-1.01
corr$(ret^{(4)}, ret^{\$,(4)})$	0.95	0.67	0.85	0.71
corr$(ret^{(20)}, ret^{\$,(20)})$	0.86	0.69	0.82	0.63

Table 8: **Model-Implied Summary Statistics for Different Shock Specifications**
The table reports quarterly model-implied statistics of selected variables. The values for inflation and the nominal rate are annual rates. Means and standard deviations are reported in percentage terms. The baseline parameter values are presented in table 4. "Baseline" indicates an economy with both price and wage rigidities and all four exogenous shocks. "Only A^p" indicates only permanent productivity shocks ($\sigma_z = \sigma_u = \sigma_{\pi^\star} = 0$). "Only Z" indicates only transitory productivity shocks ($\sigma_a = \sigma_u = \sigma_{\pi^\star} = 0$). "Only u" indicates only policy shocks ($\sigma_a = \sigma_z = \sigma_{\pi^\star} = 0$). "Only $\pi^\star$" indicates only shocks to the inflation target ($\sigma_a = \sigma_z = \sigma_u = 0$). The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $\mathrm{AR}(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi RP^{(20)}$ are the 5-year real and inflation risk premia, respectively.

	(1) Baseline	(2) Only A^p	(3) Only Z	(4) Only u	(5) Only $\pi^\star$
$\mathbb{E}[\Delta c]$	0.47	0.47	0.47	0.47	0.47
$\mathbb{E}[\pi]$	3.26	3.27	3.19	3.21	3.20
$\sigma(x)$	0.35	0.10	0.02	0.17	0.09
$\sigma(\Delta c)$	0.37	0.34	0.04	0.14	1.8×10^{-3}
$\sigma(\pi)$	1.36	0.22	0.22	0.31	1.08
$\mathrm{corr}(\Delta c, \pi)$	-0.11	-0.80	-0.59	0.24	-0.03
$\mathrm{AR}(\Delta c)$	-0.01	0.01	0.06	-0.14	0.30
$\mathbb{E}[r]$	1.19	1.21	1.88	1.87	1.88
$\mathbb{E}[r^{(20)} - r]$	0.64	0.61	-1×10^{-4}	7.1×10^{-3}	-4×10^{-4}
$\mathbb{E}[i]$	5.04	5.04	5.07	5.08	5.08
$\mathbb{E}[i^{(20)} - i]$	1.36	1.21	2×10^{-4}	7.8×10^{-3}	-3×10^{-4}
$\sigma(r)$	1.22	0.13	0.12	1.21	0.03
$\sigma(r^{(20)})/\sigma(r)$	0.16	0.24	0.59	0.14	0.13
$\sigma(i)$	1.65	0.17	0.25	0.96	1.08
$\sigma(i^{(20)})/\sigma(i)$	0.79	0.22	0.66	0.13	1.00
$\sigma(r)/\sigma(i)$	0.74	0.75	0.48	1.25	0.03
$\mathbb{E}[rTP^{(20)}]$	0.96	0.88	7×10^{-4}	0.02	0.00
$\mathbb{E}[\pi RP^{(20)}]$	1.32	1.17	6×10^{-4}	-3×10^{-4}	1×10^{-4}
$\mathrm{corr}(ret^{(4)}, ret^{\$,(4)})$	0.95	0.69	0.66	0.99	-0.42
$\mathrm{corr}(ret^{(20)}, ret^{\$,(20)})$	0.86	0.96	0.98	0.99	-0.84

Table 9: **Model-Implied Summary Statistics for Different Monetary Policy Specifications**

The table reports quarterly model-implied statistics of selected variables. The values for inflation and the nominal rate are annual rates. Means and standard deviations are reported in percentage terms. The baseline parameter values are presented in table 4. "Baseline" indicates an economy with both price and wage rigidities and all four exogenous shocks. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $\mathrm{AR}(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi RP^{(20)}$ are the 5-year real and inflation risk premia, respectively. The baseline interest rule parameters are $\imath_\pi = 1.5$, $\imath_x = 0.125$, $\rho = 0.725$, and $\phi_{\pi^\star} = 0.9995$.

	(1) Baseline	(2) $\imath_\pi = 1.7$	(3) $\imath_x = 0.25$	(4) $\rho = 0.8$	(5) $\phi_{\pi^\star} = 0.9$
$\mathbb{E}[\Delta c]$	0.47	0.47	0.47	0.47	0.47
$\mathbb{E}[\pi]$	3.26	3.29	3.22	3.29	3.23
$\sigma(x)$	0.35	0.22	0.28	0.23	0.20
$\sigma(\Delta c)$	0.37	0.37	0.36	0.38	0.37
$\sigma(\pi)$	1.36	0.97	1.47	0.85	0.47
$\mathrm{corr}(\Delta c, \pi)$	-0.11	-0.16	-0.10	-0.15	-0.32
$\mathrm{AR}(\Delta c)$	-0.01	-0.01	-0.01	-0.02	-0.01
$\mathbb{E}[r]$	1.19	1.13	1.24	1.17	1.19
$\mathbb{E}[r^{(20)} - r]$	0.64	0.88	0.45	0.92	0.65
$\mathbb{E}[i]$	5.04	5.03	5.04	5.05	5.02
$\mathbb{E}[i^{(20)} - i]$	1.36	1.52	1.22	1.39	1.35
$\sigma(r)$	1.22	1.19	1.18	1.31	1.22
$\sigma(r^{(20)})/\sigma(r)$	0.16	0.15	0.15	0.16	0.16
$\sigma(i)$	1.65	1.32	1.72	1.21	1.04
$\sigma(i^{(20)})/\sigma(i)$	0.79	0.68	0.83	0.59	0.27
$\sigma(r)/\sigma(i)$	0.74	0.90	0.68	1.09	1.18
$\mathbb{E}[rTP^{(20)}]$	1.32	1.25	0.73	1.32	1.30
$\mathbb{E}[\pi RP^{(20)}]$	0.96	1.24	1.34	1.07	0.97
$\mathrm{corr}(ret^{(4)}, ret^{\$,(4)})$	0.95	0.95	0.95	0.94	0.95
$\mathrm{corr}(ret^{(20)}, ret^{\$,(20)})$	0.86	0.91	0.85	0.89	0.90

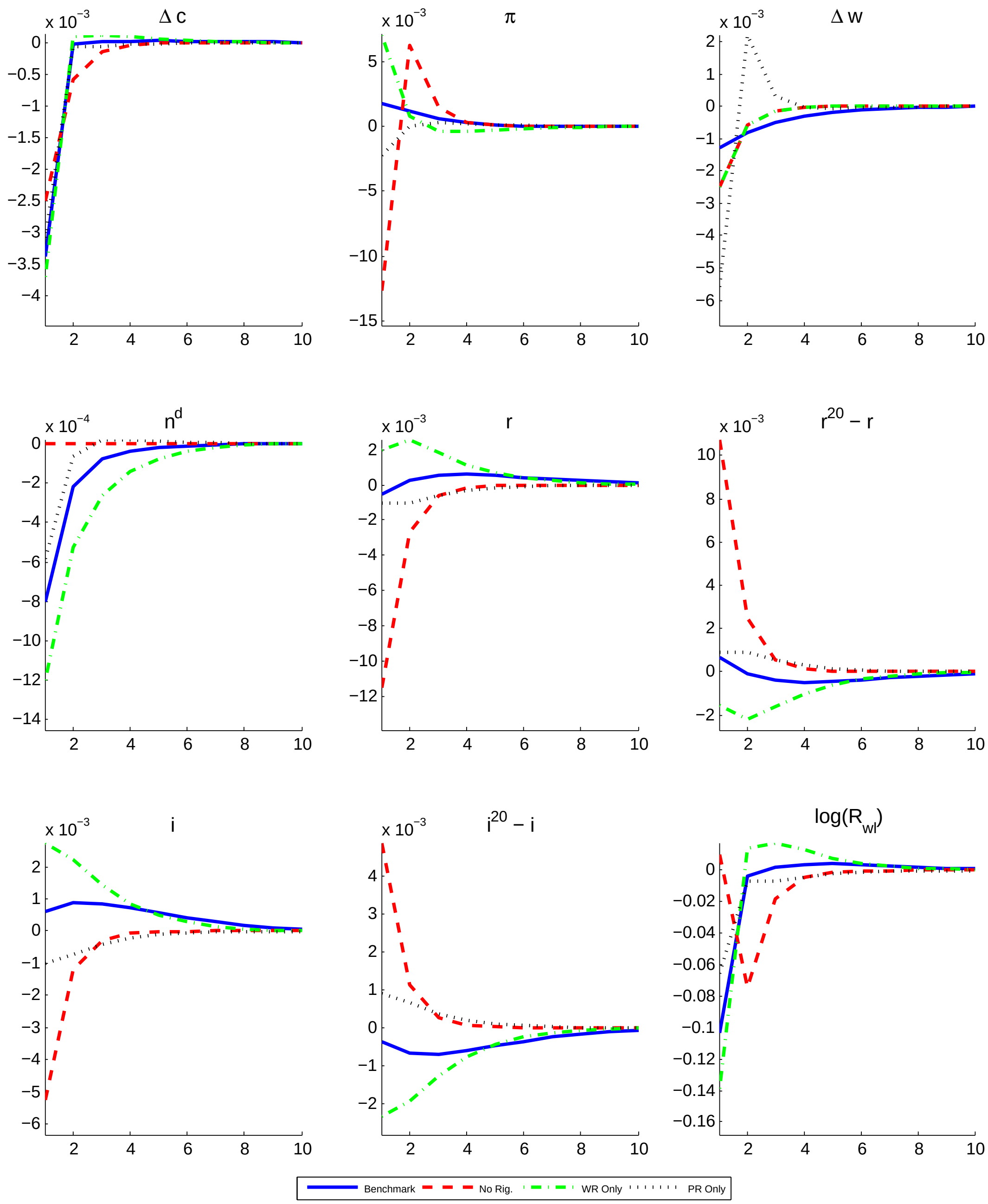

Figure 1: Impulse responses to a one-standard deviation negative permanent productivity shock for different variables. The baseline parameter values are presented in table 4. The responses for inflation, the one-quarter nominal and real rates, and the five-year nominal and real spreads are reported in annual rates.

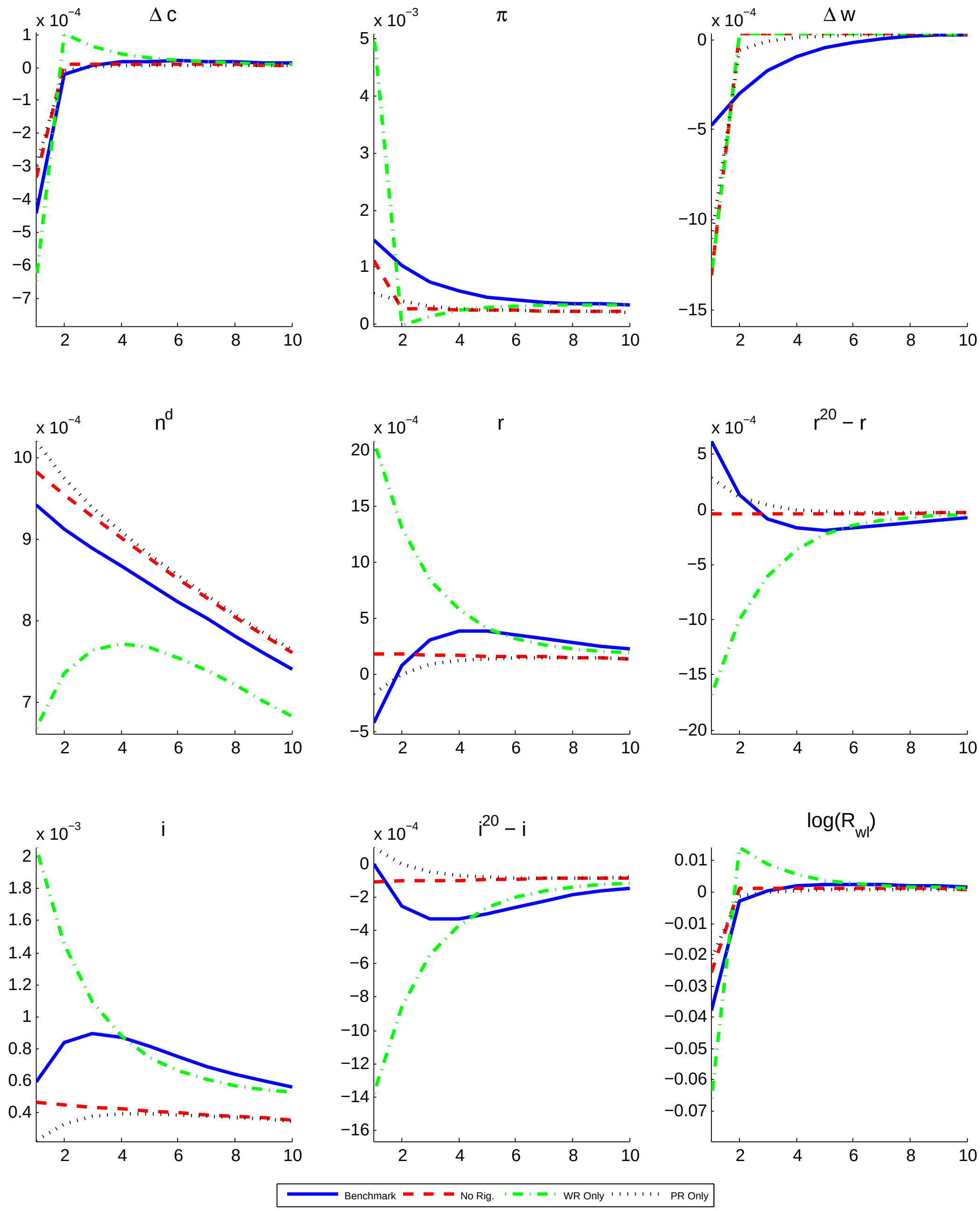

Figure 2: Impulse responses to a one-standard deviation negative transitory productivity shock for different variables. The baseline parameter values are presented in table 4. The responses for inflation, the one-quarter nominal and real rates, and the five-year nominal and real spreads are reported in annual rates.

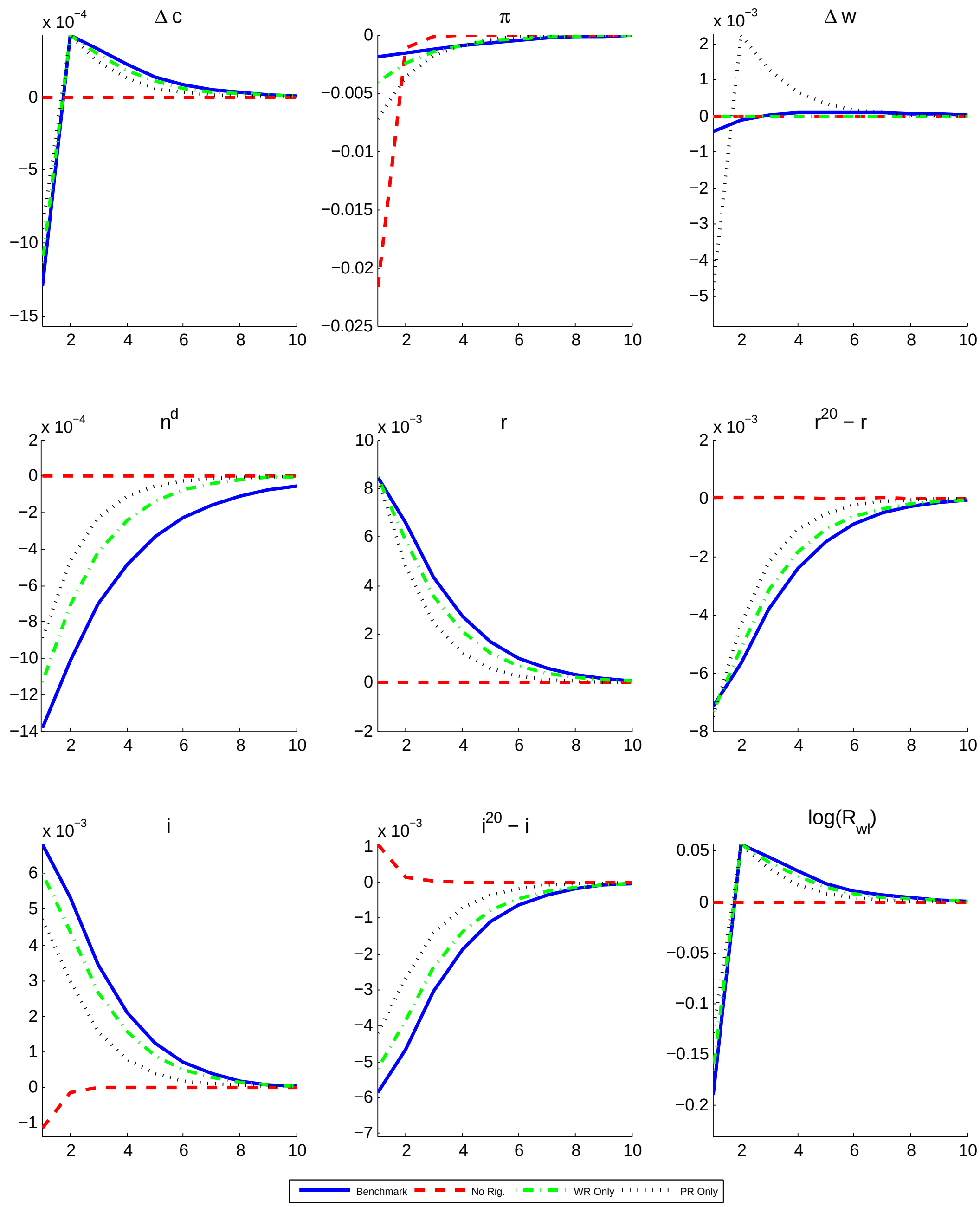

Figure 3: Impulse responses to a one-standard deviation positive policy shock for different variables. The baseline parameter values are presented in table 4. The responses for inflation, the one-quarter nominal and real rates, and the five-year nominal and real spreads are reported in annual rates.

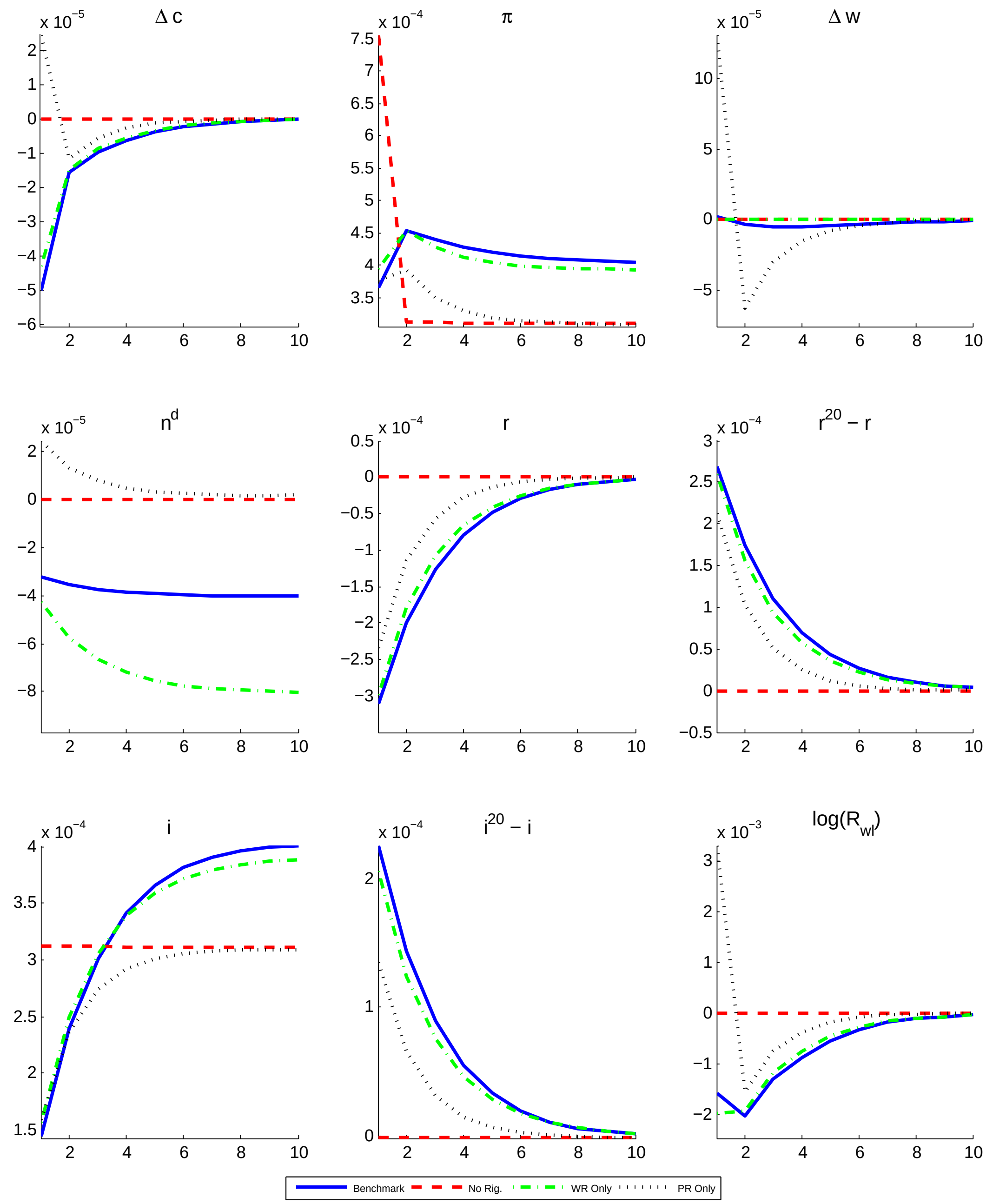

Figure 4: Impulse responses to a one-standard deviation positive inflation target shock for different variables. The baseline parameter values are presented in table 4. The responses for inflation, the one-quarter nominal and real rates, and the five-year nominal and real spreads are reported in annual rates.